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Hamiltonian Laceability in Total Graphs

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Abstract

A simple connected graph G is Hamiltonian Laceable, if there exists a Hamiltonian path between every pair of distinct vertices at an odd distance in it. G is a Hamiltonian-t-laceable (t*-laceable) if there exists a Hamiltonian path in G between every pair (at least one pair) of vertices u and v in G with the property d(u,v)=t, 1≤t≤diam G. In this paper we explore Hamiltonian laceability properties of the Total graph of the Sunlet graph, Star graph, Path graph and Cycle.

Keywords

Connected graph, Hamiltonian-t*- connected graph, Total graph, Sunlet graph, $K_{1,n}$ graph, Hamiltonian-t*-laceable graph, Hamiltonian-t-laceability number $\lambda_{(t)}$.

2000 Mathematics subject Classification: 05C45, 05C99

1. Introduction

All Graphs considered in this paper are finite, undirected, connected and simple. The vertex set and edge set of the graph G are denoted by V(G) and E(G) respectively. The Cardinalities of V(G) and E(G) are called respectively the order and size of G.

Let u and v be two vertices in G. The distance between u and v, denoted by d(u,v) is the length of a shortest u-v path in G. G is Hamiltonian laceable if there exists a Hamiltonian path between every pair of distinct vertices in it at an odd distance. G is a Hamiltonian-t-laceable (t*-laceable) if there exists a Hamiltonian path between every pair (at least one pair) of vertices u and v in G with the property d(u,v)=t, where t is positive integer such that 1≤ t ≤ diamG.

Let a_i and a_j be any two distinct vertices in a connected graph G. Let E' be the minimal set of edges not in G and P be a path in G, such that $P \cup E'$ is a Hamiltonian path in G from a_i to a_j. Then, $|E'|$ is called the t-laceability number $\lambda_{(t)}$ of G. Further the edges in E' are called the t-laceability edges.

In [2], [3] and [4] the authors have studied the Hamiltonian-t-laceability and Hamiltonian-t*-laceability properties and $\lambda_{(t)}$ for different graph structures. In this paper we explore the Hamiltonian-t-laceability properties of the Total graph of the Sunlet graph,Star graph, Path graph and Cycle.

Definition 1.1

The n- Sun let graph on 2n vertices is obtained by attaching n-pendent edges to the cycle C_n and is denoted by S_n.

Definition 1.2

Let G be a graph with vertex set V(G) and edge set E(G). The Total graph of G, denoted by T (G) and is defined as follows. The vertex set of T (G) is $V(G) \cup E(G)$. Two vertices *x, y* in the vertex set of T(G) are adjacent in T(G) in case one of the following holds.

(i) *x, y* are in V(G) and *x* is adjacent to y in G.
(ii) *x, y* are in E(G) and *x, y* are adjacent in G.
(iii) *x* is in V(G), y is in E(G), and *x, y* are incident in G.

2. Main Results

Theorem 2.1

Let G= S_n, n $\geq$3. Then

(i) T(G) is Hamiltonian-t*-laceable for t=1,2.
(ii) If n is odd T(G) is Hamiltonian-3*-laceable.
(iii) If n is even T(G) is Hamiltonian-3*-laceable with $\lambda_{(t)} = 1$.

Proof:

Let S_n be the sunlet graph on 2n vertices. Let $V(S_n)$= $\{ a_1, a_2, a_3, \text{---------}, a_n\} \cup \{ ,b_1, b_2, b_3, \text{----------}, b_n\}$ Where a_i's are the vertices of cycles taken in cyclic order and b_i's are pendant vertices such that each $a_i b_i$ is a pendent edge.

Let $V(S_n)$= $\{ a_1, a_2, a_3 \ldots\ldots\ldots, a_n\} \cup \{ ,b_1, b_2, b_3, \ldots\ldots\ldots, b_n\}$ and $E(S_n) = \{e_i' : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n-1\} \cup \{e_n\}$ where e_i is the edge $a_i a_{i+1} (1 \leq i \leq n-1)$, e_n is the edge $a_n a_1$ and e_i' is the edge $a_i b_i (1 \leq i \leq n)$ by the definition of the total graph $V(T(S_n)) = V(S_n) \cup E(S_n) = \{a_i : 1 \leq i \leq n\} \cup \{a_i' : 1 \leq i \leq n\} \cup \{b_i : 1 \leq i \leq n\} \cup \{b_i' : 1 \leq i \leq n\}$ where, a_i' and b_i' represents the edge e_i and e_i' $\{1 \leq i \leq n\}$ respectively.

Case (i): For t=1

In G, $d(b_1, a_1) = 1$ and the path $P: (b_1, b_1') \cup (b_1', a_n') \cup (a_n', a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, b_{n-1}') \cup (b_{n-1}', a_{n-2}) \cup \ldots\ldots\ldots\ldots$ $(a_3', a_3) \cup (a_3, b_3) \cup (b_3, b_3') \cup (b_3', a_2') \cup (a_2', a_2) \cup (a_2, b_2) \cup (b_2, b_2') \cup (b_2', a_1') \cup (a_1', b_1') \cup (b_1', b_1) \cup$ $(b_1, a_1) \cup (a_1, a_1') \cup (a_1', a_1)$. is a Hamiltonian path from b_1 to a_1 in G. Hence *G* is Hamiltonian-1*-laceable.

Case (ii): For t=2

In G, $d(b_1, a_2) = 2$ and the path

$$P: (b_1, b_1') \cup (b_1', a_1) \cup (a_1, a_{n-2}') \cup (a_{n-2}', a_{n-2}) \cup (a_{n-2}, b_{n-2}) \cup (b_{n-2}', a_{n-3}) \cup (b_{n-3}, b_{n-3}') \cup \ldots\ldots \cup (a_4', a_4) \cup (a_4, b_4) \cup (b_3, b_3') \cup (b_3', a_2') \cup (a_3', a_3) \cup (b_2', b_2) \cup (b_2, b_2') \cup (b_3', a_2) \cup (a_2, a_1') \cup (a_1', b_2') \cup (b_2', b_2) \cup (b_2, a_2)$$

is a Hamiltonian path from b_1 to a_2 in G. Hence T(G) is Hamiltonian-t*-laceable for t=2.

Case (iii): For t=3

If *n* is odd in G, $d(b_1, a_2') = 3$ and the path

$P:(b_1,b_1')\cup(b_1',a_1')\cup(a_1',b_2')\cup(b_2',b_2)\cup(a_2,b_2)\cup(a_2,v_{a1})\cup(a_1,a_n')\cup(a_n',a_n)\cup(a_{n-2},b_{n-2})\cup$
$(b_{n-2},b_{n-2}')\cup$.......... ....$\cup(a_3',a_3)\cup(a_3,b_3)\cup(b_3,b_3')\cup(b_3,a_2')$

is a Hamiltonian path from b_0 to a'_2 in G. Hence G is Hamiltonian-3*-laceable.

Case (iv):

If n is even and t=3

If n is even in G, $d(b_1,a_3)=3$ and the path

$P:(a_1,b_1')\cup(b_1',a_1)\cup(a_1,a_1')\cup(a_1',a_2)\cup(a_2,b_2)\cup(b_2,b_2')\cup(b_2',a_2')\cup(a_2',a_3')\cup$.......... ...$\cup$
$(a_{n-1},a_{n-1}',)\cup(a_{n-1}',,a_n)\cup(a_n,b_n)\cup(b_n,a_n')\cup(b_n',a_n')\cup(b_3,a_3)$. is a Hamiltonian path from b_1 to a_3. Where (a_n',a_3) is the Laceability edge. Hence G is Hamiltonian-3*-laceable with $\lambda_{(t)}=1$.

Theorem 2.2

Let G=$K_{1,n}$, ($n\geq3$) graph. Then, T(G) is Hamiltonian-t*- laceable for t=1 and 2.

Proof: Let $\{a_0,a_1,a_2,\ldots,a_{n-1}\}$ $V(K_{1,n})=\{a_0,a_1,a_2,\ldots,a_{n-1}\}$ where $aa_i=e_i\{1\leq i\leq n\}$ by the definition of Total graph $T(K_{1,n})$ has the vertex set $V(K_{1,n})\cup\{b_i:1\leq i\leq n-1\}$ where b_i the vertex of subdivision of the edge is e_i. also the vertex subset $\{a_0,a_1,a_2,\ldots,a_{n-1}\}$ of $K_{1,n}$ induces a clique on n vertices.

Case (i): For t=1

In G, $d(a_0,b_0)$=1 and the path $P:(a_0,a_1)\cup(a_1,a_2)\cup(a_2,a_3)\cup(a_3,a_4)\cup(a_4,a_5)\cup$........ $(a_{n-3},a_{n-2})\cup(a_{n-2},a_{n-1})\cup(a_{n-1},a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup$.......... .... $(b_3,b_2)\cup(b_2,b_1)\cup(b_1,b_0)$ is a Hamiltonian path from a_0 to b_0. Hence G is Hamiltonian-1*-laceable.

Case (ii): For t=2

In G, $d(a_0,b_1)$=2 and the path

$P:(a_0,a_1)\cup(a_1,a_2)\cup(a_2,a_3)\cup(a_3,a_4)\cup(a_4,a_5)\cup$.......... ... $(a_{n-3},a_{n-2})\cup(a_{n-2},a_{n-1})\cup(a_{n-1},a_n)\cup$
$(a_n,b_{n-1})\cup(b_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup$.......... .....$\cup(b_3,b_2)\cup(b_2,b_0)\cup(b_0,b_1)$

is a Hamiltonian path from a_0 to b_1. Hence G is Hamiltonian-2*-laceable.

Theorem 2.3

Let $G=P_n$, $n\geq3$. Then

(i) $T(G)$ is a Hamiltonian-t*-laceable for t=1
(ii) $T(G)$ is a Hamiltonian-t*-laceable for t=2 with $\lambda_{(t)}=1$
(iii) $T(G)$ is a Hamiltonian-t*-laceable for t=3 if n even and $n\geq4$
(iv) $T(G)$ is a Hamiltonian-t*-laceable for t=3 with $\lambda_{(t)}=1$, if n is odd and $n\geq5$

Proof: Let $V(P_n)=\{a_0,a_1,a_2,\ldots,a_{n-1}\}$ and let $V(T(P_n))=\{a_i:0\leq i\leq n-1\}\cup\{b_i:0\leq i\leq n-2\}$ Where, b_i is the vertex of $T(P_n)$ corresponding to edge a_ia_{i+1} of P_n

Case (i): For t=1

In G, $d(a_0,b_0)$=1 and the path

$P:(a_0,a_1)\cup(a_1,a_2)\cup(a_2,a_3)\cup(a_3,a_4)\cup(a_4,a_5)\cup.........\cup(a_{n-3},a_{n-2})\cup(a_{n-2},a_{n-1})\cup(a_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup..........\,..(b_5,b_4)\cup(b_4,b_3)\cup(b_3,b_2)\cup(b_2,b_1)\cup(b_1,b_0)$ is a Hamiltonian path from a_0 to b_0. Hence G is Hamiltonian-1*-laceable.

Case (ii): For t=2

In G, $d(a_0,b_1)$=2 and the path

$P:(a_0,b_0)\cup(b_0,a_1)\cup(a_1,a_{n-2})\cup(b_{n-2},a_{n-1})\cup(a_{n-1},a_{n-2})\cup(b_{n-3},a_{n-3})\cup(a_{n-3},a_{n-4})\cup.........\,(b_{n-8},a_{n-8})\cup(a_{n-8},a_{n--9})\cup..........\,..........\,........\,(a_4,b_3)\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,a_2)\cup(a_2,b_1)$ is a Hamiltonian path from a_0 to b_1. Hence G is a Hamiltonian-2*-laceable with $\lambda_{(t)}=1$.

Case(iii): For t=3, if $n\geq 4$

In G, $d(a_0,b_2)$=3 and the path

$P:(a_0,b_0)\cup(b_0,a_1)\cup(a_1,b_1)\cup(b_1,a_2)\cup(a_4,a_5)\cup..........\,..........\,.\cup(a_{n-3},a_{n-2})\cup(a_{n-2},a_{n-1})\cup(a_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup..........\,..........\,.........\,\cup(b_5,b_4)\cup(b_4,b_3)\cup(b_3,b_2)$ is a Hamiltonian path from a_0 to b_2. Hence G is a Hamiltonian-3*-laceable.

Case (iv): For t=3, if $n\geq 5$

In G, $d(a_0,b_2)$=3 and the path

$P:(a_0,b_0)\cup(b_0,a_1)\cup(a_1,b_1)\cup(b_1,a_2)\cup(a_2,b_{n-2})\cup(b_{n-2},a_{n-1})\cup(a_{n-1},a_{n-2})\cup(a_{n-2},b_{n-3})\cup..........\,......\cup(a_4,b_3)\cup(b_3,a_3)\cup(a_3,b_2)$ is a Hamiltonian path from a_0 to b_2. Hence G is a Hamiltonian-3*-laceable with $\lambda_{(t)}=1$.

Theorem 2.4

Let $G=C_n$ $n\geq 4$. Then $T(G)$ is Hamiltonian-t*-laceable for t=1, 2 and 3.

Proof: Let $V(C_n)=\{a_0,a_1,a_2,.........,a_{n-1}\}$ and let $V[T(C_n)]=\{a_0,a_1,a_2,a_3,---------,a_{n-1}\}\cup\{b_0,b_1,b_2,b_3,----------,b_{n-1}\}$ where a_i is the vertex of $T(Cn)$ corresponding to the edge $a_i\,a_{i+1}$ of Cn ($1\leq i\leq n-1$).

Case (i): For t=1

Sub Case (i): If n is even

In G, $d(a_0,b_0)$=1 and the path

$P:(a_0,a_1)\cup(a_1,a_2)\cup(a_2,a_3)\cup.......\cup(b_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup..........\,..........\cup(b_{n-8},b_{n-9})\cup(a_{n-2},b_{n-3})\cup..........\,...\cup(b_3,b_2)\cup(b_2,b_1)\cup(b_1,b_0)$ is Hamiltonian path from a_0 to b_0 Hence G is a Hamiltonian-1*-laceable.

Case (ii): For t=2

In G, $d(a_0,b_1)$=2 and the path

$P:(a_0,b_0)\cup(b_0,a_1)\cup(a_1,a_2)\cup(a_2,a_3)\cup(a_3,a_4)\cup .... \cup(b_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup .... \cup (b_{n-8},b_{n-9})\cup(a_{n-2},b_{n-3})\cup------- (b_3,b_2)\cup(b_2,b_1)$ is a Hamiltonian path from a_0 to b_1. Hence G is a Hamiltonian-2*-laceable.

Case (iii): For t=3

Sub Case (ii): If *n* is even

In *G*, $d(a_0,b_2)$=3 and the path

$P:(a_0,b_0)\cup(b_0,a_1)\cup(a_1,b_1)\cup(b_1,a_2)\cup(a_2,a_3)\cup(a_3,a_4)\cup .... \cup(a_{10},a_{11})\cup(b_{n-9},a_{n-8})\cup .... \cup(b_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup ...... \cup(b_4,b_3)\cup(b_3,b_2)$ is a Hamiltonian path from a_0 to b_2. Hence G is a Hamiltonian-3*-laceable.

Remark 3

Let $G=C_n$, the total graph $T(G)$ is Hamiltonian-t*-laceable for t=3 if n is odd and $n\geq 5$

In *G*, $d(a_0,b_2)$=3 and the path

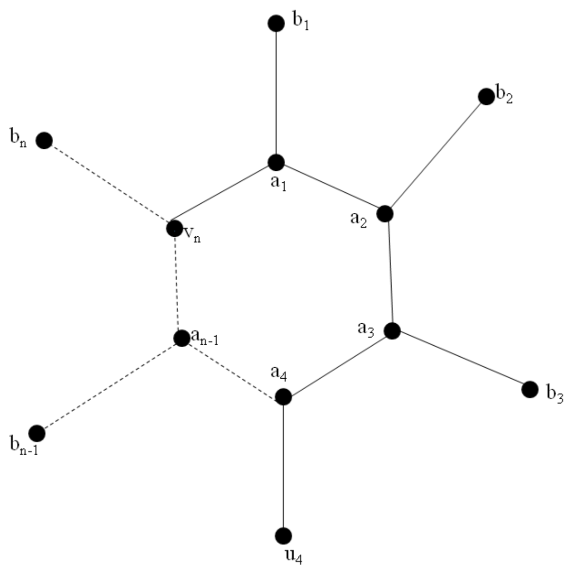
$P:(a_0,b_0)\cup(b_0,a_1)\cup(a_1,b_1)\cup(b_1,a_2)\cup(a_2,a_3)\cup(a_3,a_4)\cup .... \cup(a_{10},a_{11})\cup(b_{n-9},a_{n-8})\cup ....\cup (b_{n-1},b_{n-2})\cup(b_{n-2},b_{n-3})\cup .... \cup(b_4,b_3)\cup(b_3,b_2)$ is a Hamiltonian path from a_0 to b_2. Hence G is a Hamiltonian-3*-laceable. Hence the proof.

4. Conclusion

Here we investigate new results of Laceability in Total Graphs of Sunlet Graphs, Star Graphs, Paths and Cycles. It is possible to investigate similar results for other graph families. There is a scope to obtain similar results corresponding to

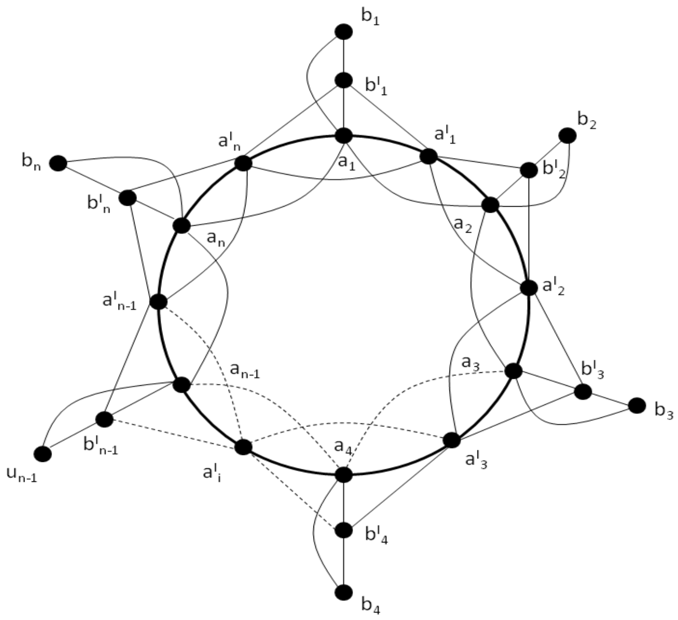
Example 1

Sunlet Graph S_n



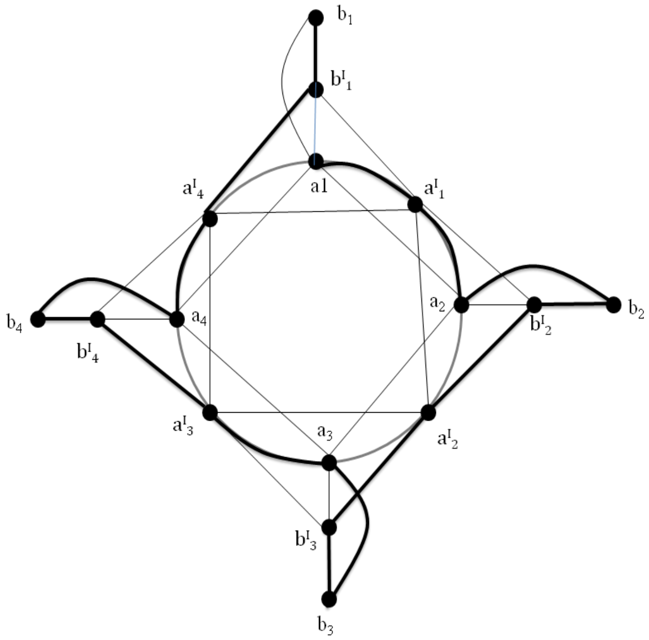
Example 2

Total graph of Sunlet Graph $T(S_n)$



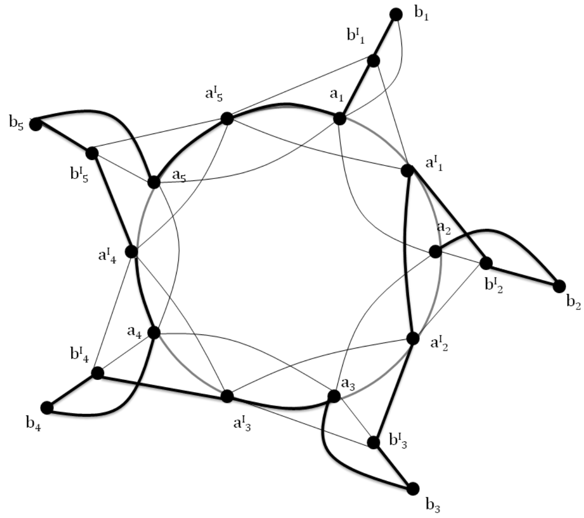
Example 3: For t=1

Hamiltonian path from the vertex b_1 to a_1 in total graph T[S_4]



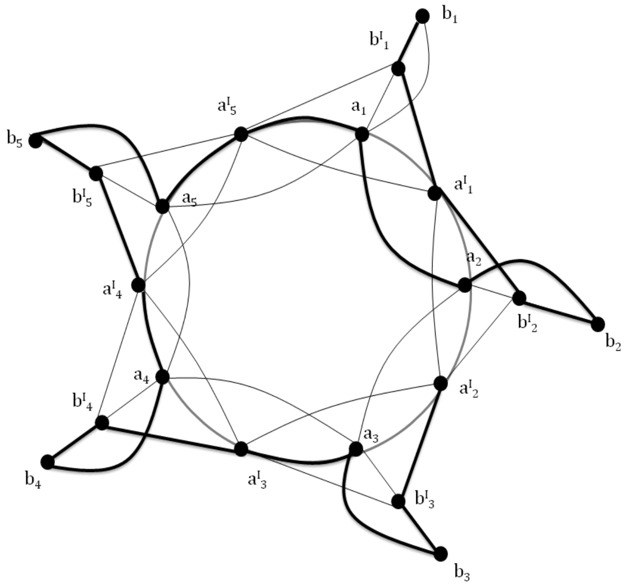
Example 4: For t=2

Hamiltonian path from the vertex b_1 to a_2 in total graph T[S_5]



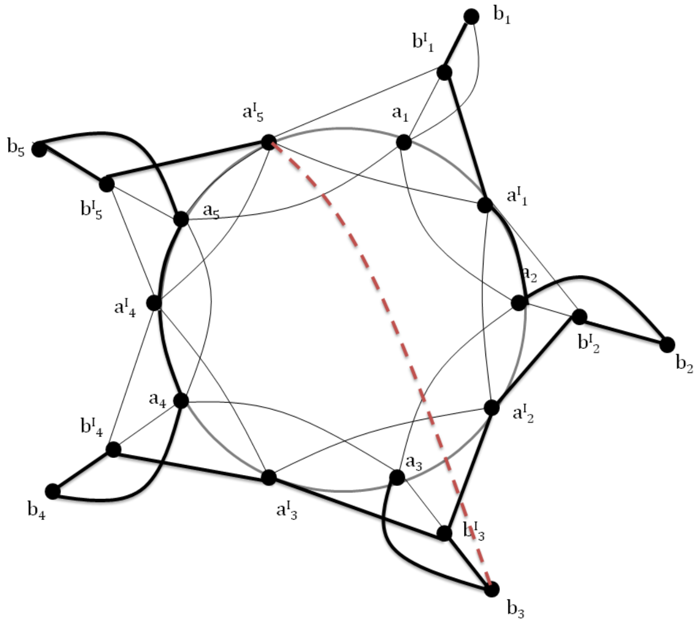
Example 5: For t=3 if n is odd

Hamiltonian path from the vertex b_1 to a'_2 in total graph T[S_5]



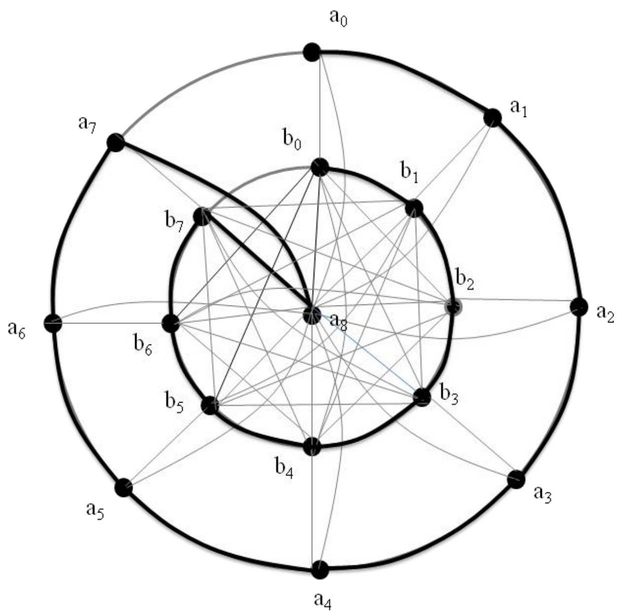
Example 6: For t=3 if n is even

Hamiltonian path from the vertex b_1 to a_3 in total graph T[S_6]



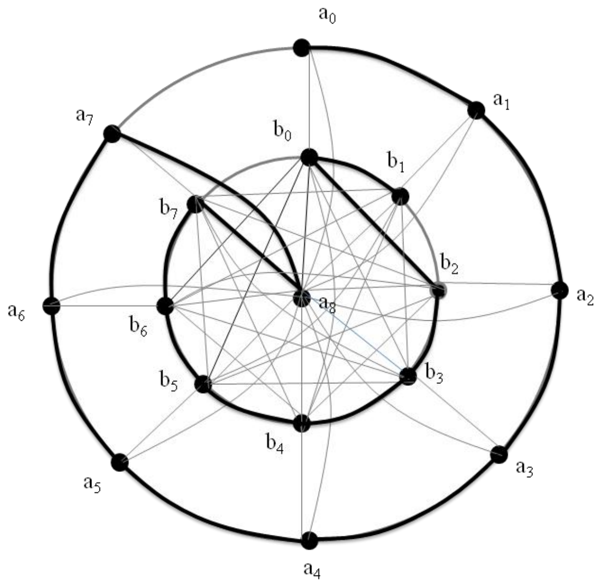
Example 7: For t=1

Hamiltonian path from the vertex a_0 to b_0 in total graph $T[K_{1,8}]$



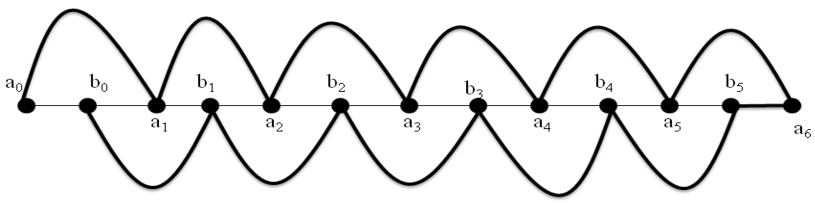
Example 8: For t=2

Hamiltonian path from the vertex a_0 to b_1 in total graph $T[K_{1,8}]$



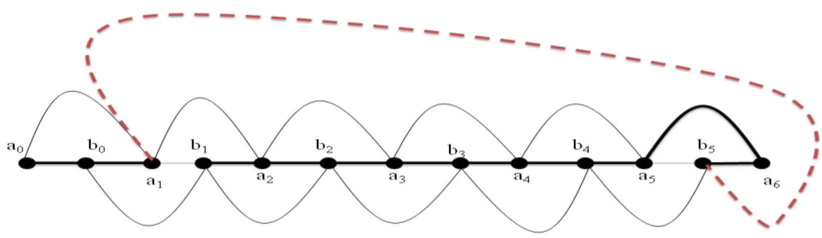
Example 9: For t=1

Hamiltonian path from the vertex a_0 to b_0 in total graph $T[P_7]$



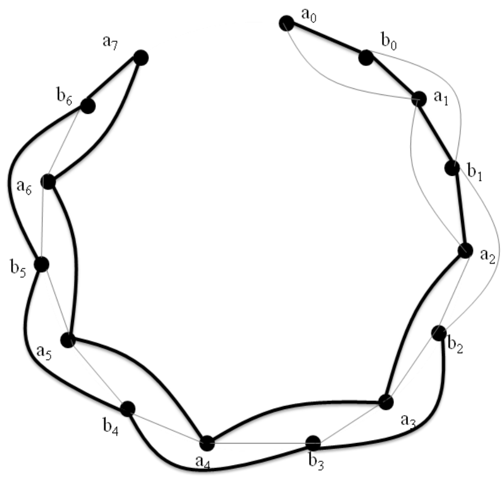
Example 10: For t=2 with $\lambda_{(t)} = 1$

Hamiltonian path from the vertex a_0 to b_1 in total graph $T[P_7]$



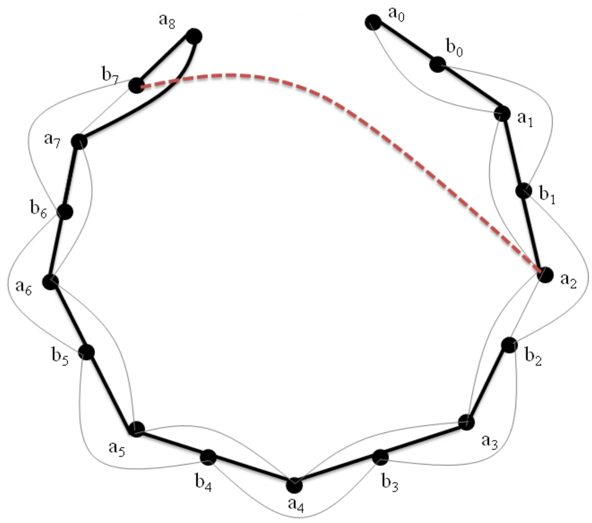
Example 11: For t=3 if $n \geq 4$ for even n

Hamiltonian path from the vertex a_0 to b_2 in total graph $T[P_8]$



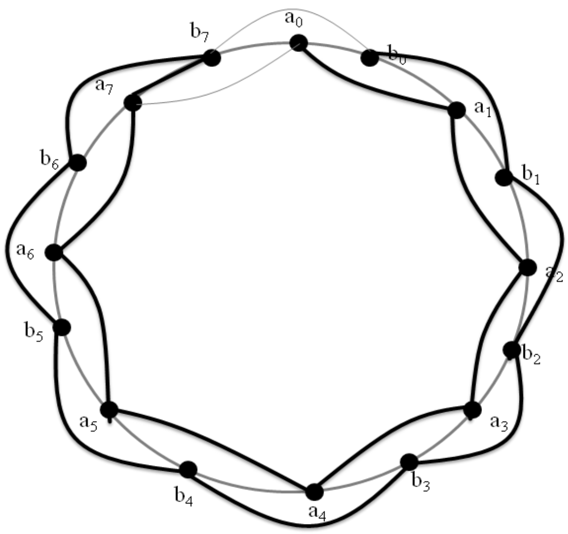
Example 12: For t=3 with, $\lambda_{(t)} = 1$ if $n \geq 5$ for odd n

Hamiltonian path from the vertex a_0 to b_2 in total graph $T[P_9]$



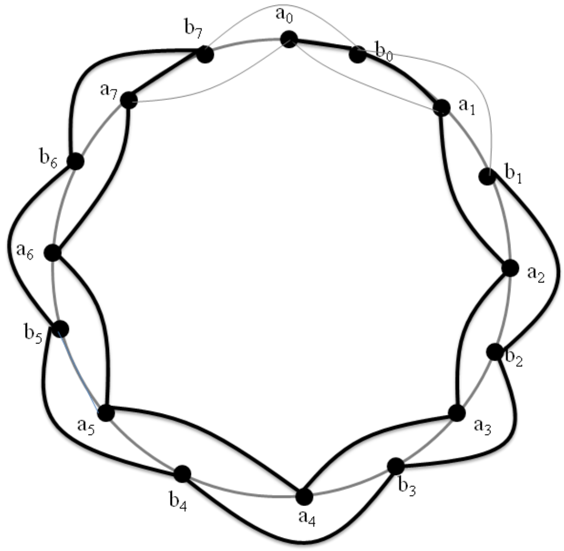
Example 13: For t=1, if n is even

Hamiltonian path from the vertex a_0 to b_0 in total graph $T[C_8]$



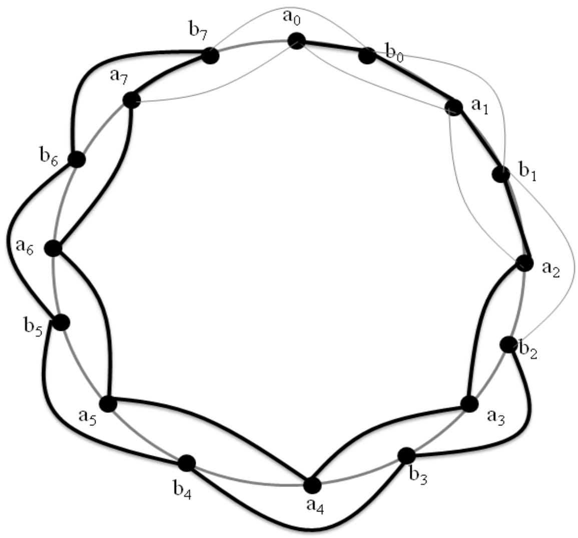
Example 14: For t=2 if n is even

Hamiltonian path from the vertex a_0 to b_1 in total graph $T[C_8]$



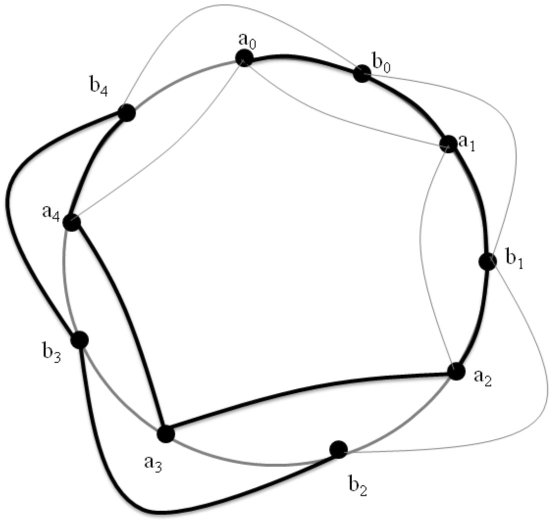
Example 15: For t=3, if n is even

Hamiltonian path from the vertex a_0 to b_2 in total graph $T[C_8]$



Example16: For t=3 if $n \geq 5$ for odd n

Hamiltonian path from the vertex a_0 to b_2 in total graph $T[C_5]$



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