[Volume 2 issue 12 December 2014]
Page No.806-811 ISSN :2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Lacunary Interpolation By g-Splines

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ABSTRACT

Th. Fauzi constructed special kinds of lacunary quintic g-splines and proved that for functions $f \in C^{(4)}$ the methods converges faster than that investigated by A.K. Verma and for functions $f \in C^{(5)}$ the order of approximation is the same as the best order of approximation using quintic g-splines.

In this paper, we construct quintic lacunary g-splines which are solutions of (0,1,4)- Interpolation problem and obtain their local approximations with functions belonging to $C^{(4)}(I)$ and $C^{(5)}(I)$. Our methods are of lower degree having better convergence property than the earlier investigations.

KEYWORDS – g- spline, lacunary interpolation piecewise polynomial, Taylor's expansion, Explicit form.

1. Let

(1.1) Δ: $0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 1$

be a partition of the interval I = [0,1] with $X_{k+1} - x_k = h_k$, $k = o(1)n - 1$. Th. Fauzi [3] constructed special kinds of lacunary quintile g-splines and proved that for functions $f \in C^{(4)}$ the methods converge faster than that investigated by A.K. Varma [1] and for functions $f \in C^{(5)}$ the order of approximation is the same as the best order of approximation using quintic g-splines. J. Gyorvari [4] considered local methods of degree six of class C[I] , which settles the problem of (0, 2, 3) and (0, 2, 4)-interpolations offering better approximation than the interpolants investigated by R. S. Misra and first author [2] . By varying continuity class and nature of the spline functions R.B. Saxena and H.C. Tripathi [5,6] obtained for functions $f \in C^{(6)}$ in the case of uniform partition the estimates of $| \tilde{s}^{(q)} - \tilde{f}^{(q)} |$ and $| \tilde{s}^{(q)} - \tilde{f}^{(q)} |$ Where $\tilde{s}_\Delta$ and $\hat{s}_\Delta$ each of degree six interpolate the data (0, 1 , 3) and (0, 2, 4), q = 0 (1) 5 choosing suitable initial and boundary conditions respectively.

In this paper, we construct quintic lacunary g-splines, which are solutions of (0, 1, 4) – Interpolation problems and obtain their local approximations with functions belonging to $C^{(4)}(I)$ and $C^{(5)}(I)$. Our methods are of lower degree having better convergence property than the earlier investigations made in [[1], [2], [4], [5], [6], [7], [8],[9]]. More over, our results have no counterpart in polynomial approximation theory. § 2. Is devoted to the study of quintic spline interpolant (0, 1, 4) for $C^{(4)}(I)$.

2. **Spline Interpolant (0, 1, 4) for f $\in C^{(5)}(I)$.**

Let $s_{1,\Delta}$ be a piecewise polynomial of degree $\in$ 5. The spline interpolant (0, 1, 4) for functions $\in C^{(5)}(I)$ is given by :

(2.1) $s_{1,\Delta}(x) = s_{1,k}(x) = \Sigma_{j=0}^{5} \frac{s_{k,j}^{(1)}}{j!}(x - x_k)^j, x_k \le x \le x_{k+1}, k = 0(1)n - 1,$

Where $s_{k,j}^{(1)}$, s are explicitly given below in terms of the prescribed data $\{f_k^{(j)}\}$, j = 0, 1, 4; K = 0(1)n, viz for k = 0(1)n-1 ,

(2.2) $s_{k,j}^{(1)} = f_k^{(j)}$, j = 0, 1, 4 .

For j = 2, 3, 5, we have

(2.3) $s_{k,5}^{(1)} = \frac{1}{h} [f_{k+1}^{(4)} - f_k^{(4)}]$,

(2.4) $s_{k,3}^{(1)} = -\frac{12}{h^3} [(f_{k+1} - f_k - hf_k^{(1)} - \frac{h^4}{4!} f_k^{(4)}) - \frac{h}{2}\left(f_{k+1}^{(1)} - f_k^{(1)} - \frac{h^3}{3!} f_k^{(4)}\right) + \frac{h^s}{80} s_{k,5}^{(1)}]$

and

(2.5) $s_{k,2}^{(1)} = \frac{2}{h^2}[f_{k+1} - f_k - hf_k^{(1)} - \frac{h^4}{4!} f_k^{(4)} - \frac{h^3}{3!} s_{k,3}^{(1)} - \frac{h^5}{5!} s_{k,5}^{(1)}]$

The coefficients $s_{k,j}^{(1)}$, j = 2, 3, 5 have been so chosen

That

$$D_L^{(p)} s_{1,k}(x_{k+1}) = D_R^{(p)} S_{1,k+1}(x_{k+1}), p = 0, 1, 4; \; k = 0(1)\, n - 1$$

Thus

$$s_{1,\Delta} \epsilon C^{(0.1,4)} [I] = \{ f : f^{(p)} \in C(I), p = 0, 1, 4 \}.$$

Is a unique quintic piecewise polynomial satisfying interpolator conditions (2.2).

If $f \in C^{(5)}$ [I], then owing to (2.3) – (2.5) and using Taylor ' s expansion, we have

(2.6) $\left| s_{k,j}^{(1)} - f_k^{(j)} \right| \leq C_{k,j}^{(1)} h^{5-j} \; \omega\left(f^{(5)} ; \mathrm{h}\right),\;$ j = 2, 3, 5; k = 0(1)n − 1

Where the constant $C_{k,j}^{(1)}$ are given by :

$$C_{k,2}^{(1)} = \frac{1}{10}\,,\; C_{k,3}^{(1)} = \frac{1}{4} \text{ and } C_{k,5}^{(1)} = 1.$$

Using (2.1) - (2.6) and a little computation gives :

<u>Theorem 2.1</u>

Let $f \in C^{(5)}$ [I] and $s_{1,\Delta} \epsilon C^{(0.1,4)}$ (I) be the unique spline interpolant (0, 1, 4) given in (2.1) - (2.5),

then

(2.7) $\| D^{(j)} (f- S_{1,\Delta}) \| L_\infty [X_k, x_{k_{+1}}] \leq c_{1,k}^{j} h^{5-j} \; \omega (f^{(5)} , h)$ j =0(1) 5; k=0 (1) n-1

Where the constants $c_{1,k}^{j}$, s are given by :

$$c_{1,k}^{0} = \frac{1}{10}\,,\quad c_{1,k}^{1} = \frac{4}{15}\,,\quad c_{1,k}^{2} = \frac{31}{60}\,,\quad c_{1,k}^{3} = \frac{3}{4}\,,\quad c_{1,k}^{4} = c_{1,k}^{5} = 1$$

<u>Almost Quartic Spline Interpolant (0, 1, 4) * for $f \in C^{(4)}$ (I).</u>

Almost quartic spline interpolant (0, 1, 4)* is a piecewise polynomial of degree 4 in each subinterval except in the last one, where it is a polynomial of degree 5. In this case, we have

(2.8)

$$S^*_{1,\Delta}(x) = S^*_{1,k}(x) = \sum_{j=0}^{4} \frac{S^{*(1)}_{k,j}}{j!}(x-x_k)^j \ , \quad x_k \le x \le x_{k+1} \ , \qquad k = 0(1)n-2$$

$$= \sum_{j=0}^{5} \frac{S^{*(1)}_{n-1,j}}{j!}(x-x_{n-1})^j \quad , x_{n-1} \le x \le x_n \quad , \ k = n-1$$

The coefficients $S^{*(1)}_{k,j}$ are explicitly given in terms of the data. In particular , for K=O(1) n-1, we prescribe

(2.9) $S^{*(1)}_{k,j} = f^{(j)}_k$, j = 0, 1, 4 .

For k = 0(1)n-2 and j = 2, 3, $S^{*(1)}_{k,j}$ are given by

(2.10) $S^{*(1)}_{k,2} = \frac{6}{h^2}$ [$(f_{k+1} - f_k - hf'_k - \frac{h^4}{4!} f^{(4)}_k) - \frac{h}{3}(f'_{k+1} - f'_k - \frac{h^3}{3!} f^{(4)}_k)$]

and

(2.11) $S^{*(1)}_{k,3} = -\frac{12}{h^3}$ [$(f_{k+1} - f_k - hf'_k - \frac{h^4}{4!} f^{(4)}_k) - \frac{h}{2}(f'_{k+1} - f'_k - \frac{h^3}{3!} f^{(4)}_k)$]

For k=n-1 and j=2, 3 and 5, we have

(2.12) $S^{*(1)}_{n-1,5} = \frac{1}{h}(f^{(4)}_n - f^{(4)}_{n-1})$

(2.13) $S^{*(1)}_{n-1,3} = -\frac{12}{h^3}$ [$(f_n - f_{n-1} - hf'_{n-1} - \frac{h^4}{4!} f^{(4)}_{n-1}) - \frac{h}{2}(f'_n - f'_{n-1} - \frac{h^3}{3!} f^{(4)}_{n-1}) + \frac{h^5}{80} S^{*(1)}_{n-1,5}$]

and

(2.14) $S^{*(1)}_{n-1,2} = -\frac{2}{h^2}$ [$(f_n - f_{n-1} - hf'_{n-1} - \frac{h^3}{3!} S^{*(1)}_{n-1,3} - \frac{h^4}{4!} f^{(4)}_{n-1} - \frac{h^5}{5!} S^{*(1)}_{n-1}$]

(2.10) and (2.11) are obtained from the condition.

(2.15) $S_{1,\Delta}^{*} \in C^{(1)}$ [I] ,

While (2.12)-(2.14) are determined from conditions (2.9) for k = n-1 in (2.8).

Analogous to (2.6) for $\in C^{(4)}$ [I] , one can establish

(2.16) $| S_{k,j}^{*(1)} - f_k^{(j)} | \le C_{k,j}^{*(1)} h^{4-j}$ ω $(f^{(4)}$, h) ,

Where the constants $C_{k,j}^{*(1)}$ are given by

$$C_{k,j}^{*(1)} = \begin{cases} \frac{1}{3} , & j = 2 \\ 1 , & j = 3 \end{cases} \qquad \text{k=0(1)n-2}$$

$$C_{k,j}^{*(1)} = \begin{cases} \frac{2}{3} , & j = 2 \\ \frac{17}{10} , & j = 3 \end{cases} \qquad \text{k=n-1}$$

Finally, similar to theorem 2.1 , we have

Theorem 2.2

Let f $\in C^{(4)}$ [I] and $S_{1,\Delta}^{*}$ be the unique almost quartic spline interpolant (0, 1, 4)* , given by (2.8) , then

(2.17)

$$\|D^{(J)}(f - S_{1,\Delta}^{*})\| \; L_\infty \, [x_k \, , x_{k+1}] \le C_{1,k}^{*(j)} \, h^{4-j} \omega(f^{(4)}, h),$$

Where the constants $C_{1,k}^{*(j)}$ are given by :

	$C_{1,k}^{*(0)}$	$C_{1,k}^{*(1)}$	$C_{1,k}^{*(2)}$	$C_{1,k}^{*(3)}$	$C_{1,k}^{*(4)}$
k=0(1)n-2	$\frac{3}{8}$	1	$\frac{11}{6}$	2	1
K=n-1	$\frac{77}{120}$	$\frac{197}{120}$	$\frac{14}{5}$	$\frac{53}{20}$	1

Acknowledgements – The Author is thankful to Prof. K.K. Mathur for the valuable suggestions of this paper.

References

(1) A.K. VARMA : Lacunary interpolation by splines-II Acta Math. Acad. Sci. Hungar., 31(1978), pp. 193-203.

(2) R. S. MISRA & K.K. MATHUR : Lacunary interpolation by splines (0; 0, 2, 3) and (0; 0, 2, 4) cases, Acta Math. Acad. Sci. Hungar, 36 (3-4) (1980), pp. 251-260.

(3) Th. FAWZY : (0, 1, 3) Lacunary interpolation by G-splines, Annales Univ. Sci., Budapest, Section Maths. XXXIX (1986), pp.63-67.

(4) J. GYORVARI : Lacunary interpolation spline functionen, Acta Math. Acad. Sci. Hungar, 42(1-2) (1983) , pp. 25-33.

(5) R.B. SAXENA & H.C. TRIPATHI : (0, 2, 3) and (0, 1, 3) – interpolation through splines, Acta Math. Hungar., 50(1-2) (1987), pp. 63-69.

(6) R.B. SAXENA & H.C. TRIPATHI : (0, 2, 3) and (0, 1, 3)- interpolation by six degree splines, Jour. Of computational and applied Maths., 18 (1987), pp. 395-101.

(7) Abbas Y. Albayati, Rostam K.S., Faraidun K. Hamasalh: Consturction of Lacunary Sixtic spline function Interpolation and their Applications. Mosul University, J. Edu. And Sci., 23(3)(2010).

(8) F. Lang and X. Xu: “A new cubic B-spline method for linear fifth order boundary value problems”. Journal of Applied Mathematics and computing, vol. 36, no. 1-2, pp-110-116, 2011.

(9) Ambrish Kumar Pandey, Q S Ahmad, Kulbhushan Singh : Lacunary Interpolation (0, 2; 3) problem and some comparison from Quartic splines: American journal of Applied Mathematics and statistics 2013, 1(6), pp- 117-120.