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On Complex K-Horadam and Gaussian K-Horadam Sequences

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ARTICLE INFO	ABSTRACT
Published Online: 29 November 2018 Corresponding Author: **Hasan GÖKBAŞ**	In this study, we consider firstly complex sequences and gaussian number sequences. Then we define complex *k*-Horadam and gaussian *k*-Horadam sequences. We give the distance between complex *k*-Horadam and gaussian *k*-Horadam numbers. We also show the generalized comlex k-Horadam and gaussian *k*-Horadam sequences. Then we have computed the limits of consecutive generalized comlex k-Horadam and gaussian *k*-Horadam sequences. We obtained the properties of complex *k*-Horadam and gaussian *k*-Horadam sequences.

KEYWORDS: complex *k*-Horadam, gaussian *k*-Horadam, generalized complex *k*-Horadam, generalized gaussian *k*-Horadam.
Mathematics Subject Classification: 11B37, 11B83.

1. Introduction

Studies show that there has been an increasing interest on number sequences and their generalizations. Number sequences, generalized of these number sequences, complex number sequences, gaussian number sequences and their interesting properties are studied by some authors.

Horadam(1963), established the complex Fibonacci number called as the gaussian Fibonacci number and investigated the complex Fibonacci polynomials. Jordan(1965), extented some relationship which are known about the common Fibonacci sequences. Berzsenyi(1977), show a closed form to gaussian Fibonacci numbers by the Fibonacci Q matrix. Harman(1981), gave an extension of Fibonacci numbers into the complex plane and generalized the methods. Koshy(2001), examined some properties of gaussian Fibonacci and Lucas numbers. Aşçı and Gürel(2013), studied gaussian Fibonacci numbers, gaussian Jacobsthal and gaussian Jacobsthal-Lucas numbers. Altınışık et all. (2015), studied the matrices whose elements are complex Fibonacci numbers. Halıcı and Öz(2016), defined gaussian Pell and gaussian Pell-Lucas numbers.

In this study, we define complex *k*-Horadam, gaussian *k*-Horadam, generalized complex *k*-Horadam and generalized gaussian *k*-Horadam sequences. In addition on this definition, We give some formulas for these sequences. Also we found the distance between the numbers generalized complex *k*-Horadam and generalized gaussian *k*-Horadam.

Definition 1: [8] Let *k* be any positive real number and *f(k)*, *g(k)* are scalar value polynomials. For $n \geq 0$ and $f^2(k)+4g(k) > 0$, the generalized *k*-Horadam sequence $(H_{k,n})_{n\in N}$ is defined by

$$H_{k,n+2} = f(k)H_{k,n+1} + g(k)H_{k,n}$$

with initial conditions $H_{k,0} = a$ and $H_{k,1} = b$.

Theorem 2: [8] For $n \geq 0$ and $f(k)+g(k)-1 \neq 0$, let $H_{k,n}$ be *nth* generalized *k*-Horadam number,

$$\sum_{i=0}^{n-1} H_{k,i} = \frac{H_{k,n} + g(k)H_{k,n-1} - H_{k,1} + (f(k)-1)H_{k,0}}{f(k)+g(k)-1}$$

Theorem 3: [8] For $S = (b-af(k))^2 - a^2 - 2(b-ar_1)(b-ar_2)\left(\frac{1-(-g(k))^n}{1+g(k)}\right)$, $H_{k,-1} = \frac{b-af(k)}{g(k)}$, $f^2(k)+2g(k)-g^2(k)-1 \neq 0$ *and* , let $H_{k,n}$ be *nth* generalized *k*-Horadam number,

$$\sum_{i=0}^{n-1} H_{k,i}^2 = \frac{H_{k,n}^2 + g^2(k)H_{k,n-1}^2 + S}{f^2(k)+2g(k)-g^2(k)-1}$$

2. Main Results

In this section, we define two $(KH_{k,n})_{n\in N}$, $(GH_{k,n})_{n\in N}$ of the special sequences. We obtain some equalities and equalities retated with this defined sequences.

Definition 4: Let k be any positive real number and *f(k), g(k)* are scalar value polynomials. For $n \geq 0$, the complex *k*-Horadam sequence $(KH_{k,n})_{n\in N}$ is defined by

$$KH_{k,n+2} = H_{k,n+1}[f(k) + if^2(k) + ig(k)] + H_{k,n}[g(k) + if(k)g(k)]$$

with initial conditions $H_{k,0} = a$ and $H_{k,1} = b$ and where i is the imaginary unit which satisfies $i^2 = -1$. When equality is arranged,

$$KH_{k,n+2} = H_{k,n+1} + iH_{k,n+3}$$

Particular cases of the previous definition are:

- ✓ If *f(k)*=2, *g(k)*=1, *a*=0 and *b*=1 the complex Pell sequence is obtained: $KH_{k,0} = i$, $KH_{k,1} = 1 + 2i$
- ✓ If *f(k)*=2, *g(k)*=1, *a*=2 and *b*=2 the complex Pell-Lucas sequence is obtained: $KH_{k,0} = 2 + 2i$, $KH_{k,1} = 2 + 6i$

Definition 5: Let k be any positive real number and *f(k), g(k)* are scalar value polynomials. For $n \geq 0$, the gaussian *k*-Horadam sequence $(GH_{k,n})_{n\in N}$ is defined by

$$GH_{k,n+2} = H_{k,n}[f^2(k) + g(k) + if(k)] + H_{k,n-1}[f(k)g(k) + ig(k)]$$

with initial conditions $H_{k,0} = a$ and $H_{k,1} = b$ and where i is the imaginary unit which satisfies $i^2 = -1$. When equality is arranged,

$$GH_{k,n+2} = H_{k,n+2} + iH_{k,n+1}$$

Particular cases of the previous definition are:

- ✓ If *f(k)*=2, *g(k)*=1, *a*=0 and *b*=1 the gaussian Pell sequence is obtained: $GH_{k,0} = i$, $GH_{k,1} = 1$
- ✓ If *f(k)*=2, *g(k)*=1, *a*=0 and *b*=1 the gaussian Pell-Lucas sequence is obtained: $GH_{k,0} = 2 - 2i$, $GH_{k,1} = 2 + 2i$

Theorem 6: For $n \geq 0$ and let $H_{k,n}$ be *nth* generalized *k*-Horadam number, let $KH_{k,n}$ be *nth* complex *k*-Horadam number and let $GH_{k,n}$ be *nth* gaussian *k*-Horadam number, we have

- ➢ $KH_{k,n+2} - GH_{k,n+2} = H_{k,n}[if(k)(f^2(k) + 2g(k) - 1)] + H_{k,n-1}[ig(k)(f^2(k) + g(k) - 1)]$
- ➢ $KH_{k,n+2} + GH_{k,n+2} = H_{k,n}[2f^2(k) + 2g(k) + if^3(k) + 2ifg(k) + if(k)] + H_{k,n-1}[2fg(k) + if^2g(k) + ig^2(k) + igk$

Proof: The desired is obtained by summing and substracting the equalities. When equalities are arranged,

- ➢ $KH_{k,n+2} - GH_{k,n+2} = i[H_{k,n+3} - H_{k,n+1}]$
- ➢ $KH_{k,n+2} + GH_{k,n+2} = 2H_{k,n+2} + i[H_{k,n+3} + H_{k,n+1}]$

Theorem 7: For $n \geq 0$ and let $H_{k,n}$ be *nth* generalized *k*-Horadam number, let $KH_{k,n}$ be *nth* complex *k*-Horadam number and let $GH_{k,n}$ be *nth* gaussian *k*-Horadam number, we have

$$|KH_{k,n+2} - GH_{k,n+2}| = H_{k,n+1}[f^2(k) + g(k) - 1] + H_{k,n}[f(k)g(k)]$$

Proof : The definition of the absolute value is obtained from the desired. When equality is arranged,

$$|KH_{k,n+2} - GH_{k,n+2}| = H_{k,n+3} - H_{k,n+1}$$

Table 1: First five terms belong to the differences $|\boldsymbol{KP - GP}|$and $|\boldsymbol{KQ - GQ}|$where KP, GP, KQ and GQ stands for complex Pell, gaussian Pell, complex Pell-Lucas and gaussian Pell-Lucas numbers.

n	***KP***	***GP***	**|*KP* - *GP*|**	***KQ***	***GQ***	**|*KQ* – *GQ*|**
0	**0**+i	0+i	***0***	**2**+2i	2-2i	***4***
1	**1**+2i	1	***2***	**2**+6i	2+2i	***4***
2	**2**+5i	2+i	***4***	**6**+14i	6+2i	***12***
3	**5**+12i	5+2i	***10***	**14**+34i	14+6i	***28***
4	**12**+29i	12+5i	***24***	**34**+82i	34+14i	***68***
5	**29**+70i	29+12i	***58***	**82**+198i	82+34i	***164***
…	…	…	...	…	…	...

Sonuç 8. It is clear from the table that $|\boldsymbol{KP}-\boldsymbol{GP}| = 2Re(KP)$ and $|\boldsymbol{KQ}-\boldsymbol{GQ}| = 2Re(KQ)$.
$\left|KH_{k,n+2}-GH_{k,n+2}\right| = H_{k,n+3}-H_{k,n+1} = 2H_{k,n+2}$

Theorem 9: For $n \geq 0$ and $f(k)+g(k)-1 \neq 0$, let $H_{k,n}$ be *nth* generalized *k*-Horadam number and let $KH_{k,n}$ be *nth* complex *k*-Horadam number, we have

$$\sum_{m=0}^{n-1} KH_{k,m+2} = \left(\frac{H_{k,n+1}+g(k)H_{k,n}-H_{k,1}+(f(k)-1)H_{k,0}}{f(k)+g(k)-1}-a\right)[f(k)+if^2(k)+ig(k)] + \left(\frac{H_{k,n}+g(k)H_{k,n-1}-H_{k,1}+(f(k)-1)H_{k,0}}{f(k)+g(k)-1}\right)[g(k)+if(k)g(k)]$$

Proof:

$$\sum_{m=0}^{n-1} KH_{k,m+2} = \sum_{m=0}^{n-1} H_{k,m+1}[f(k)+if^2(k)+ig(k)] + \sum_{m=0}^{n-1} H_{k,m}[g(k)+if(k)g(k)]$$
$$= \left(\frac{H_{k,n+1}+g(k)H_{k,n}-H_{k,1}+(f(k)-1)H_{k,0}}{f(k)+g(k)-1}-a\right)[f(k)+if^2(k)+ig(k)] + \left(\frac{H_{k,n}+g(k)H_{k,n-1}-H_{k,1}+(f(k)-1)H_{k,0}}{f(k)+g(k)-1}\right)[g(k)+if(k)g(k)]$$

So, the proof is completed.

Theorem 10: For $n \geq 0$ and $f(k)+g(k)-1 \neq 0$, let $H_{k,n}$ be *nth* generalized *k*-Horadam number and let $GH_{k,n}$ be *nth* gaussian *k*-Horadam number, we have

$$\sum_{m=0}^{n-1} GH_{k,m+2} = \left(\frac{H_{k,n}+g(k)H_{k,n-1}-H_{k,1}+(f(k)-1)H_{k,0}}{f(k)+g(k)-1}\right)[f^2(k)+g(k)+if(k)] + \left(\frac{H_{k,n-1}+g(k)H_{k,n-2}-H_{k,1}+(f(k)-1)H_{k,0}}{f(k)+g(k)-1}\right)[f(k)g(k)+ig(k)]$$

Proof: Using the same technique, we can be show the proof.

Theorem 11: For $S = (b-af(k))^2 - a^2 - 2(b-ar_1)(b-ar_2)\left(\frac{1-(-g(k))^n}{1+g(k)}\right)$ and $R = (b-af(k))^2 - a^2 - 2(b-ar_1)(b-$ *ar21−−gkn+11+g(k), f2k+2gk−g2k−1≠0,* let *Hk,n* be *nth* generalized *k*-Horadam number and let *KHk,n* be *nth* complex *k*-Horadam number, we have

$$\sum_{m=0}^{n-1} KH_{k,m+2}^2 = M^2\left(\frac{H_{k,n+1}^2+g^2(k)H_{k,n}^2+R}{f^2(k)+2g(k)-g^2(k)-1}-b^2\right) + N^2\left(\frac{H_{k,n}^2+g^2(k)H_{k,n-1}^2+S}{f^2(k)+2g(k)-g^2(k)-1}\right) + 2MN\left(\frac{f(k)\left[\frac{H_{k,n+1}^2+g^2(k)H_{k,n}^2+S}{f^2(k)+2g(k)-g^2(k)-1}\right]-H_{k,n}H_{k,n+1}+ab-a^2f(k)}{1-g(k)}\right)$$

where $M = f(k)+if^2(k)+ig(k)$ and $N = g(k)+if(k)g(k)$.

Proof:

$$\sum_{m=0}^{n-1} KH_{k,m+2}^2 = [f(k)+if^2(k)+ig(k)]^2 \sum_{m=0}^{n-1} H_{k,m+1}^2 + [g(k)+if(k)g(k)]^2 \sum_{m=0}^{n-1} H_{k,m}^2 + 2[f(k)+if^2(k)+ig(k)][g(k)+if(k)g(k)] \sum_{m=0}^{n-1} H_{k,m+1}H_{k,m}$$
$$= M^2\left(\frac{H_{k,n+1}^2+g^2(k)H_{k,n}^2+R}{f^2(k)+2g(k)-g^2(k)-1}-b^2\right) + N^2\left(\frac{H_{k,n}^2+g^2(k)H_{k,n-1}^2+S}{f^2(k)+2g(k)-g^2(k)-1}\right) + 2MN\left(\frac{f(k)\left[\frac{H_{k,n+1}^2+g^2(k)H_{k,n}^2+S}{f^2(k)+2g(k)-g^2(k)-1}\right]-H_{k,n}H_{k,n+1}+ab-a^2f(k)}{1-g(k)}\right)$$

So, the proof is completed.

Theorem 12: For $S=(b-af(k))^2-a^2-2(b-ar_1)(b-ar_2)\left(\frac{1-(-g(k))^n}{1+g(k)}\right)$ and $R=(b-af(k))^2-a^2-2(b-ar_1)(b-$ $ar21--gkn-11+g(k)$, $f2k+2gk-g2k-1\neq 0$, let Hk,n be *nth* generalized *k*-Horadam number and let GHk,n be *nth* gaussian *k*-Horadam number, we have

$$\sum_{m=0}^{n-1} GH_{k,m+2}^2 = M^2\left(\frac{H_{k,n}^2+g^2(k)H_{k,n-1}^2+S}{f^2(k)+2g(k)-g^2(k)-1}\right)+N^2\left(\frac{H_{k,n-1}^2+g^2(k)H_{k,n-2}^2+R}{f^2(k)+2g(k)-g^2(k)-1}-H_{k,-1}\right)$$
$$+2MN\left(\frac{f(k)\left[\frac{H_{k,n}^2+g^2(k)H_{k,n-1}^2+S}{f^2(k)+2g(k)-g^2(k)-1}\right]-H_{k,n-1}H_{k,n}+ab-a^2f(k)}{1-g(k)}-H_{k,-1}H_{k,0}\right)$$

where $M=f^2(k)+g(k)+if(k)$ and $N=f(k)g(k)+ig(k)$.

Proof: Using the same technique, we can be show the proof.

Definition 13: Let *k* be any positive real number and *f(k)*, *g(k)* are scalar value polynomials. For $n\ \geq 0$ and $f^2(k)+4g(k)>0$, the generalized complex *k*-Horadam sequence $(KH_{k,n})_{n\in N}$ is defined by

$$KH_{k,n+2}=f(k)KH_{k,n+1}+g(k)KH_{k,n}$$

with initial conditions $KH_{k,0}=a$ and $KH_{k,1}=b$.
Particular cases of the previous definition are:

- ✓ If *f(k)*=2, *g(k)*=1, *a*=i and *b*=1+2i the complex Pell sequence is obtained: $KH_{k,0}=i$, $KH_{k,1}=1+2i$
- ✓ If *f(k)*=2, *g(k)*=1, *a*=2+2i and *b*=2+6i the complex Pell-Lucas sequence is obtained: $KH_{k,0}=2+2i$, $KH_{k,1}=2+6i$

Definition 14: Let *k* be any positive real number and *f(k)*, *g(k)* are scalar value polynomials. For $n\ \geq 0$ and $f^2(k)+4g(k)>0$, the generalized gaussian *k*-Horadam sequence $(GH_{k,n})_{n\in N}$ is defined by

$$GH_{k,n+2}=f(k)GH_{k,n+1}+g(k)GH_{k,n}$$

with initial conditions $GH_{k,0}=a$ and $GH_{k,1}=b$.
Particular cases of the previous definition are:

- ✓ If *f(k)*=2, *g(k)*=1, *a*=i and *b*=1 the gaussian Pell sequence is obtained: $GH_{k,0}=i$, $GH_{k,1}=1$
- ✓ If *f(k)*=2, *g(k)*=1, *a*=2-2i and *b*=2+2i the gaussian Pell-Lucas sequence is obtained: $GH_{k,0}=2-2i$, $GH_{k,1}=2+2i$

Theorem 15: The ratio belong to the limits of consecutive generalized complex *k*-Horadam numbers is as follows,

$$\lim_{n\to\infty}\frac{KH_{k,n+1}}{KH_{k,n}}=\frac{f(k)+\sqrt{f^2(k)+4g(k)}}{2}$$

Proof:

$$\lim_{n\to\infty}\frac{KH_{k,n+1}}{KH_{k,n}}=\frac{f(k)+\sqrt{f^2(k)+4g(k)}}{2}=f(k)+\frac{g(k)}{\lim_{n\to\infty}\frac{KH_{k,n}}{KH_{k,n-1}}}$$

$$x=f(k)+\frac{g(k)}{x}$$

$$x=\frac{f(k)+\sqrt{f^2(k)+4g(k)}}{2}.$$

Theorem 16: The ratio belong to the limits of consecutive generalized gaussian *k*-Horadam numbers is as follows,

$$\lim_{n\to\infty}\frac{GH_{k,n+1}}{GH_{k,n}}=\frac{f(k)+\sqrt{f^2(k)+4g(k)}}{2}$$

Proof: Using the same technique, we can be show the proof.

- ✓ *f(k)*=2, *g(k)*=1, we get the ratio belong to the limits for the sequences comlex Pell, comlex Pell-Lucas, gaussian Pell and gaussian Pell-Lucas numbers as follows,

$$\frac{f(k)+\sqrt{f^2(k)+4g(k)}}{2}=\frac{2+\sqrt{4+4}}{2}=1+\sqrt{2}$$

4. Conclusion

In this paper, we firstly define complex *k*-Horadam and gaussian *k*-Horadam sequences. Then we find sum and differences of complex *k*-Horadam and gaussian *k*-Horadam sequences. We give the distance between complex *k*-Horadam and gaussian *k*-Horadam numbers. We obtained the properties of complex *k*-Horadam and gaussian *k*-Horadam sequences. We also show the generalized comlex k-Horadam and gaussian *k*-Horadam sequences. We find the limit values of consecutive generalized comlex *k*-Horadam and generalized gaussian *k*-Horadam numbers.

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