International Journal of Mathematics and Computer Research
ISSN: 2320-7167
Volume 08 Issue 07 July 2020, Page no. - 2096-2105
Index Copernicus ICV: 57.55
DOI: 10.33826/ijmcr/v8i7.05

Marshall-Olkin q-Exponential Processes

Catherine Thomas[1], Seetha Lekshmi V.[2]

[1]Dept.of Statistics, Govt. College, Kottayam, Kerala, India
[2]Dept. of Statistics, Nirmala College, Muvattupuzha, Kerala, India

ARTICLE INFO
Published Online: 25 July 2020
Corresponding Author: **Catherine Thomas**

ABSTRACT
In this paper we review the q-exponential distribution and its properties. Distributions of extreme order statistics are obtained. The Marshall-Olkin q-exponential distribution is developed and studied in detail. Estimation of parameters is also discussed. AR(1) models and max-min AR(1) models are developed and sample path properties are explored. These can be used for modeling time series data on river flow, dam levels, finance and exchange rates.

KEYWORDS: AR(1) models, Max-Min processes, Marshall-Olkin q-exponential distribution, q-exponential distribution.

1. INTRODUCTION

Tsallis(1988) introduced the q-exponential distribution in an attempt for generalizing Boltzmann-Gibbs statistics in statistical mechanics. Picoli et al (2003) considered q-exponential, Weibull and q-Weibull distributions and an empirical analysis is conducted. The family of q-distributions are useful in explaining molecular motion in fluid dynamics as well as entropy functions in communications engineering. Seetha lekshmi and Catherine (2012) developed a new count model called q-exponential count model which is a generalization of exponential count model. The q-exponential count model via the shape parameter, can capture over dispersed as well as equi-dispersed data.

Marshall and Olkin(1997) introduced a generalized family of distributions and applied the results to extend exponential and Weibull distributions. Many researchers have recently studied Marshall-Olkin family of distributions and applied in various contexts such as reliability analysis, time series modeling etc. For details see Jayakumar and Thomas (2008), Sankaran and Jayakumar (2006), Krishna et al.(2013a,b), Jose et al.(2010,2011,2014).These distributions offer wide flexibility and can be used to model data from various areas.

Autoregressive processes with non-Gaussian marginal distributions have received much attention in r ecent years. Lewis and McKenzie (1991) introduced minification processes and their general theory.
Alice and Jose(2004), Seetha lekshmi and Jose(2004, 2006), Jose and Naik(2010), Jose and Remya (2015) etc are some recent works in this respect.

This paper is organized as follows. In section 2, q-exponential distribution is reviewed. In section 3, distributions of maxima and minima are derived and it is shown that the q-exponential distribution is a compound mixture. In section 4, we introduce the Marshall-Olkin q-exponential distribution and studied its important properties. AR(1) models with q-exponential marginal distribution are introduced in section 5. As a further extension, general theory of Max-Min AR(1) processes are also developed in section 6 and generalized it to the kth order. In section 7, Max-Min process with q-exponential marginal distribution is introduced and studied. Applications are discussed in section 8.

2. q-EXPONENTIAL DISTRIBUTION

The q-exponential distribution is a generalization of the exponential distribution. The main reason for introducing q-exponential model is the switching property of the exponential form to corresponding binomial expansion as follows.

$$\lim_{q\to 1}\,[1-(1-q)z]^{\frac{1}{1-q}} = e^{-z},\quad 0<q<1$$

$$\lim_{q\to 1}\,[1+(q-1)z]^{\frac{-1}{q-1}} = e^{-z},\quad 1<q<2$$

When 0<q<1 the probability density function (pdf) of the q-exponential distribution is given by

$$f1(x) = u(2\text{-}q)[1-(1-q)(ux)]^{\frac{1}{1-q}} \; ;q < 1,u>0 \qquad (1)$$

When 1 < q < 2, the pdf of q-exponential distribution is given by

$$f2(x) = u(2\text{-}q)[1-(q-1)(ux)]^{-(\frac{1}{q-1})} \; ;q < 1,u>0 \qquad (2)$$

The important properties such as cumulative distribution function (CDF), hazard rate function (HRF), cumulative hazard rate function(CHR), mode and moments are tabulated in the Tables 1 and 2 respectively for 0 < q < 1 and 1 < q < 2.

Table 1. Properties of q-exponential distribution when 0 < q < 1

Characteristics	Functional Form
Pdf	$f1(x) = u(2\text{-}q)[1-(1-q)(ux)]^{\frac{1}{1-q}}$;q < 1,u>0,x>0
CDF	$F1(x) = 1\text{-}[1-(1-q)(ux)]^{\frac{2-q}{1-q}}$
HRF	$h(x) = \frac{u(2-q)}{1-(1-q)(ux)}$
CHR	$H(x)=(2\text{-}q)(ux)\sum_{j=0}^{\infty}[\frac{(1-q)ux]^{j+1}}{j+1}$,lxl< $\frac{1}{u(1-q)}$
s^{th}moment	$E(X^s)=\frac{2-q}{u^s(1-q)^{s+1}}\frac{\Gamma(s+1)\Gamma\frac{1}{(1-q)}+1}{\Gamma(\frac{1}{1-q})+s+2}$
Mean	$E(x) = \frac{1}{u(3-2q)}$
Variance	$V(x)=\frac{2-q}{u^2(3-2q)^2}\frac{1}{(4-3q)}$
Mode	0

Table 2. Properties of q-exponential distribution when 1 < q < 2

Characteristics	Functional Form
Pdf	$f2(x) = u(2\text{-}q)[1+(ux)(q-1)^{\frac{-1}{1-q}}$; x∈ $(0,\infty)$
CDF	$F2(x) = 1\text{-}[1+(q-1)(ux)]^{\frac{q-2}{q-1}}$
HRF	$h(x) = \frac{u(2-q)}{1+(q-1)(ux)}$
CHR	$H(x)=(\frac{2-q}{q-1})$ In[1+(q-1)(ux)]
s^{th}moment	$E(X^s)=\frac{2-q}{u^s(q-1)^{s+1}}\frac{\Gamma(s+1)\Gamma\frac{1}{(q-1)}+s-1}{\Gamma(\frac{1}{q-1})}$
Mean	$E(x) = \frac{1}{u(3-2q)}$
Variance	$V(x)=\frac{q-2}{u^2(2q-3)^2}\frac{1}{(3q-4)}$
Mode	0

3. DISTRIBUTION OF MAXIMUM AND MINIMUM

In this section, the distribution of maximum and minimum of a sequence of independent and identically distributed random variables(iid rvs) are considered.

Lemma 3.1 Let (Zi; i = 1;2;……n) be iid rvs which follows q-exponential distribution with parameter u then the distribution of the min(Z1;Z2; …Zn) is again a q-exponential distribution.

Proof: For 0 < q < 1, the survival function

$$F1(z) = 1\text{-}[1-(1-\mathrm{q})(\mathrm{uz})]^{\frac{2-q}{1-q}}$$

$$F_3(z) = \mathrm{P}[\min(\mathrm{Z1;Z2;..Zn}) > \mathrm{z}]$$

$$= \prod_{i=1}^{n} P(Z_i > z)$$

$$=[1-(1-q)(uz)]^{\frac{n(2-q)}{1-q}} \to e^{-n(uz)} \text{ as } \mathrm{q}\to 1$$

For 1 < q < 2, the survival function is

$$\bar{F}_2(\mathrm{z})=[1+(q-1)(uz)]^{\frac{-(2-q)}{q-1}}$$

$$\bar{F}_4(z) = [1+(q-1)(uz)]^{\frac{-n(2-q)}{q-1}} \to e^{-n(uz)} \text{ as } \mathrm{q}\to 1$$

It is not in the q-exponential form, but nth power of the survival function of q-exponential distribution. So this is invariabily a case of q exponential distribution.

Lemma 3.2 Let Zi, i=1,2…,n be iid rvs which follows the q-exponential distribution, then max (Z1;Z2; :::Zn) is distributed as exponentiated q-exponential distribution.

Proof: For q < 1 the CDF is

$$F1(z) = 1\text{-}[1-(1-\mathrm{q})(\mathrm{uz})]^{\frac{2-q}{1-q}}$$

$$\text{Then } F5(z) = \mathrm{P}[\max(\mathrm{Z1;Z2} \dots,\mathrm{Zn}) \le \mathrm{z}]$$

$$=\prod_{i=1}^{n}[1-(1-q)(uz)]^{\frac{2-q}{1-q}}$$

$$=[1-(1-q)(uz)]^{\frac{2-q}{1-q}}]^n \to [1-e^{-uz}]^n \text{ as } \mathrm{q}\to 1$$

3.1. The q-exponential distribution as a compound mixture

Definition 3.1. Let $\bar{G}(z/\theta),-\infty < z < \infty,-\infty < \theta < \infty$,be the conditional survive function of z given θ and let θ be a random variable with probability density function m(θ). Then a distribution with survival function $\bar{G}$(z) =$\int_{-\infty}^{+\infty} \bar{G}(z/\theta)\, m(\theta) d\theta$, ,$-\infty < z < \infty$, is called a compound distribution with mixing density m(θ).

Theorem 3.1

$\bar{G}$(z) is a compound exponential mixture.

Case (i)

For q<1 let the conditional survival function be

$$\overline{G1}(z/\theta)=\exp(-\theta)[[1-(1-\mathrm{q})(\mathrm{ux})]^{\frac{(2-q)}{(1-q)}} --1]$$

M(θ)=$e^{-\theta}$. Then the unconditional survival function is

$$\bar{G}1(\mathrm{z})=\int_0^{\infty} e^{-\theta}\, [[1-(1-\mathrm{q})(\mathrm{ux})]^{\frac{(2-q)}{(1-q)}} -\text{- }1]d\theta$$

$$=[1-[[1-q](uz)]^{\frac{2-q}{1-q}}$$

Which is the survival function of q-exponential distribution $\bar{F}1$(z) for q<1

Case(ii)

For 1<q<2, let the conditional survival function be

$$\bar{G}2(z/\theta)= \exp(-\theta)[1+[(q\text{-}1)(uz)^{\frac{q-2}{q-1}}]d\theta$$
$$=1+(q-1)(uz)^{\frac{q-2}{q-1}}$$

Which is the survival function of q-exponential distribution $\overline{F2}$(z) for 1<q<2.

The above conditional survival functions are survival functions of the extreme value distributions mixed with the exponential distributions.

4. MARSHALL-OLKIN Q-EXPONENTIAL DISTRIBUTION AND ITS PROPERTIES

In this section a new probability model known as Marshall-Olkin q-exonential distribution is developed. Various properties of the distribution and hazard rate functions are considered. The corresponding time series models are developed to illustrate its application in time series modelling.

Let

$$\bar{G}(x) = \frac{p\bar{F}(x)}{1-(1-p)\bar{F}(x)}\ , x\varepsilon R, p>0$$

Clearly when p=1, we get the standard form of the survival function.

The pdf of MO distribution is given by,

$$g(x) = \frac{pf(x)}{\left(1-(1-p)\bar{F}(x)\right)^2}, x\varepsilon R, p>0$$

The hazard rate of Marshall -Olkin distribution is given by ,

$$h(t) = \frac{r(t)}{1-(1-p)\bar{F}(t)},$$

where, $r(t) = \frac{f(t)}{\bar{F}(t)}$

The Survival function of the MOQE distribution is given by

$$\bar{G}(x) = \frac{p[1-[[1-q](uz)]^{\frac{2-q}{1-q}}}{1-(1-\mathrm{p})[1-[[1-q](uz)]^{\frac{2-q}{1-q}}} \qquad (4)$$

The density of the MOQE distribution is given by,

$$\mathrm{g(x)}=\frac{pu(2-q)[1-(1-q)(ux)]^{\frac{-1}{q-1}}}{1-(1-p)(1-(1-q)(ux))^{\frac{2-q}{1-q}})^2} \qquad (5)$$

The hazard rate function of MOQE distribution is given by,

$$h(t) = \frac{\frac{\mathrm{u}(2-\mathrm{q})}{1-(1-\mathrm{q})(\mathrm{ut})}}{[1-(1-p)]1-(\mathrm{q}-1)(\mathrm{ut})]^{\frac{\mathrm{q}-2}{\mathrm{q}-1}}}$$

The graphs of the pdf (probability density function) and HRF(Hazard Rate Function) of MO q-e distribution are given below.

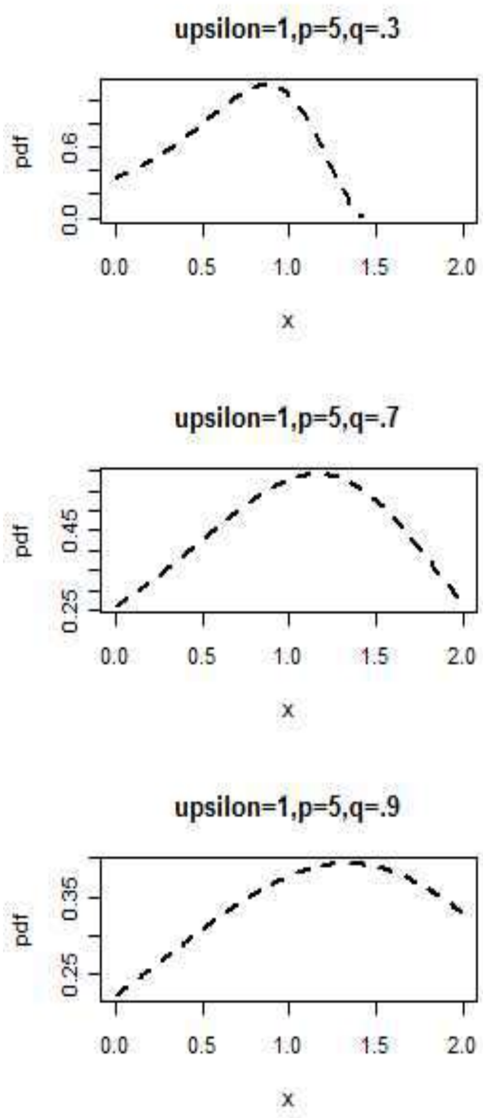

Figure1

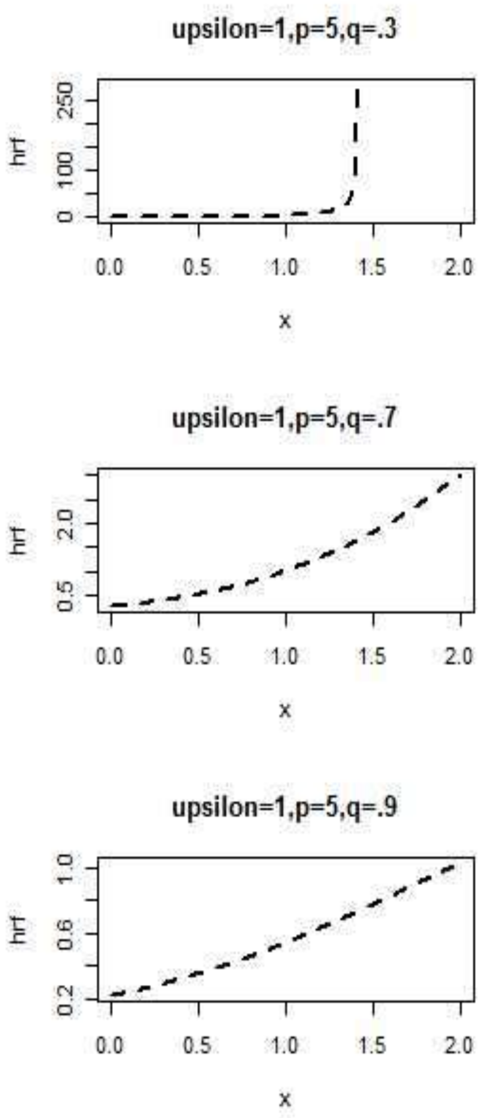

Figure2

Theorem 4.1 Let $\{X_i, i \geq 1\}$be a sequence of i.i.d.(independent and identically distributed) random variables with common survival function $\bar{F}(x)$and let N be a geometric random variable independently distributed of $\{X_i\}$such that P[N = n] = $\theta(1-\theta)^{n-1}$, $n = 1,2,\ldots$ $0 < \theta < 1$, for all $i \geq 1$. Let U_N = $min(X_1, X_2, \ldots X_n)$. Then $\{U_N\}$ is distributed as MOq-e iff $\{X_i\}$follows q-e distribution .

Proof: The survival function of the random variable U_N is

$$\bar{H}(x) = P(U_N > x) = \theta \sum_{n=1}^{\infty} [\bar{F}(x)]^n (1-\theta)^{n-1} = \frac{\theta \overline{F(x)}}{1-(1-\theta)\overline{F(x)}})$$

If X_i has the survival function of the q-e distribution then U_N has the survival function of the MOq-e distribution. The converse easily follows from(5) that

$$\bar{F}(x) = [1-(1-q)(ux)]^{\frac{2-q}{1-q}}$$

Theorem 4.2 Let $\{X_i, i \geq 1\}$be a sequence of i.i.d. random variables with common survival function $\bar{F}(x)$and let N be a geometric random variable independently distributed of $\{X_i\}$ such that P[N = n] = $\theta(1-\theta)^{n-1}, n = 1,2,\ldots$ $0 < \theta < 1$, for all $i \geq 1$.. Let $V_N = max(X_1, X_2, \ldots X_n)$. Then $\{V_N\}$ is distributed as MOqe iff $\{X_i\}$ followsq-e distribution..

Proof: The distribution function of the random variable V_N is

$$M(x) = P(V_N < x) = \theta \sum_{n=1}^{\infty} [F(x)]^n (1-\theta)^{n-1}$$

$$= \frac{\theta F_X(x)}{1-(1-\theta)F_X(x)} \qquad (6)$$

$$\bar{M}(x) = 1 - M(x) = \frac{\frac{1}{\theta} F_X(x)}{1 - \left(1 - \frac{1}{\theta} F_X(x)\right)}$$

If X_i has the survival function of the q-e distribution then V_N has the survival function of the MOq-e distribution. The converse easily follows from (6) that

$$F(x) = 1 - \text{F1}(x) = 1\text{-}[1-(1-q)(ux)]^{\frac{2-q}{1-q}}$$

5. AR(1) MODELS WITH MOQ-EXPONENTIAL MARGINAL DISTRIBUTION

Two stationary Markov processes with similar structural forms which had found useful in hydrological applications was introduced by Tavares(1977,1980).The various aspects on first order auto regressive minification processes was discussed by Lewis and Mc Kenze(1991).

In this section we develop autoregressive minification processes of order one and order k with minification structures where MOq-e distribution is the stationary marginal distribution. We call the process as MOq-e AR(1) process. Now we have the following theorem.

Theorem 5.1 Consider an AR(1) structure given by

$$X_n = \begin{cases} \varepsilon_n, & w.p \quad p_1 \\ min(X_{n-1}, \varepsilon_n) & w.p \quad 1-p_1 \end{cases}$$

where w:p. denotes 'with probability', $0 < p_1 < 1$ and $\{\varepsilon_n\}$ is a sequence of i.i.d. random variables independently distributed of X_n. Then $\{X_n\}$ is a stationary Markovian AR(1) process with MOq-e marginal if and only if $\{\varepsilon_n\}$ is distributed as q-e distribution.

Proof: From the given structure it follows that

$$\bar{F}_{X_n}(x) = p_1 \bar{F}_{\epsilon_n}(x) + (1-p_1)\bar{F}_{X_{n-1}}(x)\bar{F}_{\epsilon_n}(x)$$

Under stationary equilibrium, it reduces to

$$F(x) = \frac{p_1 \bar{F}_{\varepsilon_n}(x)}{1-(1-p_1)\bar{F}_{\varepsilon_n}(x)} \qquad (7)$$

which is the MOq-e distribution. Conversely on substituting the survival function of the innovations ε_n, we get

$$\bar{F}_{\varepsilon_n}(x) = [1-(1-q)(ux)]^{\frac{2-q}{1-q}} \qquad (8)$$

which is the survival function of q-e distribution.

The following are the sample paths for the above structure.

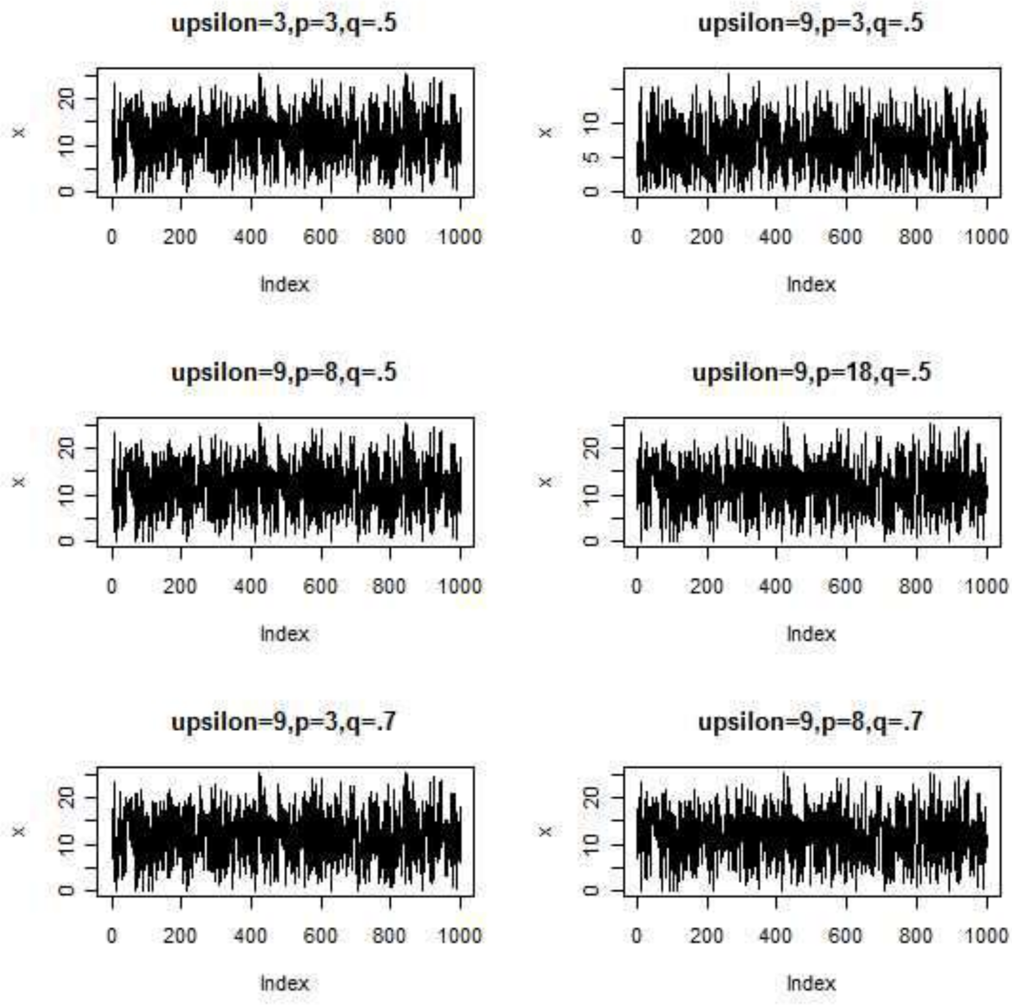

Figure3

Now we consider another AR(1) structure having three components.

Theorem 5.2 Consider an AR(1) structure given by

$$X_n = \begin{cases} X_{n-1}, & w.p. & p_2 \\ \varepsilon_n, & w.p. & p_1(1-p_2) \\ min(X_{n-1}, \varepsilon_n), & w.p. & (1-p_1)(1-p_2) \end{cases}$$

where $\{\varepsilon_n\}$ is a sequence of i.i.d. random variables independently distributed of X_n. Then $\{X_n\}$ is a stationary Markovian AR(1) process with MOq-e marginal if and only if $\{\varepsilon_n\}$ is distributed as
q-e distribution.

Proof: From the given structure it follows that

$$\bar{F}_{X_n}(x) = p_2\bar{F}_{X_{n-1}}(x) + p_1(1-p_2)\bar{F}_{\varepsilon_n}(x) + (1-p_1)(1-p_2)\bar{F}_{X_{n-1}}(x)\bar{F}_{\varepsilon_n}(x).$$

On simplification we get, the same expression as in equation (8) under stationarity. Then the result is obvious.
The following theorem generalizes the results to a k^th^ order autoregressive structure
.

Theorem 5.3 Consider an AR(k) structure given by

$$X_n = \begin{cases} \varepsilon_n, & w.p. & p_0 \\ min(X_{n-1}, \varepsilon_n), & w.p. & p_1 \\ min(X_{n-2}, \varepsilon_n), & w.p. & p_2 \\ \vdots & \vdots & \vdots \\ min(X_{n-k}, \varepsilon_n), & w.p. & p_k \end{cases}$$

where$\{\varepsilon_n\}$ is a sequence of i.i.d. random variables independently distributed of X_n, $0 < p_i < 1$, $p_1 + p_2 + \cdots + p_k = 1 - p_0$. Then the stationary marginal distribution of$\{X_n\}$ is MOq-e if and only if $\{\varepsilon_n\}$ is distributed as q-e distribution.

Proof: From the given structure the survival function is given as follows:

$$\bar{F}_{X_n}(x) = p_0\bar{F}_{\varepsilon_n}(x) + p_1\bar{F}_{X_{n-1}}(x)\bar{F}_{\varepsilon_n}(x) + \cdots + p_k\bar{F}_{X_{n-1}}(x)\bar{F}_{\varepsilon_n}(x).$$

Under stationary equilibrium, this yields

$$\bar{F}_X(x) = p_0\bar{F}_\varepsilon(x) + p_1\bar{F}_X(x)\bar{F}_\varepsilon(x) + \cdots + p_k\bar{F}_X(x)\bar{F}_\varepsilon(x).$$

This reduces to

$$\bar{F}_X(x) = \frac{p_0[1-(1-q)(ux)]^{\frac{2-q}{1-q}}}{1-(1-p_0)1-(1-q)(ux)^{\frac{q-2}{1-q}}}. \quad \text{for } q<1$$

The converse follows easily.
Then the theorem easily follows by similar arguments as in Theorem 4.2.

6. THE MAX-MIN AR(1) PROCESSES

Next we introduce a new model called the max-min process which incorporates both maximum and minimum values of the process. This has wide applications in atmospheric and oceanographic studies. The structure is given as follows.

Theorem 6.1 Consider an AR(1) structure given by

$$X_n = \begin{cases} max(X_{n-1}, \varepsilon_n), & w.p. \quad p_1 \\ min(X_{n-1}, \varepsilon_n), & w.p. \quad p_1 \\ X_{n-1}, & w.p. \quad 1-p_1-p_2 \end{cases}$$

subject to the conditions $0 < p_1, p_2 < 1, p_2 < p_1$ and $p_1 + p_2 < 1$, where$\{\varepsilon_n\}$ is a sequence of i.i.d. random variables independently distributed of X_n. Then$\{X_n\}$ is a stationary Markovian AR(1) max-min process with stationary marginal distribution $\bar{F}_X(x)$ if and only if $\{\varepsilon_n\}$ follows Marshall-Olkin distribution.

Proof: From the given structure it follows that

$$P(X_n > x) = p_1P(max(X_{n-1}, \varepsilon_n) > x) + p_2P(min(X_{n-1}, \varepsilon_n) > x)$$

$$+(1-p_1-p_2)P(X_{n-1} > x)$$

$$= p_1\left[1-\left(1-\bar{F}_{X_{n-1}}(x)\right)\left(1-\bar{F}_{\varepsilon_n}(x)\right)\right] + p_2\bar{F}_{X_{n-1}}(x)\bar{F}_{\varepsilon_n}(x)$$

$$+(1-p_1-p_2)\bar{F}_{X_{n-1}}(x).$$

Under stationary equilibrium, we get

$$\bar{F}_\varepsilon(x) = \frac{p_2\bar{F}_X(x)}{p_1 + (p_2-p_1)\bar{F}_X(x)} = \frac{p'\bar{F}_X(x)}{1-(1-p')\bar{F}_X(x)} \qquad (8)$$

Where $p' = \frac{p_2}{p_1}$. This has same functional form of Marshall-Olkin survival function. The converse can be proved by mathematical induction, assuming that $\bar{F}_{X_{n-1}}(x) = \bar{F}_X(x)$.

7. THE MAX-MIN PROCESS WITHQ-E MARGINAL DISTRIBUTION

To obtain the Q-Exponential max-min process, consider the above structure and substitute the survival function of QE xponential in equation (8). Then we get

$$\bar{F}_\varepsilon(x) = \frac{p'[1-(1-q)(ux)]^{\frac{2-q}{1-q}}}{1-(1-p')(1-(1-q)(ux)^{\frac{2-q}{1-q}}}, \qquad 0<q<1$$

which is the survival function of the Marshall-Olkin QExponential distribution where $p' = \frac{p_2}{p_1}$, $p_2 < p_1$ and $p_1 + p_2 < 1$.

Now consider a more general autoregressive structure which includes maximum, minimum as well as the innovations and the process values.

Theorem 7.1 Consider an AR(1) structure given by

$$X_n = \begin{cases} max(X_{n-1}, \varepsilon_n), & w.p \quad p_1 \\ min(X_{n-1}, \varepsilon_n), & w.p \quad p_2 \\ \varepsilon_n, & w.p \quad p_3 \\ X_{n-1}, & w.p \quad 1-p_1-p_2-p_3 \end{cases}$$

With the condition that $0 < p_1, p_2, p_3 < 1, p_2 < p_1$ *and* $0 < p_1 + p_2 + p_3 < 1$, where $\{\varepsilon_n\}$ is a sequence of i.i.d. random variables independently distributed of X_n. Then $\{X_n\}$ is a stationary Markovian AR(1) max-min process with stationary marginal distribution $\bar{F}_X(x)$ if and only if $\{\varepsilon_n\}$ follows Marshall-Olkin distribution.

Proof: From the given structure it follows that

$$P(X_n > x) = p_1 P(max(X_{n-1}, \varepsilon_n) > x) + p_2 P(min(X_{n-1}, \varepsilon_n) > x)$$

$$+p_3 P(\varepsilon_n > x) + (1-p_1-p_2-p_3)P(X_{n-1} > x).$$

This simplifies to

$$\bar{F}_{X_n}(x) = p_1\left[1-\left(1-\bar{F}_{X_{n-1}}(x)\right)\left(1-\bar{F}_{\varepsilon_n}(x)\right)\right] + p_2\bar{F}_{X_{n-1}}(x)\bar{F}_{\varepsilon_n}(x)$$
$$+p_3\bar{F}_{\varepsilon_n}(x) + (1-p_1-p_2-p_3)\bar{F}_{X_{n-1}}(x).$$

Under stationary equilibrium, this reduces to

$$\bar{F}_\varepsilon(x) = \frac{(p_2+p_3)\bar{F}_X(x)}{p_1+p_3+(p_2-p_1)\bar{F}_X(x)} = \frac{\beta\bar{F}_X(x)}{1-(1-\beta)\bar{F}_X(x)} \qquad (9)$$

where $= \frac{p_2+p_3}{p_1+p_3}$. This has the same functional form of the Marshall-Olkin survival function. The converse follows as before.

Remark: By substituting the survival function of the QE distribution in equation(9) we obtain

$$\bar{F}_\varepsilon(x) = \frac{\beta[1-(1-q)(ux)]^{\frac{2-q}{1-q}}}{1-(1-\beta)(1-(1-q)(ux)^{\frac{2-q}{1-q}}} , \qquad 0<q<1$$

,

The above model is a more generalized form having four components. Hence it can be used to model a variety of situations.

8. APPLICATIONS

The MOq-e distribution studied in this paper can be used for modeling data from various areas such as statistical mechanics, financial contexts, communications engineering, entropy studies etc. The max-min autoregressive processes can be used for modeling time series data from hydrological, financial and reliability contexts. They accommodate four components with respect to innovations, processes, minimum as well as maximum of the process values and offers wide flexibility in modeling real data sets

REFERENCES

1. Alice, T., and Jose, K.K. (2004). Bivariate semi-Pareto minification processes,Metrika 59,305-313.
2. Jayakumar, K., and Thomas, M. (2008). On a generalization of Marshall- Olkin scheme and its application to Burr type XII distribution, Statistical Papers,49, 421-439.
3. Jose, K.K., Krishna, E., and Ristic,M.M. (2014). On Record Values and Reliability Properties of Marshall-Olkin Extended Exponential Distribution, Journal of Applied Statistical Science, 21(1), 83-100.
4. Jose, K.K., Naik, S.R., and Ristic, M.M. (2010). Marshall-Olkin q-Weibul distribution and max-min processes, Statistical Papers, 51,4, 837-851.

5. .Jose,K.K., Ristic,M.M., and Ancy, J. (2011). Marshall-Olkin Bivariate Weibull distributions and processes, Statistical Papers, 52, 789-798.
6. .Jose, K.K., and Remya (2015). Negative binomial Marshall-OlkinRayleigh distribution and its applications, Economic. Quality Control,30,2,189-198.
7. Krishna, E., Jose, K.K., Alice,T. and Ristic, M.M. (2013 a). Marshall-Olkin Fréchet distribution, Communications in Statistics-Theory and Methods, 42, 4091-4107.
8. Krishna, E.,Jose, K.K. and Ristic, M.M. (2013 b). Applications of Marshall-Olkin Fréchet distribution,Communications in Statistics-Simulation and Computation, 42, 76-89.
9. Lewis, P. A. W. and McKenzie, E. (1991). Minification process and their transformations, Journal of Applied Probability Theory, 28, 45-57.
10. Marshall, A.W. ,and Olkin, I. (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families, Biometrica, 84, 641-652.
11. Picoli, S., Mendes, R.S., and Malacarne, L.C.(2003).q-exponential,Weibul, q-Weibul distributions: an empirical analysis ,Physica A, 324,3-4,678-688.
12. Sankaran, P.G., and Jayakumar, K. (2006). On proportional odds model. Statistical Papers,49, 779-789.
13. Seethalekshmi, V. and Jose, K.K.(2004). An autoregressive process with geometric a-Laplace marginals, Statistical Papers, 45,337-350.
14. Seethalekshmi, V., and Jose, K.K.(2006). Autoregressive processes with pakes andgeometric Pakes generalized Linnik marginals, Statistics and Probability Letters,76,318-326
15. Seetha Lekshmi, V. and Catherine, T. (2012 b). On q- Exponential Count Models, Journal of Applied Statistical Science,20,2.
16. Travers, L.V.(1977). The exact distribution of extremes of a non Gaussian process. Stochastic Process Appliations, 5,151-156.
17. Tavares, L.V.(1980). An exponential markovian stationary process, Journal Applied Probability, 17,1117-1120.
18. Tsallis, C. (1988). Possible generalisation of Boltzmann-Gibbs statisics,Journal of Statistical Physics, 52,1-2,479 -487.