International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 08 Issue 08 August 2020, Page no. - 2130-2136
Index Copernicus ICV: 57.55
DOI: 10.47191/ijmcr/v8i8.04

Bulk Arrival Markovian Queueing System with Two Types of Services and Multiple Vacations

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ARTICLE INFO

Published Online:
24 August 2020

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ABSTRACT

In this paper, we have a bulk arrival queueing Markovian model with two types of services first come first serve and bulk service. This model is also assumed the two servers such as main server and standby server with multiple vacations. If number of customers in the queue is less than ‘a’ then server will provide the FCFS service or goes for a vacation, if number of customers are more than or equal to ‘a’ then server will provide the bulk. After coming to the vacation, when the server will find number of customers are less than ‘a’ then he will provide the FCFS service to the customers. We have obtained the queue size distribution of this considered model and also obtained the performance measures such as idle time for the server, queue length, busy period by using the supplement variable technique.

KEYWORDS: *bulk queue, vacation, FCFS, batch service, performance measures, finite waiting capacity, SVT*

I. INTRODUCTION

There are most of the observations in queueing theory have been obtained by considering models where customers arrive individually and are served individually and also in some models it is frequently observed that the customers arrive in bulk and they receive the service in batches. These situations can be adequately modeled by bulk-arrival and bulk-service queues Bulk system have wide range of applications in areas such as transportation system, traffic, and telecommunication. Several authors worked on the bulk service queue with service time distribution depending on service batch size under a general bulk service rule. In this work, we study a bulk arrival queueing system with first come first serve and batch service. In this single arrival system: arrivals occur according to a Poisson process with rate λ. Services provided to the customers are either first come first serve or bulk according to the availability of customers.

In past years researchers have been developed important results in theory of queue with server vacations. Vacation models have been extensively used in various situations such as computer, communication and production system. In this work, we have determined some results for batch service models with server vacations. In this paper we analyze the M / G(a,b) /1 queue with multiple vacations discipline. There are two service patterns are describes; first one is customers are served by first come first serve rule and second one is Customers are served in batches according to the general bulk service rule in which, if there are less than ‘a’ customers in system then server will not provide the batch service. Server will provide the batch service to at least ‘a’ customers, at most ‘b’ customers. If server finds less than' a' customers in the system, he goes away for a vacation. After some time when server returns and finds less than' a' customers waiting then he provide first come first service.

Many researchers have worked on the Markovian queueing models with bulk arrival and batch services. Haridass and Arumuganathan (2008) analyzed the operating characteristics of an M X /G/1 queueing system with unreliable server and single vacation. The server is failed, while it is providing service, and the arrival rate of customers depends on the up and down states of the server. Failure time of server is exponentially distributed and the repair times follow general distribution.. The model is related to the embedded Markov chain technique and level crossing analysis. They have determined the expected number of customers in the system, expected length of busy period and probability generating function of the steady state system size at an arbitrary time. F. Nuts (1967) gave the theory of general bulk service rule first in which customer arrive according to the Poisson process and served in batches according to the general distribution with general bulk service rule. Choi and Han (1994) analyzed a G/Ma,b /1 queue with multiple vacations and bulk service rule. They determined the queue length probabilities at arrival time points and arbitrary time points by using the supplementary variable technique.

Krishnamoorthy and Ushakumari (2000) considered a Markovian queueing system with batch service. They have analyzed the system size probabilities in transient and steady states, waiting time distribution, busy period distribution, queue length by using the Little's formula. N. Bansal (2003) worked on the single server processor-sharing queue for the case of bulk arrivals then determine an expression for the expected response time of a job, with service times follows the generalized hyper-exponential distribution. Chang et. al. (2004) determined the steady-state departure-epoch probabilities by using the embedded Markov chain method. They also expressed various performance measures such as moments of the number of customers in the queue, loss probability, and the probability that server is busy. Haridass and Arumuganathan (2008) examined the operating characteristics of an M X /G/1 queueing system with unreliable server and single. Finally they gave the probability generating function of the steady state system equations, expected number of customers in the system, expected length of busy period and idle period. Chen et. al. (2010) worked on the modified Markovian bulk-arrival and bulk-service queue with state-dependent control and studied the equilibrium behaviour. They gave the results for the probability generating function of the equilibrium distribution, Queue length behavior. Ke et. al. (2010) have given a brief summary of the most recent research works on vacation queueing systems in the past 10 years. Parveen and Begum (2013) worked on a single server, bulk service queue with general arrival pattern and multiple working vacation. They derived the steady state Probability distribution at pre arrival epoch and arbitrary epoch, mean queue length. Tonui et. al. (2014) worked on the single –server Markovian queuing model with first come first server and infinite population by using sensitivity analysis. Kumar and Shinde (2018) focused on the bulk arrival and bulk service queueing model. Performance measure have been carried out viz average number of customers in the queue, average number of customers in the system, average waiting time of customers in queue, average waiting time of customers in the system, response time and efficiency of the server corresponding to customers.

This work is given in the following sections: In section 2, we have explained the Mathematical model. In Section 3, we have defined all used notations. In Section 4, we have explained the queue size distribution for the developed queueing model. In Section 5, we have defined the probability generating function (PGF) of the queue size distribution. In Section 6, the various performance measures for the queueing models are determined. Conclusion is given in Section 7.

II. METHODOLOGY

We have considered the queueing model whose arrival follows a Poisson process with rate λ. The main server and stand-by servers serve the customers under the general bulk service rule and first come first serve rule. The general bulk service rule defines that server will give service only when at least 'c' units are present in the queue, and server will provide the bulk service to maximum 'b' (b > a) customers. When main server will complete the batch service, and there are less 'c' customers are present in the queue, then the server will provide the first come first service. After completing the first come first service if there are less than 'a' customers in the queue then main server will goes for the vacation. During the vacation period of main server, if number of customers in queue is greater than 'a' and less then 'c' then standby server will provide the first come first service to customers. And if number of customers in queue is greater than 'c' and less then 'b' then standby server will provide the bulk service to customers again if number of customers in the queue is less then 'a' then standby server will be idle. In this case all customers receive service without waiting lot amount of time. The main server and standby server will provide the first come first service by the exponential distribution with rate μ and main server and standby server will provide the bulk service by the hyper-exponential distribution with rate ξ. The server can take multiple of vacations until the queue size reaches at least a. In addition, we assume that the service time of the main server and stand-by server and vacation time of the main server are independent of each other and follow a general (arbitrary) distribution.

III. NOTATIONS

$\mathcal{H}(x)/\hbar(x)$: Cumulative Distribution Function and Probability generating function of the FCFS Service by the main Server.

$\mathbb{C}(x)/\mathfrak{C}(x)$: Cumulative Distribution Function and Probability generating function of the batch Service by the main Server.

$\mathcal{L}(x)/\ell(x)$: Cumulative Distribution Function and Probability generating function of the remaining time of the vacation by main Server.

$\mathbb{Q}(x)/\sigma(x)$: Cumulative Distribution Function and Probability generating function denote the idle time spend in the system by the standby system.

$\wp(x)/p(x)$: Cumulative Distribution Function and Probability generating function of the FCFS Service by the standby Server.

$\Im(x)/\daleth(x)$: Cumulative Distribution Function and Probability generating function of the batch Service by the standby Server.

Let $\mathcal{E}(t)$ = {0, 1, 2, 3, 4, 5} denotes the FCFS service by main server, batch service by main server, remaining time of the vacation by main server And idle time; FCFS service and batch service by the standby server at time t respectively.

$N_s(t)$: Number of customers in the service station at time t

$N_q(t)$: Number of customers in the queue at time t.

IV. QUEUE SIZE DISTRIBUTION

$$\zeta_{ij}(x,t)\delta x = P_r\{N_q(t) = i, N_s(t) = 1, x \le \sigma(x) \le x+\delta x, \mathcal{E}(t) = 3\}, 1 < a-1, j \ge 0$$

$$\alpha_{ij}(x,t)\delta x = P_r\{N_s(t) = i, N_q(t) = j, x \le \hbar(x) \le x+\delta x, \mathcal{E}(t) = 0\}, a \le i \le c, j \ge 0$$

$$\beta_{ij}(x,t)\delta x = P_r\{N_s(t) = i, N_q(t) = j, x \le \complement(x) \le x+\delta x, \mathcal{E}(t) = 1\}, c \le i \le b, j \ge 0$$

$$\gamma_{ij}(x,t)\delta x = P_r\{N_s(t) = i, N_q(t) = j, x \le \ell(x) \le x+\delta x, \mathcal{E}(t) = 2\}, i \ge 0, j \ge 0$$

$$\eta(x,t)\delta x = P_r\{N_s(t) = i, N_q(t) = j, x \le p(x) \le x+\delta x, \mathcal{E}(t) = 4\}, a \le i \le c, j \ge 0$$

$$\xi(x,t)\delta x = P_r\{N_s(t) = i, N_q(t) = j, x \le \daleth(x) \le x+\delta x, \mathcal{E}(t) = 5\}, c \le i \le b, j \ge 0$$

Steady state equations are given as

$$-\frac{d}{dx}\zeta_{ij}(x) = -\lambda\zeta_{ij}(x) + \sigma(x)\gamma_{0,0}(0) \qquad \dots(1)$$

$$-\frac{d}{dx}\zeta_{ij}(x) = -\lambda\zeta_{ij}(x) + \sigma_i(x)\gamma_{0,0}(0) + \lambda\sum_{k=1}^{j}\zeta_{ij-a}(x)\psi_{\omega,} i < a-1, j > 1 \qquad \dots(2)$$

$$-\frac{d}{dx}\alpha_{ij}(x) = -\lambda\alpha_{ij}(x) + \sum_{m=a}^{c}\alpha_{mi}(0)\,\hbar(x) + \hbar(x)\gamma_{0,0}(0) + \hbar(x)\zeta_{0,0}(0), \ a \le i \le c, j < a-1 \qquad \dots(3)$$

$$-\frac{d}{dx}\alpha_{ij}(x) = -\lambda\alpha_{ij}(x) + \lambda\sum_{k=1}^{j}\alpha_{ij-k}(x)\psi_{\omega,} \qquad \dots(4)$$

$$-\frac{d}{dx}\alpha_{ij}(x) = -\lambda\alpha_{ij}(x) + \sum_{m=a}^{c}\alpha_{mc+j}(0)\,\hbar(x) + \lambda\sum_{k=1}^{j}\alpha_{ij-k}(x)\psi_{\omega,} + \hbar(x)\zeta_{c+j}(0), \ a \le i \le c, c \le j \le b \qquad \dots(5)$$

$$-\frac{d}{dx}\beta_{ij}(x) = -\lambda\beta_{ij}(x) + \sum_{n=c}^{b}\beta_{ni}(0)\,m(x) + \complement(x)\gamma_{0,0}(0) + \complement(x)\zeta_{0,0}(0) + \complement(x)\alpha_{0,0}(0), \ c \le i \le b, j < a-1 \qquad \dots(6)$$

$$-\frac{d}{dx}\beta_{ij}(x) = -\lambda\beta_{ij}(x) + \lambda\sum_{n=c}^{b}\beta_{i(j-k)}(x)\,\psi_{\omega,}\ c \le i \le b, c \le j \le b \qquad \dots(7)$$

$$-\frac{d}{dx}\beta_{ij}(x) = -\lambda\beta_{ij}(x) + \sum_{n=c}^{b}\beta_{nb+j}(0)\,\complement(x) + \complement(x)\zeta_{b+j} + \complement(x)\alpha_{b+j} + \complement(x)\gamma_{b+j}, c \le i \le b, \ a \le j \le c \qquad \dots(8)$$

$$-\frac{d}{dx}\eta_{ij}(x) = -\lambda\eta_{ij}(x) + p(x)\xi_{0,0}(0) + p(x)\zeta_{0,0}(0), \ a \le i \le c, j < a-1 \qquad \dots(9)$$

$$-\frac{d}{dx}\eta_{ij}(x) = -\lambda\eta_{ij}(x) + \lambda\sum_{n=c}^{b}\eta_{i(j-k)}(x)\,\psi_{\omega,}\ a \le i \le c, a \le j \le c \qquad \dots(10)$$

$$-\frac{d}{dx}\eta_{ij}(x) = -\lambda\eta_{ij}(x) + \sum_{m=a}^{c}\eta_{mc+j}(0)\,p(x) + p(x)\zeta_{c+j} + p(x)\xi_{c+j}, \ a \le i \le c, c \le j \le b \qquad \dots(11)$$

$$-\frac{d}{dx}\xi_{ij}(x) = -\lambda\xi_{ij}(x) + \daleth(x)\eta_{0,0}(0) + \daleth(x)\zeta_{0,0}(0), \ c \le i \le b, j < a-1 \qquad \dots(12)$$

$$-\frac{d}{dx}\xi_{ij}(x) = -\lambda\xi_{ij}(x) + \sum_{n=c}^{b}\xi_{i(j-k)}(x)\,\psi_{\omega,} \qquad \dots(13)$$

$$-\frac{d}{dx}\xi_{ij}(x) = -\lambda\xi_{ij}(x) + \sum_{n=c}^{b}\xi_{nb+j}(0)\,\daleth(x) + \daleth(x)\eta_{b+j} + \daleth(x)\zeta_{b+j}, c \le i \le b, a \le j \le b \qquad \dots(14)$$

Now by taking the Laplace Transform of the equations (1) – (14), we get

$$S\zeta_{ij}(s) - \zeta_{ij}(0) = \lambda\zeta_{ij}(s) - \sigma(s)\gamma_{0,0}(0) \qquad \dots(15)$$

$$S\zeta_{ij}(s) - \zeta_{ij}(0) = \lambda\zeta_{ij}(s) - \sigma_j(s)\gamma_{0,0}(0) - \lambda\sum_{k=1}^{j}\zeta_{ij-w}(s)\,\psi_w \qquad \dots(16)$$

$$S\alpha_{ij}(s) - \alpha_{ij}(0) = \lambda\alpha_{ij}(s) - \sum_{m=a}^{c}\alpha_{mi}(0)\,\hbar(0) - \hbar(s)\gamma_0(0) - \hbar(s)\zeta_0(0) \qquad \dots(17)$$

$$S\alpha_{ij}(s) - \alpha_{ij}(0) = \lambda\alpha_{ij}(s) - \lambda\sum_{k=1}^{j}\alpha_{i(j-k)}(s)\,\psi_w \qquad \dots(18)$$

$$S\alpha_{ij}(s) - \alpha_{ij}(0) = \lambda\alpha_{ij}(s) - \sum_{m=a}^{c}\alpha_{mc+j}(0)\,\hbar(s) - \lambda\sum_{k=1}^{j}\alpha_{i(j-k)}(s)\,\psi_w - \zeta_{c+j}(0)\hbar(s) \qquad \dots(19)$$

$$S\beta_{ij}(s) - \beta_{ij}(0) = \lambda\beta_{ij}(s) - \sum_{n=c}^{b}\beta_{nj}(0)\,\complement(s) - \complement(s)\gamma_{0,0}(0) - \complement(s)\zeta_0(0) - \complement(s)\alpha_{0,0} \quad \ldots(20)$$

$$S\beta_{ij}(s) - \beta_{ij}(0) = \lambda\beta_{ij}(s) - \sum_{n=c}^{b}\beta_{nb+j}(0)\,\complement(s) - \complement(s)\gamma_{b+j}(0) - \complement(s)\zeta_{b+j}(0) - \complement(s)\alpha_{b+j} \quad \ldots(21)$$

$$S\beta_{ij}(s) - \beta_{ij}(0) = \lambda\beta_{ij}(s) - \lambda\sum_{n=c}^{b}\beta_{i(j-k)}(0)\,\psi_w \quad \ldots(22)$$

$$S\eta_{ij}(s) - \eta_{ij}(0) = \lambda\eta_{ij}(s) - p(s)\xi_0(0) - \zeta_0 p(s) \quad \ldots(23)$$

$$S\eta_{ij}(s) - \eta_{ij}(0) = \lambda\eta_{ij}(s) - \sum_{m=a}^{c}\eta_{im(j-k)}(s)\,\psi_w \quad \ldots(24)$$

$$S\eta_{ij}(s) - \eta_{ij}(0) = \lambda\eta_{ij}(s) - \sum_{m=a}^{c}\eta_{mc+j}(0)\,p(s) - \zeta_{c+j}(0)p(s) - \xi_{c+j}(0)p(s) \quad \ldots(25)$$

$$S\xi_{ij}(s) - \xi_{ij}(0) = \lambda\xi_{ij}(s) - \complement(s)\eta_0(0) - \complement(s)T_0 \quad \ldots(26)$$

$$S\xi_{ij}(s) - \xi_{ij}(0) = \lambda\xi_{ij}(s) - \sum_{n=c}^{b}\xi_{nb+j}(0)\,\daleth(s) - \eta_{b+j}(0)\daleth(s) - \zeta_{b+j}(0)\daleth(s) \quad \ldots(27)$$

$$S\xi_{ij}(s) - \xi_{ij}(0) = \lambda\xi_{ij}(s) - \sum_{n=c}^{b}\xi_{ni(j-k)}(s)\,\psi_w \quad \ldots(28)$$

V. PROBABILITY GENERATING FUNCTION

In order to find the system size distribution, we have define the following probability generating functions (PGF)

$$\left.\begin{aligned}
\zeta_i(z,s) &= \textstyle\sum_{j=0}^{\infty}\zeta_{ij}(s)z^j \ \ and\ \ \zeta_i(z,0) = \sum_{j=0}^{\infty}\zeta_{ij}(0)z^j\\
\alpha_i(z,s) &= \textstyle\sum_{j=0}^{\infty}\alpha_{ij}(s)z^j \ \ and\ \ \alpha_i(z,0) = \sum_{j=0}^{\infty}\alpha_{ij}(0)z^j\\
\beta_i(z,s) &= \textstyle\sum_{j=0}^{\infty}\beta_{ij}(s)z^j \ \ and\ \ \beta_i(z,0) = \sum_{j=0}^{\infty}\beta_{ij}(0)z^j\\
\gamma_i(z,s) &= \textstyle\sum_{j=0}^{\infty}\gamma_{ij}(s)z^j \ \ and\ \ \gamma_i(z,0) = \sum_{j=0}^{\infty}\gamma_{ij}(0)z^j\\
\eta_i(z,s) &= \textstyle\sum_{j=0}^{\infty}\eta_{ij}(s)z^j \ \ and\ \ \eta_i(z,0) = \sum_{j=0}^{\infty}\eta_{ij}(0)z^j\\
\xi_i(z,s) &= \textstyle\sum_{j=0}^{\infty}\xi_{ij}(s)z^j \ \ and\ \ \xi_i(z,0) = \sum_{j=0}^{\infty}\xi_{ij}(0)z^j
\end{aligned}\right\} \quad \ldots(29)$$

Now by equation (15) and (16)

$$\sum_{j=0}^{\infty}\zeta_{ij}(s-\lambda) = \sum_{j=0}^{\infty}\zeta_{ij}(0) + \sigma(s)[\gamma_0(0)+\gamma_j(0)] - \lambda\sum_{k=1}^{j}\zeta_{1j-k}(s)\psi_w$$

$$\sum_{j=0}^{\infty}\zeta_{ij}(s-\lambda)z^j + \lambda\sum_{k=1}^{j}\zeta_{1j-k}(s)\psi_w\,z^j = \sum_{j=0}^{\infty}\zeta_{ij}(0)z^j + \sigma(s)[\gamma_0(0)+\gamma_j(0)]z^j$$

$$(s-\lambda+\lambda y(z))\zeta_i(z,s) = \zeta_i(z,0) - q(s)\gamma_i(z,0) \quad \ldots(30)$$

Again from (17), (18) and (19)

$$\sum_{j=0}^{\infty}\alpha_{ij}(s)(s-\lambda) + \lambda\sum_{k=1}^{j}\alpha_{i(j-k)}(s)\psi_w = \sum_{j=0}^{\infty}\alpha_{ij}(0) + \hbar(s)\left[-\sum_{m=a}^{c}\alpha_{mi(0)}\gamma_0(0) - \zeta_0(0) - \sum_{m=a}^{c}\alpha_{mc+j}(0) - \zeta_{c+j}(0)\right]$$

$$\alpha_i(z,s)(s-\lambda+\lambda\gamma(z)) = \alpha_i(z,0) + \hbar(s)\left(-\sum_{m=a}^{c}\alpha_{mi}(0) - \gamma_0(0) - \zeta_0(0) - \sum_{m=0}^{c}\alpha_{mc+j}(0) - \zeta_{c+j}(0)\right) \quad \ldots(31)$$

$$\beta_i(z,s)(s-\lambda+\lambda y(z)) = \beta_i(z,0) + \complement(s)\left[-\gamma_0 - \zeta_0 - \alpha_0 - \sum_{n=c}^{b}\beta_{nj}(0) - \sum_{n=c}^{b}\beta_{nb+j}(0) - \zeta_{b+j} - \alpha_{b+j} - \gamma_{b+j}\right] \quad \ldots(32)$$

$$\eta_i(z,s)(s-\lambda+\lambda y(z)) = \eta_i(z,0) + p(s)\left[-\xi_0 - \zeta_0 - \sum_{m=a}^{c}\eta_{nc+j}(0) - \zeta_{c+j} - \xi_{c+j}\right] \quad \ldots(33)$$

$$\xi_i(z,s)(s-\lambda+\lambda y(z)) = \xi_i(z,0) + \daleth(s)\left[-\eta_0 - \zeta_0 - \sum_{c=a}^{b}\xi_{nb+j}(0) - \zeta_{b+j} - \eta_{b+j}\right] \quad \ldots(34)$$

Now put $s = \lambda - \lambda y(z)$ in equation (30) then we get

$$T_i(z,0) = \sigma(s)\gamma_i(z,0) \quad \ldots(35)$$

Similarly from equation (31)

$$\alpha_i(z,0) = \hbar(s)\left(-\sum_{m=a}^{c}\alpha_{mj}(0) - \sum_{m=0}^{c}\alpha_{mc+j}(0) - \zeta_{c+j}(0) - \gamma_0(0) - \zeta_0(0)\right) \quad \ldots(36)$$

Now similarly from equation (32), we get

$$\beta_i(z,0) = \complement(s)\left[\sum_{n=c}^{b}\beta_{nj}(0) + \sum_{n=c}^{b}\beta_{nb+j}(0) + \zeta_{b+j}(0) + \alpha_{b+j} + \gamma_{b+j} + \gamma_0 + \zeta_0 + \alpha_0\right] \quad \ldots(37)$$

From equation (33), we get

$$\eta_i(z,0) = p(s)\left[\sum_{m=a}^{c}\eta_{mc+j}(0) + \zeta_{c+j}(0) + \xi_{c+j} + \zeta_0 + \xi_0\right] \quad \ldots(38)$$

From equation (34), we get

$$\xi_i(z,0) = \daleth(s)\left[\sum_{n=c}^{b}\xi_{nb+j}(0) + \eta_0 + \zeta_0 + \zeta_{b+j} + \eta_{b+j}\right] \quad \ldots(39)$$

Put the value of $\zeta_i(z,0), \alpha_i(z,0), \beta_i(z,0), \eta_i(z,0)$ and $\xi_i(z,0)$ into the general equations; then we get the following equations

$$\zeta_i(z,s) = \frac{[\sigma(\lambda-\lambda y(z)-\sigma(s)]\gamma_i(z,0)}{(s-\lambda+\lambda y(z))} \quad \ldots(40)$$

$$\alpha_i(z,s) = \frac{[\hbar(\lambda-\lambda y(z)-\hbar(s)]\left[\sum_{m=a}^{c}\alpha_{mj}(0)+\sum_{m=a}^{c}\alpha_{mc+j}(0)-\gamma_0(0)-\zeta_0(0)-\zeta_{c+j}(0)\right]}{(s-\lambda+\lambda y(z))} \quad \ldots(41)$$

$$\beta_i(z,s) = \frac{[\complement(\lambda-\lambda y(z)-\complement(s)]\left[\sum_{n=c}^{b}\beta_{nj}(0)+\sum_{n=c}^{b}\beta_{nb+j}(0)+\zeta_{b+j}(0)+\alpha_{b+j}(0)+\gamma_{b+j}(0)+\gamma_0-\zeta_0-\alpha_0\right]}{(s-\lambda+\lambda y(z))} \quad \ldots(42)$$

$$\eta_i(z,s) = \frac{[p(\lambda-\lambda y(z)-p(s)]\left[\sum_{m=a}^{c}\eta_{mc+j}(0)+\zeta_{c+j}(0)+\xi_{c+j}(0)+\xi_0-\zeta_0\right]}{(s-\lambda+\lambda y(z))} \quad \ldots(43)$$

$$\xi_i(z,s) = \frac{[\daleth(\lambda-\lambda y(z)-\daleth(s)]\left[\sum_{n=c}^{b}\xi_{nb+j}(0)+\zeta_{b+j}(0)+\eta_{b+j}(0)+\eta_0+\zeta_0\right]}{(s-\lambda+\lambda y(z))} \quad \ldots(44)$$

Probability generating function of queue size:

$$P(z) = \sum_{i=1}^{a-1}\gamma_i(z,0) + \sum_{i=a}^{c-1}(\alpha_i(z,0) + \eta_i(z,0)) + \sum_{i=c}^{b}(\beta_i(z,0) + \xi_i(z,0)) \quad \ldots(45)$$

This value can be obtained by putting $s = 0$

$$P(z) = \left[\begin{array}{c} \sum_{i=1}^{a-1}\left[[p(\lambda-\lambda y(z)]\left[\sum_{m=a}^{c}\eta_{mc+j}(0)+\zeta_{c+j}(0)+\xi_{c+j}(0)+\xi_0+\zeta_0\right]\right] \\ +\sum_{i=a}^{c-1}\left[\begin{array}{c}[\hbar(\lambda-\lambda y(z)]\left[\sum_{m=a}^{c}\alpha_{mj}(0)+\sum_{m=a}^{c}\alpha_{mc+j}(0)-\gamma_0(0)-\zeta_0(0)-\zeta_{c+j}(0)\right] \\ +[p(\lambda-\lambda y(z)]\left[\sum_{m=a}^{c}\eta_{mc+j}(0)+\zeta_{c+j}(0)+\xi_{c+j}(0)+\xi_0+\zeta_0\right]\end{array}\right] \\ \sum_{i=c}^{b}\left[\begin{array}{c}[\complement(\lambda-\lambda y(z))]\left[\sum_{n=c}^{b}\beta_{nj}(0)+\sum_{n=c}^{b}\beta_{nb+j}(0)+\zeta_{b+j}(0)+\alpha_{b+j}(0)+\gamma_{b+j}(0)+\gamma_0-\zeta_0-\alpha_0\right] \\ +[\daleth(\lambda-\lambda y(z)]\left[\sum_{n=c}^{b}\xi_{nb+j}(0)+\zeta_{b+j}(0)+\eta_{b+j}(0)+\eta_0+\zeta_0\right]\end{array}\right]\end{array}\right] \Big/ (\lambda y(z)-\lambda)$$

VI. PERFORMANCE MEASUREMENTS

1. Expected Queue Length:

Expected queue Length can be obtained by putting $lim_{z\to 1}$in above equation then we get

$$\lim_{z\to 1}P(z) = \left[\begin{array}{c}\sum_{i=1}^{a-1}\left[[p(\lambda-\lambda y_1)][\psi_j(0)]\right] + \sum_{i=a}^{c-1}\left[[\hbar(\lambda-\lambda y_1][\phi_{cj}(0)] + [p(\lambda-\lambda y_1][\epsilon_{cj}(0)]\right] \\ +\sum_{i=c}^{b}\left[[\complement(\lambda-\lambda y_1)][\omega_{bj}(0)] + [\daleth(\lambda-\lambda y_1)][\zeta_{bj}(0)]\right]\end{array}\right] \Big/ (\lambda y_1-\lambda)$$

2. Expected Length of idle period:

Let I be the Random variable of idle period. Let v be the random variable defined by

$$v = \begin{cases} 0; \text{if queue length is at least } 'a' \text{ after vacation} \\ 1; \text{if queue length is less then } 'a' \text{ after single service} \\ 2; \text{if queue length is single server less then } 'b' \end{cases}$$

$$E(I) = E'(I)P(u=0) + \big(E(v) + E(I)\big)P(u=1)E'(I)P(u=2)$$

Where $E'(I)$ is the Expected idle time of the standby server.

$E(v)$ is the Expected vacation time of the main server.

$E(I)$ is the Expected idle time of the main server.

$E'(I)$ can be evaluated by equation (35)

$$\zeta_i(z,0) = \sigma(s)\gamma_j(z,0)\ for\ i < a-1$$

$$P(u=0) = \sum_{i=a}^{c-1} \alpha_i(z,0) + \sum_{i=c}^{b} \beta_i(z,0)$$

$$E'(I) = \frac{E(v)}{P(u=0)} + \frac{E(I)}{P(u=2)}$$

$$P(u=2) = \sum_{i=1}^{a-1} \gamma_i(z,0) + \sum_{i=a}^{c-1} \alpha_i(z,0) + \sum_{i=c}^{b-1} \beta_i(z,0)$$

Then from the above equation we have

$$E'(I) = \frac{E(v)}{\sum_{i=a}^{c-1} \alpha_i(z,0) + \sum_{i=c}^{b} \beta_i(z,0)} + \frac{E(I)}{\sum_{i=1}^{a-1} \gamma_i(z,0) + \sum_{i=a}^{c-1} \alpha_i(z,0) + \sum_{i=c}^{b-1} \beta_i(z,0)}$$

3. Expected Length of busy period:

Let B be the random variable of busy period. Let τ be the random variable which is defined by

$$\tau = \begin{cases} 0; if\ queue\ length\ is\ less\ than\ a\ after\ single\ vacation \\ A; if\ queue\ length\ is\ at\ least\ a\ after\ the\ batch\ service \\ B; if\ queue\ length\ is\ less\ then\ c\ after\ the\ batch\ service \\ C; if\ the\ queue\ length\ is\ greater\ than\ b\ after\ vacation \end{cases}$$

$$E(B) = \frac{E(s)}{P(J=0)} = \frac{E(s)}{\sum_{i=1}^{a-1} \gamma_i(z,0)}$$

Where $E(s)$ is the mean service time.

4. Probability that the server is on vacation, single server and batch service:

$$P(\gamma) = E(v) \sum_{i=1}^{a-1} \gamma_i(z,0)$$

$P(B)$ can be given as from the above equation

$$P(B) = \sum_{i=0}^{c-1} \alpha_i(z,0) + \sum_{i=c}^{b} \beta_i(z,0)$$

VII. CONCLUSION

In this paper, bulk arrival Markovian queueing systems with two types of service pattern, first come first serve and bulk service with multiple vacations is considered. We have used the probability generating function technique to determine the system size distribution and various performance measures such as idle time, waiting time, expected length of busy period, expected queue length of the given queueing queue size distribution at an arbitrary time is obtained. The advantage of this work is to determine the important terms related to the first come first serve service and bulk service which gives complete satisfaction to all the arriving customers. The essential part of standby server and main sever has been clearly all around characterized in this model which defines an important term in the greater part of the lining framework categories. This shows a dormant helpful daily life application in supermarket, railway ticket counter and call centers. The main aim is to reduce the customer's waiting time and improve the customer's satisfaction rate.

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