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Improved General Class of Ratio Type Estimators

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Abstract

In survey sampling, the utilization of auxiliary information is very helpful in designing the estimators that can estimate the population mean with greater degree of precision. In this paper, general class of improved ratio type estimator (may be biased or unbiased) for estimation of population mean have been proposed. The relative bias and relative mean squared error of the proposed estimators have been worked out upto order O (n^{-1}) and O (n^{-2}) respectively. The efficiencies of the proposed estimators have been compared with the conventional ratio estimator and the estimator proposed by Sharma *et al.,* (2010) and were found more efficient. Empirical results also showed that the proposed estimators are more efficient than the conventional ratio estimator and the estimator proposed by Sharma *et al.*, (2010).

Keywords: *Auxiliary information, ratio type estimator, relative mean squared error, relative bias and simulation*

1. Introduction

It is well known fact that if the auxiliary information is utilized at the estimation stage, the precision of estimates of the population mean of study variable under study can be efficiently increased. If the study variable Y is positively correlated with auxiliary variable X (Cochran, 1940) and the auxiliary variate satisfy the condition (i) if $C_x/2C_y < \rho \le +1$ and both Y and X are positive or negative (ii) if $-C_x/2C_y < \rho \le +1$ and either Y or X is negative (Singh and Chaudhary, 1995), ratio method can be employed. Rao (1966), Sahoo and Swain (1989), Pandey and Dubey (1989), Singh and Narain (1989) considered almost unbiased ratio estimators. Naik and Gupta (1991) proposed a general class of estimators for estimating the populations mean using auxiliary information. Several modifications have been made in the conventional ratio estimator to achieve higher precision. The main contributions available in the literature in this regard has been made by Sukhatme (1954), Cochran (1977), Hartley and Ross (1954), Beale (1962), Tin (1965), Chakrabarty (1979), Birader and Singh (1995), Sharma *et al.*, (2010) etc.

2. Methodology

Consider a population of size N and a random sample of size n is drawn from it for both auxiliary variable X and study variables Y. Further, the sample means $\bar{x}$ and $\bar{y}$ are unbiased estimators of population means $\bar{X}$ and $\bar{Y}$ respectively while s_x^2 and s_y^2 are unbiased estimators of population variances σ_x^2 and σ_y^2 respectively. Similarly, let s_{xy} be an unbiased estimator of population covariance σ_{xy}. There are several techniques to evaluate the moments and cross moments of U_x, U_y, V_x, V_y and W

where, $U_x = \frac{\bar{x}-\bar{X}}{\bar{X}}$, $U_y = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $V_x = \frac{S_x^2-\sigma_x^2}{\bar{X}^2}$, $V_y = \frac{S_y^2-\sigma_y^2}{\bar{Y}^2}$

and $W = \frac{S_{xy}-\sigma_{xy}}{\bar{X}Y}$

(Kendall and Stuart, 1952; Sukhatme and Sukhatme, 1970; Bhatnagar, 1981). Further,

$C_{ab}= \frac{1}{N-1}\sum^{N}\left(\frac{x_i-\bar{X}}{\bar{X}}\right)^a\left(\frac{y_i-\bar{Y}}{\bar{Y}}\right)^b$ where, a and b are non – negative integers. E (T), RB (T) and RM (T) denote expected value, relative bias and relative mean squared error of an estimator T, respectively. Also,

$$\theta = \left(\frac{C_{02}}{C_{20}}\right)^{1/2} \quad \text{and} \quad \rho = \frac{C_{11}}{(C_{02})^{1/2}(C_{20})^{1/2}} .$$

3. Existing Estimators

The following ratio type estimator has proposed by Cochran (1940) for estimating the population mean

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X} \quad (1)$$

where, $\bar{y}$ and $\bar{x}$ are unbiased estimators of $\bar{Y}$ and $\bar{X}$, the population means of the characteristics under study and auxiliary characteristics respectively. The relative bias and relative mean squared error of the conventional ratio type estimator is as

$$RB(\bar{y}_r) = \frac{1}{n}(C_{20} - C_{11}) \quad (2)$$

$$RM(\bar{y}_r) = \frac{1}{n}(C_{02} - 2C_{11} + C_{20}) + \frac{1}{n^2}[(2(2C_{21} - C_{12} - C_{30}) + 3(3C_{20}^2 - 6C_{20}C_{11} + 2C_{11}^2 + C_{20}C_{02})]. \quad (3)$$

Sharma *et al.*, (2010) proposed the following general class of ratio type estimators as

$$t_s = \bar{y}_r + \frac{1}{n}\bar{y}_p\left[p\frac{s_{\bar{y}}^2}{\bar{y}^2} + q\frac{s_{xy}}{\bar{x}\bar{y}}\right] \quad (4)$$

The relative bias and relative mean squared error of t_s upto order O (n^{-1}) and O (n^{-2}) are as

$$RB(t_s)=RB(\bar{y}_r) + \frac{1}{n}(pC_{02} + qC_{11}) \quad (5)$$

$$RM(t_s) = RM(\bar{y}_r) + \frac{p}{n^2}[2C_{03} - 2C_{12} + 2C_{02}C_{11} - 2C_{02}^2] + \frac{q}{n^2}[2C_{20}C_{11} - 2C_{11}^2 + C_{12} - C_{21}] + \frac{1}{n^2}[pC_{02} + qC_{11}]^2 \quad (6)$$

4. Proposed Class Of Ratio Type Estimators

The following general improved class of ratio type estimators for $\bar{Y}$ have been proposed as

$t^* = \bar{y}_r + \frac{p}{n}\bar{y}\left[\frac{\bar{x}s_{\bar{x}}^2}{\bar{X}^3} + q\frac{s_{xy}}{\bar{X}\bar{y}}\right]$, where, p and q are scalars and specifying the estimators. (7)

The relative bias and relative mean squared error of the estimator t^* upto order O (n^{-1}) and O (n^{-2}) respectively, as

$$RB(t^*) = RB(\bar{y}_r) + \frac{p}{n}(C_{20} + qC_{11}), \quad (8)$$

$$RM(t^*) = RM(\bar{y}_r) + \frac{p}{n^2}[2C_{21} + 2C_{02}C_{20} - 2C_{20}C_{11} - 2C_{30}] + \frac{pq}{n^2}[2C_{20}C_{11} - 2C_{11}^2 + 2C_{12} - 2C_{21}] + \frac{1}{n^2}[pC_{20} + pqC_{11}]^2. \quad (9)$$

5. Comparative study of the proposed estimator with respect to existing ratio estimators

From (8) and (2), it can be observed that the estimator t^* has smaller bias than conventional ratio type estimator $\bar{y}_r$, if $p < 0$ and $C_{20} + qC_{11} > 0$.

From (9) and (3), it is seen that the estimator t^* and $\bar{y}_r$ have identical relative mean squared error upto order O (n^{-1}). They differ in terms of O (n^{-2}). The comparison of relative mean squared error of t^* and $\bar{y}_r$, upto order O (n^{-2}) showed that estimator t^* will be more efficient than $\bar{y}_r$, if

$$p[2C_{21}+2C_{02}C_{20}-2C_{20}C_{11}-2C_{30}]+pq[2C_{20}C_{11}-2C_{11}^2+2C_{12}-2C_{21}]+p^2[C_{20}+qC_{11}]^2<0. \quad (10)$$

Under bivariate normal distribution, expression (10) becomes

$$p[2C_{02}C_{20}-2C_{20}C_{11}]+pq[2C_{20}C_{11}-2C_{11}^2]+p^2[C_{20}+qC_{11}]^2<0. \quad (11)$$

The expressions (8) and (5) showed that estimator t^* has smaller bias than estimator t_s , if $pC_{20}+pqC_{11}<pC_{02}+qC_{11}$.

From (9) and (6), it is seen that the estimator t^* and t_s have identical relative mean squared error upto order O (n^{-1}). They differ in terms of O (n^{-2}). Further, the comparison of relative mean squared error of t^* and t_s showed that t^* is more efficient than t_s , if

$$p[2C_{02}C_{20}-2C_{20}C_{11}-2C_{30}+2C_{21}-C_{03}^2+2C_{12}-2C_{02}C_{11}+C_{02}^2]+q[2C_{20}C_{11}-2C_{11}^2+2C_{12}-2C_{21}]+pq[2C_{11}^2-2C_{20}C_{11}-C_{12}+C_{21}]+p^2[C_{20}+qC_{11}]^2-[pC_{02}+qC_{11}]^2<0. \quad (12)$$

Under bivariate normal distribution, expression (12) reduces to

$$p[2C_{02}C_{20}-2C_{20}C_{11}-2C_{02}C_{11}+C_{02}^2]+q[2C_{20}C_{11}-2C_{11}^2]+pq[2C_{11}^2-2C_{20}C_{11}]+p^2[C_{20}+qC_{11}]^2-[pC_{02}+qC_{11}]^2<0. \quad (13)$$

The special cases have been considered under different values of scalars p and q as

Case I: If $p=0$, the proposed ratio type estimator is similar to $\bar{y}_r$. Thus, $\bar{y}_r$ is a particular member of the proposed class of ratio type estimators.

Case II: Consider q=0 in (7), the estimator t^* reduces to

$$t^*_{(p,0)}=\bar{y}_r+\frac{p}{n}\bar{y}\frac{\bar{x}s_x^2}{\bar{X}^3}. \quad (14)$$

The relative bias and relative mean squared error of the estimator t^* upto order O (n^{-1}) and O (n^{-2}) respectively, as

$$RB\left(t^*_{(p,0)}\right)=RB(\bar{y}_r)+\frac{p}{n}C_{20}, \quad (15)$$

$$RM\left(t^*_{(p,0)}\right)=RM(\bar{y}_r)+\frac{p}{n^2}[2C_{21}+2C_{02}C_{20}-2C_{20}C_{11}-2C_{30}+pC_{20}^2]. \quad (16)$$

From (15) and (2), it is observed that the relative bias of $t^*_{(p,0)}$ is smaller than that of estimator $\bar{y}_r$ upto order O (n^{-1}), if $p<0$.

From (16) and (3), the relative mean squared error of the estimator $\bar{y}_r$ is identical to the estimator $t^*_{(p,0)}$ upto order O (n^{-1}). They differ in terms of order O (n^{-2}).

Further, it is found that estimator $t^*_{(p,0)}$ is more efficient than $\bar{y}_r$ upto order O (n^{-2}), if

$$p[2C_{21}+2C_{02}C_{20}-2C_{20}C_{11}-2C_{30}+pC_{20}^2]<0. \quad (17)$$

Under bivariate normal distribution, expression (17) reduces to

$$p[2C_{02}C_{20}-2C_{20}C_{11}+pC_{20}^2]<0. \quad (18)$$

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Thus, the estimator ${t^*}_{(p,0)}$ will be more efficient than $\bar{y}_r$, if $p < 0$ and $\rho < \frac{2\theta^2-1}{2\theta}$ which results to θ lies between $0.710 < \theta < 1.366$.

It has been found that the estimator ${t^*}_{(p,0)}$ have smaller bias than $t_{s(p,0)}$, if $p < 0$ and $C_{20} > C_{02}$.

The comparison of the estimators ${t^*}_{(p,0)}$ and $t_{s(p,0)}$ showed that both are equally efficient upto order O (n^{-1}) but for order O (n^{-2}), estimator ${t^*}_{(p,0)}$ is more efficient than $t_{s(p,0)}$, if

$$p[2C_{21} + 2C_{02}C_{20} - 2C_{20}C_{11} - 2C_{30} + pC_{20}^2 - 2C_{03}^2 + 2C_{12} - 2C_{02}C_{11} + 2C_{02}^2 - pC_{02}^2] < 0. \tag{19}$$

For bivariate normal population, expression (19) reduces to

$$p[2C_{02}C_{20} - 2C_{20}C_{11} + pC_{20}^2 - 2C_{02}C_{11} + 2C_{02}^2 - pC_{02}^2] < 0. \tag{20}$$

The estimator ${t^*}_{(p,0)}$ will perform better than $t_{s(p,0)}$, if $p < 0$ and $0.581 < \theta < 1.00$.
Thus, the proposed class will be efficient than the existing $\bar{y}_r$ and $t_{s(p,0)}$ for $p < 0$ and $0.710 < \theta < 1.00$.
Case III: For $p < 0$ and $q < 0$, consider the scalars p and q as p= -1 and q= -1, the estimator t^* will reduce to ${t^*}_{(-1,-1)} = \bar{y}_r + \frac{1}{n}\bar{y}\left[\frac{s_{xy}}{\bar{X}\bar{y}} - \frac{\bar{x}s_x^2}{\bar{X}^3}\right],$ (21)

The bias of the proposed estimator is equal to zero. Thus, the estimator is unbiased and the relative variance of the estimator ${t^*}_{(-1,-1)}$ will be as

$$RV\left({t^*}_{(-1,-1)}\right) = RM(\bar{y}_r) - 2C_{02}C_{20} + 2C_{20}C_{11} - C_{11}^2 + C_{20}^2 + 2C_{30} - 4C_{12} + 2C_{21}. \tag{22}$$

Under bivariate normal population, the above expression (22) reduces to

$$RV\left({t^*}_{(-1,-1)}\right) = RM(\bar{y}_r) - 2C_{02}C_{20} + 2C_{20}C_{11} - C_{11}^2 + C_{20}^2. \tag{23}$$

From (22) and (3), it can be seen that the estimator ${t^*}_{(-1,-1)}$ is more efficient than $\bar{y}_r$, if $\rho < \frac{3\theta^2-1}{2\theta}$ and $0.578 < \theta < 1.000$.

Further, the estimator ${t^*}_{(-1,-1)}$ and $t_{s(-1,-1)}$ have identical relative mean squared error upto order O (n^{-1}). They differ in terms of order O (n^{-2}). The estimator ${t^*}_{(-1,-1)}$ will be more efficient than $t_{s(-1,-1)}$, if

$$4C_{20}C_{11} - 2C_{20}C_{02} - 4C_{11}^2 + C_{20}^2 - 3C_{02}^2 + 2C_{30} + 2C_{03} - 4C_{12} < 0. \tag{24}$$

The expression (24) under bivariate normal population reduces to

$$4C_{20}C_{11} - 2C_{20}C_{02} - 4C_{11}^2 + C_{20}^2 < 0. \tag{25}$$

The estimator ${t^*}_{(-1,-1)}$ will perform better than $t_{s(-1,-1)}$, if

$\rho > \frac{3\theta^4+6\theta^2-1}{4\theta}$ and the value of θ lies between $0.394 < \theta < 0.715$.

Thus, the proposed ratio type estimator ${t^*}_{(-1,-1)}$ is unbiased and efficient than $\bar{y}_r$ and $t_{s(-1,-1)}$,

if $0.578 < \theta < 0.715$.

Case IV: For $p < 0$ and $q > 0$, consider p=-1 and q=1, the estimator t^* will become

$${t^*}_{(-1,1)} = \bar{y}_r - \frac{1}{n}\bar{y}\left[\frac{\bar{x}s_x^2}{\bar{X}^3} + \frac{s_{xy}}{\bar{X}\bar{y}}\right]. \tag{26}$$

The relative bias and relative mean squared error of the estimator $t^*_{(-1,1)}$ are as

$$RB\left(t^*_{(-1,1)}\right) = RB(\bar{y}_r) - \frac{1}{n}(C_{20} + C_{11}), \quad (27)$$

$$RM\left(t^*_{(-1,1)}\right) = RM(\bar{y}_r) + \frac{1}{n^2}[3C_{11}^2 + C_{20}^2 + 2C_{20}C_{11} - 2C_{20}C_{02} + 2C_{30} - 2C_{12}]. \quad (28)$$

From (27) and (2), it is observed that the relative bias of $t^*_{(-1,1)}$ is smaller than that of estimator $\bar{y}_r$ upto order O (n^{-1}), if $C_{20} > -C_{11}$.

From (28) and (3), the relative mean squared error of the estimator $\bar{y}_r$ is identical to the estimator $t^*_{(-1,1)}$ upto order O (n^{-1}). They differ in terms of order O (n^{-2}). Further, it is found that estimator $t^*_{(-1,1)}$ is more efficient than $\bar{y}_r$, if

$$3C_{11}^2 + C_{20}^2 + 2C_{20}C_{11} - 2C_{20}C_{02} + 2C_{30} - 2C_{12} < 0. \quad (29)$$

For bivariate normal distribution, the expression (29) becomes

$$3C_{11}^2 + C_{20}^2 + 2C_{20}C_{11} - 2C_{20}C_{02} < 0. \quad (30)$$

which gives $\theta^2 + 1 > 2\rho\theta$ which is true for $\theta > 0$.

It is observed that that estimator $t^*_{(-1,1)}$ have smaller relative bias upto order O (n^{-1}) than $t_{s(-1,1)}$, if $C_{20} + 2C_{11} > C_{02}$. The comparison of the estimator showed that $t^*_{(-1,1)}$ and $t_{s(-1,1)}$ have identical relative mean squared error upto order O (n^{-1}). They differ in terms of order O (n^{-2}). Further, it is found that estimator $t^*_{(-1,1)}$ is more efficient than $t_{s(-1,1)}$, if

$$4C_{11}^2 + C_{20}^2 - 3C_{02}^2 - 2C_{20}C_{02} + 4C_{02}C_{11} - 2C_{30} + 2C_{03} - 6C_{12} + C_{21} < 0 \quad (31)$$

For bivariate normal population, the expression (31) gives

$$\rho > \frac{3\theta^4 - 2\theta^2 - 1}{4\theta^3} \text{ and } \quad 1.001 < \theta < 1.770.$$

Thus, the estimator $t^*_{(-1,1)}$ will be more efficient than $t_{s(-1,1)}$, if $\theta > 0$.

6. EMPIRICAL STUDY:

Improved ratio types of estimators of population mean have been theoretically developed. Their efficiencies have been tested by generating two population datasets P_1 and P_2 through simulation using SAS software Different types of estimators have been developed by using the values of p and q with respect to different cases. The estimators holding properties of unbiasedness, most efficient and consistency have been proposed.

Table 1: Descriptive statistics of the variable under study and auxiliary variable for P_1 and P_2

	P_1		P_2	
Variable	Population mean	Population variance	Population mean	Population variance
Y	21.943	229.667	11.923	133.327
X	10.432	39.358	0.503	0.243
ρ_{XY}	0.853		0.918	

From table 1, it has been observed that the correlation was 0.853, the mean and variance of the study variable Y were 21.943 and 229.667; of auxiliary variable X were 10.432 and 39.358 respectively for P_1. In case of P_2, the correlation between the two variables (Y and X) was found to be 0.918 whereas the mean and variance of the study variable Y were 11.923 and 133.327 and of auxiliary variable X were 0.503 and 0.243 respectively.

Table 2: Relative bias of the proposed product estimator $t^*_{(-3,1)}$ and $t^*_{(-1,-1)}$ with respect to conventional ratio estimator and estimator proposed by Sharma *et al.*, (2010) for P_1 and P_2.

Estimator	P_1			P_2		
	30	60	120	30	60	120
$\bar{y}_r$	0.01016	0.00681	0.00252	0.05566	0.01257	0.01016
$t_{s\,(-3,1)}$	-0.04515	-0.02356	-0.00884	-0.36366	-0.03532	-0.01659
$t_{s\,(-1,-1)}$	0.02033	0.01362	0.00504	-0.05430	-0.00503	0.00319
$t^*_{(-3,1)}$	-0.02131	-0.01499	-0.00648	-0.02281	-0.03250	-0.01158
$t^*_{(-1,-1)}$	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

From the table 2, it has been observed that for both populations the proposed estimator $t^*_{(-3,1)}$ was negatively biased and was observed to be 0.02131, 0.01499 and 0.00648 at sample sizes 30, 60 and 120 for P_1 whereas in case of P_2, it was 0.02281, 0.03250 and 0.01158 at sample sizes 30, 60 and 120 respectively. In both populations biases were found to be decreasing as the sample size increases. Further, the bais of the proposed estimator $t^*_{(-1,-1)}$ was found to be zero in both the populations, hence it is unbiased in nature.

Table 3:Relative mean squared error / relative variance of the proposed product estimator $t^*_{(-3,1)}$ and $t^*_{(-1,-1)}$ with respect to conventional ratio estimator and estimator proposed by Sharma *et al.*, (2010) for P_1 and P_2

Estimator	P_1			P_2		
	30	60	120	30	60	120
$\bar{y}_r$	0.02970	0.01738	0.00623	0.24678	0.02983	0.02134
$t_{s(-3,1)}$	0.03578	0.01907	0.00645	0.51606	0.03406	0.02269
$t_{s(-1,-1)}$	0.03130	0.01783	0.00629	0.28649	0.03096	0.02149
$t^*_{(-3,1)}$	0.02902	0.01706	0.00622	0.22523	0.02977	0.02125
$t^*_{(-1,-1)}$	0.02969	0.01732	0.00622	0.24026	0.02958	0.02110
Percent relative efficiencies of proposed estimators						
$t^*_{(-3,1)}$ w.r.t. $\bar{y}_r$	(102.34)	(101.85)	(100.22)	(109.57)	(100.18)	(100.43)
$t^*_{(-3,1)}$ w.r.t. $t_{s(-3,1)}$	(123.31)	(111.80)	(103.64)	(229.13)	(114.40)	(106.78)
$t^*_{(-1,-1)}$ w.r.t. $\bar{y}_r$	(100.04)	(100.30)	(100.23)	(102.71)	(100.83)	(101.15)
$t^*_{(-1,-1)}$ w.r.t. $t_{s(-1,-1)}$	(105.29)	(102.91)	(101.06)	(119.24)	(104.66)	(101.87)

*() values in the parenthesis indicate the percent relative efficiency of the proposed ratio estimator w.r.t. to different ratio estimators.

Table 3 revealed that in case of population P_1, that the proposed estimator $t^*_{(-3,1)}$ have smaller relative mean squared error than the conventional ratio estimator and the estimator proposed by Sharma *et al.*, (2010). The value of the relative mean squared error of the proposed estimator $t^*_{(-3,1)}$ at samples of sizes 30, 60 and 120 were 0.02902, 0.01706 and 0.00622 respectively. The percent relative efficiency of the proposed estimator $t^*_{(-3,1)}$ was found to lie between 100.221 to 102.339 with respect to conventional ratio estimator and with respect to $t_{s(-3,1)}$, it was found to lie between 103.64 to 123.31. For the unbiased proposed estimator ${t^*}_{(-1,-1)}$, the values of relative variance were 0.02970, 0.01732 and 0.00622 at sample of sizes 30, 60 and 120 respectively. The percent relative efficiency of the proposed estimator ${t^*}_{(-1,-1)}$ was found to lie between 100.000 to 101.296 with respect to conventional ratio estimator and with respect to $t_{s(-1,-1)}$, it was lying between 101.06 to 105.29.

Moreover, in case of population P_2, the proposed estimator ${t^*}_{(-3,1)}$ have smaller relative mean squared error than the conventional ratio estimator and the estimator proposed by Sharma *et al.*, (2010). Further, the value of the relative mean squared error of the proposed estimator ${t^*}_{(-3,1)}$ at samples of sizes 30, 60 and 120 were 0.22523, 0.02977 and 0.02125 respectively. The percent relative efficiency of the proposed estimator ${t^*}_{(-3,1)}$ was found to lie between 100.426 to 109.569 with respect to conventional ratio estimator and was lying between 106.78 to 229.13 with respect to $t_{s(-3,1)}$. For the unbiased proposed estimator ${t^*}_{(-1,-1)}$, the values of relative variance were 0.24026, 0.02958 and 0.02110 at samples of sizes 30, 60 and 120 respectively. The percent relative efficiency of the proposed estimator ${t^*}_{(-1,-1)}$ was found to lie between 100.827 to 102.714 with respect to conventional ratio estimator and lying between 101.87 to 119.24 with respect to $t_{s(-1,-1)}$.

7. SUMMARY AND CONCLUSIONS

In this paper, an improved general class of ratio type estimator (may be biased or unbiased) for estimation of population mean have been developed by making use of the auxiliary information using simple random sampling scheme. Their relative biases and relative mean squared errors have been theoretically as well as empirically examined. The empirical results through simulation have shown that the proposed estimator $\boldsymbol{t}^*_{(-3,1)} = \bar{\boldsymbol{y}}_r - \frac{1}{n}\bar{\boldsymbol{y}}_p\left[\frac{s_x^2}{\bar{X}^2} + \frac{s_{xy}}{\overline{xy}}\right]$ is biased and more efficient than the conventional ratio type estimator and the estimator proposed by Sharma *et al.*, (2010) on the basis of percent relative efficiency (PRE) whereas the proposed estimator $\boldsymbol{t}^*_{(-1,-1)} = \bar{\boldsymbol{y}}_r - \frac{1}{n}\bar{\boldsymbol{y}}_p\left[\frac{s_x^2}{\bar{X}^2} - \frac{s_{xy}}{\overline{xy}}\right]$ was unbiased and more efficient than the conventional ratio type estimator and the estimator proposed by Sharma *et al.*, (2010).

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