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Fuzzy Laplace Transform With Fuzzy Fractional Differential Equation

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ABSTRACT

This paper deals with fuzzy Laplace transform to obtain the solution of fuzzy fractional differential equation (FFDEs) under Riemann Liouville H-differentiability .This is in contrast to conventional solution that either require a quantity of fractional derivative of unknown solution at the initial point(Riemann Liouville) or a solution with increasing length of their support (Hukuhara),using the fuzzy Laplace transform to solve differential equation with fractional order ($0 < \beta<1$).The best of our knowledge,there is limited research devoted to the analytical method to solve the FFDEs under Riemann Liouville H-differentiability. An analytical solution is presented to confirm the capability of proposed method.

Introduction:

Fractional calculus is a mathematical branch investigating the properties of derivatives and integrals of non-integer orders.It applied in modeling of many physical and chemical processes and in engineering [4, 6, 9] . Podlubny and kilbas[10,12,] gave the idea of fractional calculus and consider Riemann Liouville differentiability to solve FFDEs. Agarwal [2] proposed the concept of solutions for fractional differential equations with uncertainty.

Laplace transform is the one of the interesting transforms used for solving fuzzy differential equation .Solving fuzzy fractional differential equation, fuzzy initial and boundary value problems we use fuzzy laplace transform to reduce the problem. The advantage of fuzzy laplace transform is to solve the problem directly without determining a general solution.

Here we have seen some basic definition and Riemann Liouville H- differentiabilityin section 2. In section 3, fuzzy Laplace transforms are introduced and we discuss the properties .The solutions of FFDEs are determined by fuzzy laplace transform under Riemann Liouville H- differentiability and solve the example in section 4. In section 5, conclusion is drawn.

2.Definition:2.1[8]

Fuzzy number is a mapping u:R→[0,1] with the following properties:

1. u is upper semi continuous,

2. u is fuzzy convex ,i.e., $u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$ for all $x, y \in$R , λ∈[0,1],

3. u is normal,i.e., ∃ x_0 ∈R for which $u(x_0) = 1$,

4.supp $u = \{x \in R/u(x) > 0\}$ is the support of the u,and its closure cl(supp u) is compact.

Definition:2.2[13,14]

A fuzzy number u in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r), 0 \leq r \leq 1$,which satisfy the following requirements:

1.$\underline{u}(r)$ is a bounded non- decreasing left continuous function in (0,1],and right continuous at 0,

2. $\overline{u}(r)$ is a bounded non-increasing left continuous function in (0,1],and right continuous at0, 3. $\underline{u}(r) \leq \overline{u}(r)$,$0 \leq r \leq 1$.

Definition:2.3(Zadeh's extension principle)

Addition operation on E is defined by

$$(u+v)(x) = \sup_{y\in R} \min\{u(y), v(x-y)\}, x \in R$$

and scalar multiplication of a fuzzy number is given by

$$(k\odot u)(x) = \begin{cases} u\left(x/_k\right) & , \quad k > 0 \\ \tilde{0} & , \quad k = 0 \end{cases}$$

Where $\tilde{0}\epsilon E$

Please note that the function $f: A \rightarrow E$, A ⊆R so called fuzzy valued function .However an arbitrary function f, where $f: A \rightarrow R$, A ⊆R so called real valued function. The r- cut representation of fuzzy valued function f can be expressed by $f(x;r) = \left[\underline{f}(x;r), \overline{f}(x;r)\right]$ and $0 \leq r \leq 1$.

Theorem:2.1[15] Let f be fuzzy valued function on[a,∞) represented by $\left(\underline{f}(x;r), \overline{f}(x;r)\right)$.For any fixed r∈[0,1], assume $\underline{f}(x;r)$ and $\overline{f}(x;r)$ are Riemann- integrable on [a,b] for every b≥a, and assume there are two positive functions $\underline{M}$(r),$\overline{M}$(r) such that $\int_a^b \left|\underline{f}(x;r)\right| dx \leq \underline{M}$(r) and $\int_a^b \left|\overline{f}(x;r)\right| dx \leq \overline{M}$(r) for every b≥ a.Then $f(x)$ is improper fuzzy Riemann integrable on [a,∞)and the improper fuzzy Riemann integral is a fuzzy number. Further more, we have $\int_a^\infty f(x;r)\, dx = \left[\int_a^\infty \underline{f}(x;r)dx\, , \int_a^\infty \overline{f}(x;r)dx)\right]$

Definition:2.4 Let $x, y \in E$. If there exists z∈ E such that $x = y + z$, then z is called the H- difference of x and y,and it is denoted by $x \ominus y$.

Riemann Liouville H- differentiability:[7]

C^F[a,b] as the space of all continuous fuzzy valued function on [a,b]. Also we denote the space of all Lebesque integrable fuzzy valued function on [a,b] by $L^F[a,b]$.

Definition:2.5 Let $f \in C^F$[a,b]∩ $L^F[a,b]$,x_0 in (a,b) and $\Phi(x) = \frac{1}{\Gamma(1-\beta)} \int_a^x \frac{f(t)dt}{(x-t)^\beta}$ .We say that f is Riemann Liouville H- differentiable about order 0< $\beta < 1$ at x_0, if there exists an element$({}^{RL}D_{a^+}^\beta f\,)(x_0) \in E$ such that for h>0 sufficiently small

(i) $({}^{RL}D_{a^+}^\beta f\,)(x_0) = \lim_{h\to 0^+} \frac{\Phi(x_0+h)\ominus\Phi(x_0)}{h} = \lim_{h\to 0^+} \frac{\Phi(x_0)\ominus\Phi(x_0-h)}{h}$

(or)

(ii) $({}^{RL}D_{a^+}^\beta f\,)(x_0) = \lim_{h\to 0^+} \frac{\Phi(x_0)\ominus\Phi(x_0+h)}{-h} = \lim_{h\to 0^+} \frac{\Phi(x_0-h)\ominus\Phi(x_0)}{-h}$

(or)

(iii) $({}^{RL}D_{a^+}^\beta f\,)(x_0) = \lim_{h\to 0^+} \frac{\Phi(x_0+h)\ominus\Phi(x_0)}{h} = \lim_{h\to 0^+} \frac{\Phi(x_0-h)\ominus\Phi(x_0)}{-h}$

(or)

(iv) $({}^{RL}D_{a^+}^\beta f\,)(x_0) = \lim_{h\to 0^+} \frac{\Phi(x_0)\ominus\Phi(x_0+h)}{-h} = \lim_{h\to 0^+} \frac{\Phi(x_0)\ominus\Phi(x_0-h)}{h}$

We say that the fuzzy valued function f is $({}^{RL}(i) - \beta\,)$ differentiable if it is differentiable as in the definition2.5 case(i), and f is$({}^{RL}(ii) - \beta\,)$ differentiable if it is differentiable as in the definition 2.5 of case (ii) and so on for other cases.

Theorem:2.2[17]

Let $f \in C^F$[a,b]∩ $L^F[a,b]$,x_0 in (a,b) and 0 < β <1.Then (i)Let us consider f is $({}^{RL}(i)-\beta)$ differentiable fuzzy valued function, then $({}^{RL}D_{a^+}^{\beta}f)(x_0;r) = \left[({}^{RL}D_{a^+}^{\beta}\underline{f})(x_0;r), ({}^{RL}D_{a^+}^{\beta}\overline{f})(x_0;r)\right]$, $0\leq r\leq 1$

(ii) Let us consider f is $({}^{RL}(ii)-\beta)$ differentiable fuzzy valued function, then $({}^{RL}D_{a^+}^{\beta}f)(x_0;r) = \left[({}^{RL}D_{a^+}^{\beta}\underline{f})(x_0;r), ({}^{RL}D_{a^+}^{\beta}\overline{f})(x_0;r)\right]$, $0\leq r\leq 1$

Where $({}^{RL}D_{a^+}^{\beta}\underline{f})(x_0;r) = \left[\frac{1}{\Gamma(1-\beta)}\frac{d}{dx}\int_a^x \frac{\underline{f}(t;r)dt}{(x-t)^{\beta}}\right]_{x=x_0}$ (1)

$$({}^{RL}D_{a^+}^{\beta}\overline{f})(x_0;r) = \left[\frac{1}{\Gamma(1-\beta)}\frac{d}{dx}\int_a^x \frac{\overline{f}(t;r)dt}{(x-t)^{\beta}}\right]_{x=x_0} \quad (2)$$

3.Fuzzy Laplace transforms

Definition:3.1[16]

Let f be continuous fuzzy valued function. Suppose that $f(x)\odot e^{-px}$ is improper fuzzy Riemann integrable on $[0,\infty)$, then $\int_0^{\infty} f(x)\odot e^{-px}\,dx$ is called fuzzy Laplace transforms and denoted by $L[f(x)] = \int_0^{\infty} f(x)\odot e^{-px}\,dx$ (p>0 and integer) (3)

Using Theorem2.1 we have $0\leq r\leq 1$;

$$\int_0^{\infty} f(x;r)\odot e^{-px}\,dx = \left[\int_0^{\infty} \underline{f}(x;r)\odot e^{-px}\,dx\ ,\int_0^{\infty} \overline{f}(x;r)\odot e^{-px}\,dx\ \right]$$

Using the classical Laplace transform, $l\left[\underline{f}(x;r)\right] = \int_0^{\infty} \underline{f}(x;r)e^{-px}\,dx$ and $l\left[\overline{f}(x;r)\right] = \int_0^{\infty} \overline{f}(x;r)e^{-px}\,dx$

Then we get $L[f(x;r)] = \left[l\left[\underline{f}(x;r)\right], l\left[\overline{f}(x;r)\right]\right]$

Definition :3.2 hypergeom (n,d,z) is the generalized hypergeometric function F(n,d,z), also known as Barnes extended hypergeometric function .For scalar a ,b and c, hypergeom([a,b],c,z) is a Gauss hypergeometric function $2F_1(a,b;c;z)$. The Gauss hypergeometric function $2F_1(a,b;c;z)$ is defined in the unit disc as the sum of the hypergeometric series

$$2F_1(a,b;c;z) = \sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(c)_k}\frac{z^k}{k!}, |z|<1$$

Definition:3.3 The pochhammer symbol $(a)_k$ is defined by

$$(a)_0 = 1,$$
$$(a)_n = a(a+1)\ldots\ldots(a+n-1), n\in N$$

Definition :3.4 A two parameters function of Mittag -Leffler type is defined by the series expansion

$$E_{\alpha,\beta}(z) = \sum_{r=0}^{\infty}\frac{z^r}{\Gamma(\alpha r+\beta)} \quad (\alpha,\beta>0)$$

An error function is defined by $erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}dt.$

Theorem:3.1 [16]

Let f &g are continuous fuzzy valued functions.Suppose that c_1 &c_2 are constants. $L[(c_1\odot\odot f(x))\oplus(c_2\odot\odot g(x))] = (c_1\odot\odot L[f(x)])\oplus(c_2\odot\odot L[g(x)])$

Lemma:3.2[16]

Let f be continuous fuzzy valued function on $[0,\infty)$ and $\lambda\in R$ then $L[\lambda\odot\odot f(x)] = \lambda\odot\odot L[f(x)]$

Derivative theorem:3.2

Suppose that $f\in C^F[0,\infty)\cap L^F[0,\infty)$. Then

$$L\left[({}^{RL}D_{a^+}^{\beta} f)(x)\right] = s^{\beta} L[f(t)] \ominus ({}^{RL}D_{a^+}^{\beta-1} f)(0), \quad (4)$$

if f is $({}^{RL}(i) - \beta)$ differentiable, and

$$L\left[({}^{RL}D_{a^+}^{\beta} f)(x)\right] = -({}^{RL}D_{a^+}^{\beta-1} f)(0) \ominus -\left(s^{\beta} L[f(t)]\right) \quad (5)$$

if f is $({}^{RL}(ii) - \beta)$ differentiable.

4.Fuzzy fractional differential equations under Riemann Liouville H- differentiability:

Let $f \in C^F[a,b] \cap L^F[a,b]$ and consider the fuzzy fractional differential equation of order $0 < \beta < 1$ with the initial condition and $x_0 \in (a,b)$.

$$\begin{cases} ({}^{RL}D_{a^+}^{\beta} y)(x) = f[x, y(x)], \\ ({}^{RL}D_{a^+}^{\beta-1} y)(x_0) = ({}^{RL}{y_0}^{(\beta-1)}) \in E \end{cases} \quad (6)$$

Determining the solutions:

Here we use fuzzy Laplace transform and its inverse to derive the solution .By taking Laplace transform on both sides ,we get

$$L[({}^{RL}D_{a^+}^{\beta} y)(x)] = L[f(x, y(x))], \quad (7)$$

Based on the Riemann Liouville H- differentiability, we have the following cases:

Case(i) Let us consider $y(x)$ is a $({}^{RL}(i) - \beta)$ differentiable function then the equation (7) is extended based on the its lower and upper functions as follows

$$s^{\beta} l\left[\underline{y}(x;r)\right] - ({}^{RL}D_{a^+}^{\beta-1} \underline{y})(0;r) = l\left[\underline{f}(x, y(x); r)\right] \quad 0 \le r \le 1$$

$$s^{\beta} l[\overline{y}(x;r)] - ({}^{RL}D_{a^+}^{\beta-1} \overline{y})(0;r) = l[\overline{f}(x, y(x); r)] \quad 0 \le r \le 1 \quad (8)$$

Where $\underline{f}(x, y(x); r) = \min\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$

$\overline{f}(x, y(x); r) = \max\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$

To solve the linear system(8) ,we assume that $H_1(p;r)$, $k_1(p;r)$ are the solutions

$l\left[\underline{y}(x;r)\right] = H_1(p;r)$

$l[\overline{y}(x;r)] = k_1(p;r)$

By using inverse Laplace transform $\underline{y}(x;r)$ & $\overline{y}(x;r)$ are computed as follows,

$\underline{y}(x;r) = l^{-1}[H_1(p;r)]$

$$\overline{y}(x;r) = l^{-1}[k_1(p;r)] \quad (9)$$

Case(ii)

Let us consider $y(x)$ is a $({}^{RL}(ii) - \beta)$ differentiable function then the equation(7)can be written as follows

$$\begin{cases} -({}^{RL}D_{a^+}^{\beta-1} \underline{y})(0;r) - (-s^{\beta} l[\underline{y}(x;r)]) = l\left[\underline{f}(x, y(x); r)\right] \\ -({}^{RL}D_{a^+}^{\beta-1} \overline{y})(0;r) - (-s^{\beta} l[\overline{y}(x;r)]) = l[\overline{f}(x, y(x); r)] \end{cases} \quad 0 \le r \le 1 \quad (10)$$

Where $\underline{f}(x, y(x); r) = \min\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$

$\overline{f}(x, y(x); r) = \max\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$

To solve the linear system(10) ,we assume that $H_2(p;r)$, $k_2(p;r)$ are the solutions

$l\left[\underline{y}(x;r)\right] = H_2(p;r)$

$l[\overline{y}(x;r)] = k_2(p;r)$

By using inverse Laplace transform $\underline{y}(x;r)$ & $\overline{y}(x;r)$ are computed as follows,

$\underline{y}(x;r) = l^{-1}[\,H_2(p;r)\,]$

$\overline{y}(x;r) = l^{-1}[\,k_2(p;r)\,]$ (11)

Example:1

Let us consider the following fuzzy fractional differential equation

$$\begin{cases} ({}^{RL}D^{\beta}_{0^+}y)(x) = \lambda \odot y(x) + e^x, \quad 0 < \beta,\ x < 1 \\ ({}^{RL}D^{\beta-1}_{0^+}y)(0) = ({}^{RL}y_0{}^{(\beta-1)}) \in E \end{cases} \tag{12}$$

Solution:

Case(i): Suppose $\lambda \in R^+ = (0,+\infty)$, then applying Laplace transform on both sides

$$L[({}^{RL}D^{\beta}_{0^+}y)(x)] = L[\,\lambda \odot y(x) + e^x], \tag{13}$$

$$L[({}^{RL}D^{\beta}_{0^+}y)(x)] = L[\,\lambda \odot y(x)] + L[e^x],$$

Using$({}^{RL}(i) - \beta)$ differentiability,we get

$$\begin{cases} s^{\beta} l\left[\underline{y}(x;r)\right] - ({}^{RL}D^{\beta-1}_{a^+}\underline{y})(0;r) = \lambda l\left[\underline{y}(x;r)\right] + \frac{1}{S-1} \\ s^{\beta} l[\overline{y}(x;r)] - ({}^{RL}D^{\beta-1}_{a^+}\overline{y})(0;r) = \lambda l[\overline{y}(x;r)] + \frac{1}{S-1} \end{cases} \tag{14}$$

$$\Rightarrow \quad (s^{\beta} - \lambda) l\left[\underline{y}(x;r)\right] = ({}^{RL}D^{\beta-1}_{a^+}\underline{y})(0;r) + \frac{1}{S-1}$$

$$(s^{\beta} - \lambda)\, l\,[\overline{y}(x;r)] = ({}^{RL}D^{\beta-1}_{a^+}\overline{y})(0;r) + \frac{1}{S-1}$$

$$l\left[\underline{y}(x;r)\right] = ({}^{RL}D^{\beta-1}_{a^+}\underline{y})(0;r)\frac{1}{(s^{\beta}-\lambda)} + \frac{1}{(S-1)\ (s^{\beta}-\lambda)}$$

$$l[\overline{y}(x;r)] = ({}^{RL}D^{\beta-1}_{a^+}\overline{y})(0;r)\frac{1}{(s^{\beta}-\lambda)} + \frac{1}{(S-1)\ (s^{\beta}-\lambda)} \tag{15}$$

Applying inverse transform on bothsides,

$$\underline{y}(x;r) = ({}^{RL}D^{\beta-1}_{a^+}\underline{y})(0;r) l^{-1}\left[\frac{1}{(s^{\beta}-\lambda)}\right] + l^{-1}\left[\frac{1}{(S-1)\ (s^{\beta}-\lambda)}\right]$$

$$\overline{y}(x;r) = ({}^{RL}D^{\beta-1}_{a^+}\overline{y})(0;r) l^{-1}\left[\frac{1}{(s^{\beta}-\lambda)}\right] + l^{-1}\left[\frac{1}{(S-1)\ (s^{\beta}-\lambda)}\right] \tag{16}$$

1st term in equation(16) $l^{-1}\left[\frac{1}{(s^{\beta}-\lambda)}\right] = l^{-1}[s^{-\beta}\,(1 - \lambda s^{-\beta})^{-1}]$

$$= l^{-1}[s^{-\beta}\,(1 + \lambda s^{-\beta} + (\lambda s^{-\beta})^2 + (\lambda s^{-\beta})^3 + \cdots)]$$

$$= l^{-1}[s^{-\beta}\,\textstyle\sum_{r=0}^{\infty} (\lambda s^{-\beta})^r]$$

$$= \textstyle\sum_{r=0}^{\infty} \lambda^r\, l^{-1}\left[s^{-\beta r-\beta}\right]$$

$$= \sum_{r=0}^{\infty} \lambda^r \frac{x^{\beta r+\beta-1}}{\Gamma(\beta r+\beta)} \qquad \left[\ l^{-1}[s^{-n}] = \frac{t^{n-1}}{(n+1)!} = \frac{t^{n-1}}{\Gamma(n)}\right]$$

$$= x^{\beta-1} \sum_{r=0}^{\infty} \frac{(\lambda x^{\beta})^r}{\Gamma(\beta r+\beta)}$$

$$= x^{\beta-1}\, E_{\beta,\beta}(\lambda x^{\beta})$$

Convolution theorem in laplace transform we have 2nd term in eq(16)

$$l^{-1}\left[\frac{1}{(S-1)\ (s^{\beta}-\lambda)}\right] = \int_0^x (x-t)^{(\beta-1)}\, E_{\beta,\beta}(\lambda(x-t)^{\beta})\, e^t dt$$

(16)⇒

$$\underline{y}(x;r) = ({}^{RL}\underline{y_0}{}^{(\beta-1)}) \odot x^{\beta-1}\, E_{\beta,\beta}(\lambda x^{\beta}) + \int_0^x (x-t)^{(\beta-1)}\, E_{\beta,\beta}(\lambda(x-t)^{\beta})\, e^t dt$$

$$\overline{y}(x;r) = ({}^{RL}\overline{y}_0{}^{(\beta-1)}) \odot x^{\beta-1}\, E_{\beta,\beta}(\lambda x^{\beta}) + \int_0^x (x-t)^{(\beta-1)}\, E_{\beta,\beta}(\lambda(x-t)^{\beta})\, e^t dt \tag{17}$$

Case(ii) Suppose $\lambda \in R^- = (-\infty, 0)$, then using$({}^{RL}(ii) - \beta)$ differentiability the solution will obtain similar to equ(17).

For the special case , let us consider β=0.5 , λ=1and $({}^{RL}D_{0^+}^{-0.5}y)(0;r) = [1+r, 3-r]$in case(i)

$\underline{y}(x;r)$=$[1+r,3-r]\odot x^{-\frac{1}{2}}\, E_{\frac{1}{2},\frac{1}{2}}(x^{\frac{1}{2}})$+ $\int_0^x (x-t)^{(-\frac{1}{2})}\, E_{\frac{1}{2},\frac{1}{2}}(x-t)^{\frac{1}{2}})\, e^t dt$

$\overline{y}(x;r)$=$[1+r,3-r]\odot x^{-\frac{1}{2}}\, E_{\frac{1}{2},\frac{1}{2}}(x^{\frac{1}{2}})$+ $\int_0^x (x-t)^{(-\frac{1}{2})}\, E_{\frac{1}{2},\frac{1}{2}}(x-t)^{\frac{1}{2}})\, e^t dt$ (18)

Now consider 1st term in eq (18)

$$x^{-\frac{1}{2}} E_{\frac{1}{2},\frac{1}{2}}(x^{\frac{1}{2}}) = x^{-\frac{1}{2}}\left[\sum_{k=0}^{\infty}\frac{(x^{\frac{1}{2}})^k}{\Gamma(\frac{k+1}{2})}\right] = \frac{1}{\sqrt{x}}\left[\frac{1}{\Gamma(\frac{1}{2})}+\frac{x^{\frac{1}{2}}}{\Gamma(1)}+\frac{x^1}{\Gamma(\frac{3}{2})}+\cdots\right]$$

$$= \left[\frac{1}{\sqrt{\pi x}}+\left(\frac{x^0}{\Gamma(1)}+\frac{x^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}+\cdots.\right)\right]$$

$$= \frac{1}{\sqrt{\pi x}}+\sum_{k=0}^{\infty}\frac{x^{(\frac{k}{2})}}{\Gamma(\frac{k}{2}+1)} = \frac{1}{\sqrt{\pi x}}+E_{\frac{1}{2},1}(x^{\frac{1}{2}})$$

$$= \frac{1}{\sqrt{\pi x}}+e^{(x^{\frac{1}{2}})2}\ \text{erfc}\ (-x^{\frac{1}{2}})$$

$$= \frac{1}{\sqrt{\pi x}}+ e^x \text{erfc}\ (-\sqrt{x})$$

2nd term in eq(18)

$$\int_0^x (x-t)^{(-\frac{1}{2})}\, E_{\frac{1}{2},\frac{1}{2}}(x-t)^{\frac{1}{2}})\, e^t dt = \int_0^x (x-t)^{(-\frac{1}{2})}\sum_{k=0}^{\infty}\frac{(x-t)^{\frac{k}{2}}}{\Gamma(\frac{k+1}{2})}\, e^t dt$$

$$= \int_0^x \sum_{k=0}^{\infty}\frac{(x-t)^{\frac{k-1}{2}}}{\Gamma(\frac{k+1}{2})}\, e^t dt$$

$$= \int_0^x \frac{(x-t)^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})}e^t dt + \int_0^x \frac{(x-t)^0}{\Gamma(1)}e^t dt + \int_0^x \frac{(x-t)^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}e^t dt + \int_0^x \frac{(x-t)^1}{\Gamma(2)}e^t dt + \int_0^x \frac{(x-t)^{\frac{3}{2}}}{\Gamma(\frac{5}{2})}e^t dt + \int_0^x \frac{(x-t)^2}{\Gamma(3)}e^t dt + \cdots$$

$$= \frac{x^{\frac{1}{2}}}{(\frac{1}{2})!}\ \text{hypergeom}(1,1.5,x) + \left(\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)$$

$$+ \frac{x^{\frac{3}{2}}}{(\frac{3}{2})!}\ \text{hypergeom}(1,2.5,x) + \left(\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+..\right)$$

$$+ \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!}\text{hypergeom}(1,3.5,x) + \left(\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\cdots\right)+\ldots\ldots\ldots$$

$$= \sum_{n=0}^{\infty}\frac{x^{(n+\frac{1}{2})}}{(n+\frac{1}{2})!}\ \text{hypergeom}(1, n+\tfrac{3}{2}, x) + \sum_{n=1}^{\infty}\frac{n x^n}{n!}$$

(18)⇒

$$\underline{y}(x;r) = [1+r,3-r]\odot\left(\frac{1}{\sqrt{\pi x}}+ e^x\text{erfc}\ (-\sqrt{x})\right) + \sum_{n=0}^{\infty}\frac{x^{(n+\frac{1}{2})}}{(n+\frac{1}{2})!}\ \text{hypergeom}(1, n+\tfrac{3}{2}, x) + \sum_{n=1}^{\infty}\frac{n x^n}{n!}$$

$$\overline{y}(x;r) = [1+r,3-r]\odot\left(\frac{1}{\sqrt{\pi x}}+ e^x\text{erfc}\ (-\sqrt{x})\right) + \sum_{n=0}^{\infty}\frac{x^{(n+\frac{1}{2})}}{(n+\frac{1}{2})!}\ \text{hypergeom}(1, n+\tfrac{3}{2}, x) + \sum_{n=1}^{\infty}\frac{n x^n}{n!}$$

→ (19)

$\underline{y}(x;r) = [1+r](\frac{1}{\sqrt{\pi x}} + e^{x}\text{erfc }(-\sqrt{x})) + \sum_{n=0}^{\infty}\frac{x^{(n+\frac{1}{2})}}{(n+\frac{1}{2})!}\text{hypergeom}(1, n+\frac{3}{2}, x) + \sum_{n=1}^{\infty}\frac{nx^{n}}{n!}$

$\overline{y}(x;r) = [3-r](\frac{1}{\sqrt{\pi x}} + e^{x}\text{erfc }(-\sqrt{x})) + \sum_{n=0}^{\infty}\frac{x^{(n+\frac{1}{2})}}{(n+\frac{1}{2})!}\text{hypergeom}(1, n+\frac{3}{2}, x) + \sum_{n=1}^{\infty}\frac{nx^{n}}{n!}$ (20)

5.Conclusion:

In this paper ,solving FFDEs of order 0< β <1using fuzzy laplace transforms under Riemann Liouville –H differentiability.We solved example problem involving exponential term.

Conflict of interest: none declared

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