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Estimators of Lindley-Type for the Multivariate Normal Mean In the Bayesian Case

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ABSTRACT

This article concerns the estimation of the mean θ of a multivariate normal distribution $X \sim N_p(\theta, \sigma^2 I_p)$ in which the variance σ^2 is unknown and estimated by the chi-square variable $S^2 \sim \sigma^2 \chi_n^2$. First, we consider the estimators of Lindley-Type that shrink the components of the Maximum Likelihood Estimator (MLE) X to the random variable $\overline{X}$. Secondly, we consider the mean θ as a random variable and construct the modal Bayes estimator δ^{MB}, we then study the minimaxity of the estimator δ^{MB} and the asymptotic behavior of risks ratios of δ^{MB} to the MLE when the dimension of the parameters space p and the sample size n tend simultaneously to infinity.

KEYWORDS: Modal Bayes estimator, multivariate normal distribution, quadratic risk, shrinkage estimator.

1. INTRODUCTION

One common problem in multivariate statistical analysis and Bayesian statistics is the estimation of the mean parameters of a multivariate normal distribution. This latter has attracted the attention of several researchers. When the dimension of the parameter space is greater, the performance of MLE method is not satisfactory. For more information in this context, we refer readers to Stein (1956), and James and Stein (1961), Lindley (1962), Efron and Morris (1972) and Efron and Morris (1973). Among several methods, shrinkage estimation is one of the widely used for improving the MLE. Furthermore a large amount of research have been carried out to develop the properties of shrinkage estimators and to compare them with MLE, we cite for example Baranchick (1970), Bock (1975), Efron and Morris (1977), Hamdaoui et al. (2020). The majority of these authors studied the estimation by shrinkage estimators, of the mean θ of a multivariate normal distribution $N_p(\theta, \sigma^2 I_p)$ in $\Re^p$. In these works one estimates the mean θ by shrinkage estimators deduced from the empirical mean estimator, which are better in quadratic loss than the empirical mean estimator.

More precisely, if X represents an observation or a sample of multivariate normal distribution $N_p(\theta, \sigma^2 I_p)$, the aim is to estimate θ by an estimator δ relatively at the quadratic loss function:

$$L(\delta, \theta) = \|\delta - \theta\|_p^2, \qquad (1.1)$$

where $\|.\|_p$ is the usual norm in $\Re^p$. We associate its risk function:

$$R(\delta, \theta) = E_\theta\left(\|\delta - \theta\|_p^2\right). \qquad (1.2)$$

James and Stein (1961), introduced a class of estimators improving $\delta_0 = X$, when the dimension of the space of the observations $p \geq 3$, denoted by :

$$\delta_j^{JS} = \left(1 - \frac{(p-2)}{(n+2)} \frac{S^2}{\|X\|^2}\right) X_j \ , \ j = 1,\ldots,p \,, \qquad (1.3)$$

where $S^2 \sim \sigma^2 \chi_n^2$ is an estimator of σ^2, independent of X .

Baranchik (1964), proposed the positive-part version of James-Stein estimator, an estimator dominating the James-Stein estimator when $p \geq 3$,

$$\delta_j^{JS+} = \max\left(0, \left(1 - \frac{(p-2)}{(n+2)} \frac{S^2}{\|X\|^2}\right)\right) X_j \ , \ j = 1,\ldots,p \,. \qquad (1.4)$$

Casella and Hwang (1982), studied the case where σ^2 is known $(\sigma^2 = 1)$ and showed that if the limit of the ratio $\frac{\|\theta\|^2}{p}$, when p tends to infinity is a constant $c > 0$, then

$$\lim_{p \to +\infty} \frac{R(\delta_{JS}, \theta)}{R(X, \theta)} = \lim_{p \to +\infty} \frac{R(\delta_{JS}^{+}, \theta)}{R(X, \theta)} = \frac{c}{1+c} \,.$$

Sun (1995), has considered the following model: $(y_{ij} / \theta_j, \sigma^2) \sim N(\theta_j, \sigma^2)$; $i = 1,\ldots,n$, $j = 1,\ldots,m$ where $E(y_{ij}) = \theta_j$ for the group j and $var(y_{ij}) = \sigma^2$ is unknown. In this case, the risk of the maximum likelihood estimator, denoted δ_0, is $R(\delta_0, \theta) = \frac{m\sigma^2}{n}$ and the James-Stein estimator is written:

$$\delta^{JS} = (\delta_1^{JS}, \ldots, \delta_m^{JS})^t \text{, where } \delta_j^{JS} = \left(1 - \frac{(m-3)}{(N+2)} \frac{S^2}{T^2}\right)(\bar{y}_j - \bar{y}) + \bar{y} \,, \ j = 1,\ldots,m \,,$$

$$\text{and } S^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} (y_{ij} - \bar{y}_j)^2 \,, \ T^2 = n \sum_{j=1}^{m} (\bar{y}_j - \bar{y})^2 \,, \ \bar{y}_j = \frac{\sum_{i=1}^{n} y_{ij}}{n} \text{ and } \bar{y} = \frac{\sum_{j=1}^{m} \bar{y}_j}{m} \,, \ N = (n-1)m \,.$$

He showed that for any estimator of the form $\delta = (\delta_1, \ldots, \delta_m)^t$,

$$\text{where: } \delta_j = (1 - \psi(S^2, T^2))(\bar{y}_j - \bar{y}) + \bar{y} \,, \ j = 1,\ldots,m \,.$$

$$\text{If } \lim_{m \to +\infty} \frac{\sum_{j=1}^{m} (\theta_j - \bar{\theta})^2}{m} = q \text{ exists, then } \lim_{m \to +\infty} \frac{R(\delta, \theta)}{R(X, \theta)} \geq \frac{q}{q + \frac{\sigma^2}{n}}$$

and $\lim_{m \to +\infty} \frac{R(\delta^{JS}, \theta)}{R(X, \theta)} = \frac{q}{q + \frac{\sigma^2}{n}}$. Namely $\frac{q}{q + \frac{\sigma^2}{n}}$ constitutes a lower bound for the ratio $\lim_{m \to +\infty} \frac{R(\delta, \theta)}{R(\delta_0, \theta)}$ and is equal to

$$\lim_{m \to +\infty} \frac{R(\delta^{JS}, \theta)}{R(\delta_0, \theta)} .$$

He also showed that this bound is attained for a class of estimators defined by:

$$\delta_j = (1 - \varphi(S^2, T^2) \frac{S^2}{T^2})(\bar{y}_j - \bar{y}) + \bar{y} \,, \ j = 1,\ldots,m \,,$$

where φ satisfies certain conditions. This bound is also attained for any estimator dominating the James-Stein estimator, in particular the positive-part version of the James-Stein estimator.

Finally, we note that if n tends to infinity then the ratio $\frac{q}{q+\frac{\sigma^2}{n}}$ tends to 1, and thus the risk of the James-Stein estimator is that of δ_0 (when n and m tend simultaneously to infinity).

Hamdaoui and Benmansour (2015), considered the same model given by Casella and Hwang (1982), namely $X \sim N_p(\theta, \sigma^2 I_p)$ but in this time the parameter σ^2 is unknown and estimated by the statistic $S^2 : S^2 \sim \sigma^2 \chi_n^2$ independent to variable X. The authors studied the following class of shrinkage estimators $\delta_\psi = \delta_{JS} + l\psi(S^2, \|X^2\|)X$, they showed that, if $\lim\limits_{p\to+\infty} \frac{\|\theta\|^2}{p\sigma^2} = c(>0)$ and the shrinkage function ψ satisfies some conditions, the risks ratio $\frac{R(\delta_\psi, \theta)}{R(X,\theta)}$ tends to constant $\frac{c}{1+c}$ (<1) when n and p tend simultaneously to infinity. Finally, they deduced that, when n and p tend simultaneously to infinity and under the same condition $\lim\limits_{p\to+\infty} \frac{\|\theta\|^2}{p\sigma^2} = c(>0)$, the risks ratio of any shrinkage estimator δ_ψ dominating the James-Stein estimator δ^{JS}, to the maximum likelihood estimator X, tends to constant $\frac{c}{1+c}$ (<1), in particularly the risks ratios $\frac{R(\delta^{JS}, \theta)}{R(X,\theta)}$ and $\frac{R(\delta^{JS+}, \theta)}{R(X,\theta)}$.

When the dimension p is finite, Brandwein and Strawderman (2012) considered the following model $(X,U) \sim f\left(\|X-\theta\|^2 + \|U\|^2\right)$, where $\dim X = \dim\theta = p$ and $\dim U = k$. The classical example of this model is of course, the normal model of density $\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{p+k} e^{-\frac{\|X-\theta\|^2}{2\sigma^2}}$. They showed that, if the function g satisfies certain conditions, the estimator $\delta_g = X + \left\{\frac{\|U\|^2}{k+2}\right\} g(X)$ dominate X, so that δ_g is minimax.

Maruyama (2014), has also studied the minimaxity of shrinkage estimator when the dimension of parameter's space is finite. Then he considered the following model $z \sim N_d(\theta, I_d)$ and the so called l_p -norm given by: $\|z\|_p = \left\{\sum_{i=1}^{i=d} |z_i|^p\right\}^{\frac{1}{p}}$, $p > 0$. He studied the minimaxity of shrinkage estimators defined as follows: $\hat{\theta}_\phi = (\hat{\theta}_{1\phi}, \ldots, \hat{\theta}_{d\phi})$ with : $\hat{\theta}_{i\phi} = \left(1 - \phi(\|z\|_p) / \|z\|_p^{2-\alpha} |z_i|^\alpha\right) z_i$ where $0 \le \alpha \le (d-2)/(d-1)$ and $p > 0$.

Note that the risk functions of these estimators are calculated relatively to the usual quadratic loss function defined above.

In this work we adopt the model $X \sim N_p(\theta, \sigma^2 I_p)$ where the parameter σ^2 is unknown and estimated by the statistic S^2: $S^2 \sim \sigma^2 \chi_n^2$ independent of the observations X. Note that $R(X;\theta) = p\sigma^2$ is the risk of the MLE. Our aim is to estimate the mean θ by the shrinkage estimators of the form $\delta^\psi = \left(1 - \psi(S^2, \|X^2\|) \frac{S^2}{\|X^2\|}\right)(X - \overline{X}) + \overline{X}$, that shrink the components of the MLE X to the random variable $\overline{X}$. This paper is organized as follows, In Section 1, we recall some results obtained in Benkhaled and Hamdaoui (2019). Under the usual quadratic loss function defined in (1.1), the authors showed that if the shrinkage function ψ satisfies some conditions, the estimator δ^ψ is minimax. They also proved that if the condition $\lim\limits_{p\to+\infty} \frac{\|\theta\|^2}{p\sigma^2} = c(>0)$) is satisfies, the limit of risks ratio of shrinkage estimators δ^ψ to the MLE X, equal to $\frac{c}{1+c}$ when n and p tend simultaneously to infinity, thus they assured the minimaxity property of this estimator when the the parameters space p and the sample size n are large. In Section 2, we trait the Bayesian case, then we suppose that the mean θ is a random variable and construct the modal Bayes estimator. We study the minimaxity of this estimator when the dimension of the parameters space is finite, and the asymptotic behavior of risks ratio of this estimator to the MLE when the dimension of the parameters space p and the sample size n tend simultaneously to infinity.

2. PRELIMINAIRIES.

We recall that if X is a multivariate Gaussian random $N_p(\theta, \sigma^2 I_p)$ in $\Re^p$ then $\frac{\|X\|^2}{\sigma^2} \sim \chi_p^2(\lambda)$ where $\chi_p^2(\lambda)$ denotes the non-central chi-square distribution with p degrees of freedom and non-centrality parameter $\lambda = \frac{\|\theta\|^2}{2\sigma^2}$. We also recall the following results that we will use in the next.

Let the model: $X/\theta,\sigma^2 \sim N_p(\theta,\sigma^2 I_p)$, where the parameters θ and σ^2 are unknown and σ^2 is estimated by the statistic $S^2 : S^2 \sim \sigma^2\chi_n^2$ independent to the random variable X. The aim is to estimate the mean $\theta = (\theta_1, \theta_2, \ldots, \theta_p)^t$ by the shrinkage estimators of the form:

$$\delta_j^\phi = \left(1-\phi\left(S^2,T^2\right)\right)\left(X_j - \overline{X}\right) + \overline{X}, \quad j = 1,\ldots,p, \qquad (2.1)$$

where: $\bar{X} = \frac{1}{p}\sum_{i=1}^p X_i$, $T^2 = \sum_{i=1}^p (X_i - \bar{X})^2$ and the two random variables S^2 and T^2 are independent.

Lemma 2.1. (Benkhaled and Hamdaoui (2019)) *Assume $K \sim P\left(\sum_{i=1}^p \left(\theta_i - \overline{\theta}\right)^2 / 2\sigma^2\right)$, then for any functions of two variables f and g, such that all expectations of a) and b) exist, we have*

$$a)\ E\left\{f\left(S^2,T^2\right)\right\} = E\left\{f\left(\sigma^2\chi_n^2, \sigma^2\chi_{p-1+2K}^2\right)\right\},$$

$$b)\ E\left\{g\left(S^2,T^2\right)\sum_{i=1}^p \left(X_i - \overline{X}\right)\left(\theta_i - \overline{\theta}\right)\right\} = 2\sigma^2 E\left\{Kg\left(\sigma^2\chi_n^2, \sigma^2\chi_{p-1+2K}^2\right)\right\},$$

where $K \sim P\left(\sum_{i=1}^p \left(\theta_i - \overline{\theta}\right)^2 / 2\sigma^2\right)$ being the Poisson distribution of parameter $\sum_{i=1}^p \left(\theta_i - \overline{\theta}\right)^2 / 2\sigma^2$ and $\overline{\theta} = \frac{1}{p}\sum_{i=1}^p \theta_i$.

Proposition 2.2. (Benkhaled and Hamdaoui (2019)) *Let δ^ϕ is given in (2.1), then for any $p \geq 4$ we have*

$$i)\ R\left(\delta^\phi, \theta\right) = \sigma^2 E\left\{\phi_K^2 \chi_{p-1+2K}^2 - 2\phi_K\left(\chi_{p-1+2K}^2 - 2K\right) + p\right\}, \qquad (2.2)$$

$$ii)\ R\left(\delta^\phi, \theta\right) \geq B_p(\theta), \text{ where}$$

$$\phi_K = \phi\left(\sigma^2\chi_n^2, \sigma^2\chi_{p-1+2K}^2\right) \text{ and } B_p(\theta) = \sigma^2\left\{p - \frac{\left(\chi_{p-1+2K}^2 - 2K\right)^2}{\chi_{p-1+2K}^2}\right\},$$

$$iii)\ \text{if } c = \lim_{p\to+\infty} \sum_{i=1}^p \left(\theta_i - \overline{\theta}\right)^2 / p\sigma^2 \text{ exists, then}$$

$$\lim_{p\to+\infty} \frac{B_p(\theta)}{R(X,\theta)} = \lim_{p\to+\infty} \frac{B_p(\theta)}{p\sigma^2} = \lim_{p\to+\infty} b_p(\theta) = \frac{c}{1+c}.$$

The proof of this Proposition is based to the Lemma 2.1.

Now, we consider the special case when $\phi\left(S^2,T^2\right) = d\frac{S^2}{T^2}$, where d is a constant, then the estimator given in (3.1) is written as :

$$\delta_j^d = \left(1 - d\frac{S^2}{T^2}\right)\left(X_j - \overline{X}\right) + \overline{X}, \quad j = 1,\ldots,p.$$

From Proposition 2.2, we have

$$R(\delta^d,\theta)=\sigma^2\left\{p-\left[2dn(p-3)-d^2n(n+2)\right]E\left(\frac{1}{p-2+2K}\right)\right\}.$$

We note that when $d=0$, the estimator δ^0 becomes the maximum likelihood estimator X, its risk equal $p\sigma^2$.

In this case, the James-Stein estimator is obtained by minimizing the risk $R(\delta^d,\theta)$, thus the James-Stein estimator is given by:

$$\delta_j^{JS}=\left(1-\frac{p-3}{n+2}\frac{S^2}{T^2}\right)\left(X_j-\overline{X}\right)+\overline{X},\quad j=1,\ldots,p\ , \qquad (2.3)$$

its risk function is

$$R(\delta^{JS},\theta)=\sigma^2\left\{p-\frac{n}{n+2}(p-3)^2E\left(\frac{1}{p-2+2K}\right)\right\}, \qquad (2.4)$$

$$\text{where } K\sim P\left(\sum_{i=1}^{p}\left(\theta_i-\overline{\theta}\right)^2/2\sigma^2\right).$$

Proposition 2.3. (Benkhaled and Hamdaoui (2019))

a) If $p\geq 4$, *the James-Stein estimator* δ^{JS} *given in (2.3) is minimax.*

b) If $\lim_{p\to+\infty}\sum_{i=1}^{p}\left(\theta_i-\overline{\theta}\right)^2/p\sigma^2=c(>0)$, *then*

$$\lim_{n,p\to+\infty}\frac{R\left(\delta^{JS},\theta\right)}{R\left(X,\theta\right)}=\frac{c}{1+c}.$$

Remark 2.4. *From Propositions 2.2 and 2.3, we note that the risks ratios of any shrinkage estimator* δ^{ϕ} *of the form (2.1) dominating the James-Stein estimator* δ^{JS}, *to the maximum likelihood estimator attains the limiting lower bound* $B_m=\frac{c}{1+c}$, *when* n *and* p *tend simultaneously to infinity.*

Next, we consider the general form of shrinkage estimators of Lindley-type, defined by :

$$\delta_j^{\varphi}=\left(1-\varphi\left(S^2,T^2\right)\frac{S^2}{T^2}\right)\left(X_j-\overline{X}\right)+\overline{X},\quad j=1,\ldots,p. \qquad (2.5)$$

For the next, we recall the result of minimaxity of the estimator of Lindley-type given in (2.5) and the limit of risks ratio of this estimator to the MLE X.

2.1. Minimaxity

Proposition 2.5. (Benkhaled and Hamdaoui (2019)) *Assume that* δ^{φ} *is given in (2.5), such that* $p\geq 4$ *and* φ *satisfies:*

a) $\varphi\left(S^2,T^2\right)$ *is monotone non-decreasing in* T^2,

$$b)\ 0\leq\varphi\left(S^2,T^2\right)\leq\frac{2(p-3)}{(n+2)}.$$

A sufficient condition so that the estimator δ^{φ} *is minimax is, for any k (k=0, 1, 2,…),*

$$E\left\{\varphi\left(\sigma^2\chi_{n+4}^2,\sigma^2\chi_{p-1+2k}^2\right)\right\}\leq E\left\{\varphi\left(\sigma^2\chi_{n+2}^2,\sigma^2\chi_{p-1+2k}^2\right)\right\}.$$

Remark 2.6. *It is clear that the James-Stein estimator given in (2.3) satisfies the conditions of Proposition 2.5, thus the James-Stein estimator is minimax.*

2.2. Limit of risks ratios

Proposition 2.7. (Benkhaled and Hamdaoui (2019)) *Assume that* δ^{φ} *is given in (2.5), such that* $p\geq 4$ *and* φ *satisfies :*

$$H)\ \left|\frac{p-3}{n+2}-\varphi\left(S^2,T^2\right)\right|\leq g\left(S^2\right)\ \ a.s,$$

where $E\left\{\left[g\left(\sigma^2\chi^2_{n+4}\right)\right]^2\right\}=O\left(\frac{1}{n^2}\right)$ *when n is in the neighborhood of* $+\infty$.

If $\lim_{p\to+\infty}\sum_{i=1}^{p}\left(\theta_i-\bar{\theta}\right)^2/p\sigma^2=c(>0)$, *then*

$$\lim_{n,p\to+\infty}\frac{R\left(\delta^{\varphi},\theta\right)}{R\left(X,\theta\right)}=\frac{c}{1+c}.$$

3. MAIN RESULTS

3.1. The Modal Bayes Estimator

We recall that, if $X/\theta,\sigma^2\sim N_p\left(\theta,\sigma^2I_p\right)$, where the parameter σ^2 is known and the prior distribution $\theta_j/\nu,\tau^2\sim N\left(\nu,\tau^2\right)$ ($j=1,...,p$), with the hyper parameters ν and τ^2 are known. Then the Bayes estimator is given by:

$$\delta_j^B=\left(1-\frac{\sigma^2}{\sigma^2+\tau^2}\right)\left(X_j-\nu\right)+\nu,\quad j=1,...,p. \qquad (3.1)$$

Now, assume that $X/\theta,\sigma^2\sim N_p\left(\theta,\sigma^2I_p\right)$, where the parameters θ and σ^2 are unknown and σ^2 is estimated by $S^2\sim\sigma^2\chi^2_n$ and the prior distribution $\theta_j/\nu,\tau^2\sim N\left(\nu,\tau^2\right)$ ($j=1,...,p$), with the hyper parameters ν and τ^2 are unknown. Then

$$L\left(X/\nu,\tau^2,\sigma^2\right)=\frac{1}{\left[2\pi\left(\sigma^2+\tau^2\right)\right]^{p/2}}\exp\left(-\frac{1}{2}\frac{\sum_{i=1}^{p}\left(X_i-\nu\right)^2}{\sigma^2+\tau^2}\right).$$

The function $L\left(X/\nu,\tau^2,\sigma^2\right)$ taking its maximum value if

$\hat{\nu}=\bar{X}$ and $\hat{\tau}^2=\left(U^2-\sigma^2\right)^+=\max\left(0,U^2-\sigma^2\right)$,

where

$$\bar{X}=\frac{1}{p}\sum_{i=1}^{p}X_i \text{ and } U^2=\frac{1}{p}\sum_{i=1}^{p}\left(X_i-\bar{X}\right)^2=\frac{1}{p}T^2.$$

As σ^2 is unknown and $\frac{S^2}{n+2}$ is an asymptotically unbiased estimator of σ^2 (i.e. $\lim_{n\to+\infty}E\left(\frac{S^2}{n+2}\right)=\sigma^2$), then we take the following statistics:

$$\hat{\nu}=\bar{X},\ \hat{\sigma}^2=\frac{S^2}{n+2}\text{ and }\hat{\tau}^2=\left(U^2-\frac{S^2}{n+2}\right)^+=\max\left(0,U^2-\frac{S^2}{n+2}\right).$$

Thus, the modal Bayes estimator is,

$$\delta_j^{MB}=\left(1-\frac{S^2}{S^2+(n+2)\left(U^2-\frac{S^2}{n+2}\right)^+}\right)\left(X_j-\bar{X}\right)+\bar{X},\quad j=1,...,p. \qquad (3.2)$$

It is clear that the estimator defined in (3.2) is of the form (2.5) with $\varphi\left(S^2,T^2\right)=\min\left(\frac{p}{n+2},\frac{T^2}{S^2}\right)$ and it can be easily shown that if $p\geq 6$, the estimator δ^{MB} satisfies conditions of Proposition 2.5, hence δ^{MB} is minimax. We also note that the estimator δ^{MB} satisfies conditions of Proposition 2.7, it suffices to take $g\left(S^2\right)=\frac{2p-3}{n+2}$, thus the risk ratio of this estimator to the maximum likelihood estimator attain the limiting lower bound $B_m=\frac{c}{1+c}$.

CONCLUSION

In this work, we considered the model $X \sim N_p(\theta, \sigma^2 I_p)$ where the parameter σ^2 is unknown and estimated by the statistic $S^2 : S^2 \sim \sigma^2 \chi_n^2$ independent of the observations X. Note that $R(X;\theta) = p\sigma^2$ is the risk of the MLE. Our aim is to estimate the mean θ by the shrinkage estimators of the form $\delta^{\psi} = \left(1 - \psi\left(S^2, \|X^2\|\right)\frac{S^2}{\|X^2\|}\right)\left(X - \overline{X}\right) + \overline{X}$, that shrink the components of the MLE X to the random variable $\overline{X}$. Under the usual quadratic loss function, we studied the minimaxity of these estimators when the dimension of the parameter's space p is finite. We also studied the limit of risks ratios of estimator δ^{ψ}, to the maximum likelihood estimator, when n and p tend simultaneously to infinity. Finaly, we considered that the mean θ is a random variable and constructed the modal Bayes estimators δ^{MB}. We showed that if the dimension of the parameters space p, the estimator δ^{MB} is minimax and we prove that the limit of risks ratio of this estimator to the maximum likelihood estimator when the dimension of the parameters space p and the sample size n tend to infinity tends to the value $\frac{c}{1+c}$ (<1) thus, we assured that the modal Bayes estimators δ^{MB} is minimax even if n and p tend simultaneously to infinity . An idea would be to see whether one can obtain similar results in the general case of the symmetrical spherical model.

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