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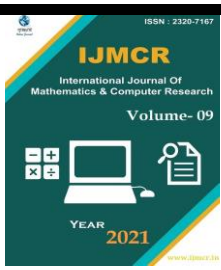

Time Impact on Residue in a Non-homogeneous Equation of Statics in the Theory of Elastic Mixture

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ABSTRACT
Residual stress in continuum has not been quantified because time relationship with residues has not been proven analytically. This is achieved in this paper by analyzing a two component mixture with the non-homogeneous equation of statics in the theory of elastic mixture, and second order differential equations with variable coefficients. A dry mixture of sand and cement is transformed into a continuum, which is been determined as an entire or a meromorphic function, as a result of the existence of residues that are contained in the principal component of the mixture obtained directly from the earth. The time relationship with residue, in these two functions are determined. Our result shows that time places a limit on residues, making the meromorphic function prone to implosion.

KEYWORDS: Entire function, Meromorphic function, and Implosion.

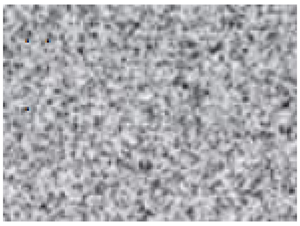
1 Introduction

The theories of mixtures in the framework of continuum mechanics have been developed throughout the sixties and seventies, and subsequent development in various constitutive theories and continuum analysis are too numerous to document [1]. A mixed problem of elasticity was solved in [2, 3] for a convex polygon and for doubly connected domain with a polygonal boundary. Also, linear and non-linear static bodies were solved by [4]. [2,5] discussed the entropy flux of transversely isotropic elastic bodies of homogeneous type, while [6, 7, 8] gave a solution of a non-classical problem of oscillation of two component mixtures. A fundamental solution of the system of differential equations of stationary oscillation of two-temperature elastic mixture theory was provided by [9]. [10] worked on the problem of boundary valued equations for force term in a non-homogeneous equation of statics in the theory of elastic mixture in [10], while [11] gave the biharmonic solution for the forcing term in a non-homogeneous equation of statics in the theory of elastic mixture.

From the available literature, time impact on residue in a non-homogeneous equation of statics in the theory of elastic mixture has not been solved. In this paper, we analyze the time impact on residues in a two component mixture using special functions. A dry mixture of sand and cement is transformed into a continuum, which is been determined as an entire, or a meromorphic functions, due to the existence of residues that are domiciled in the principal component of the mixture obtained directly from the earth . This study is therefore, aimed at analyzing the time impact on these two forms of functions, resulting from a single mixture, using second order differential equations with variable coefficient, and graphs.

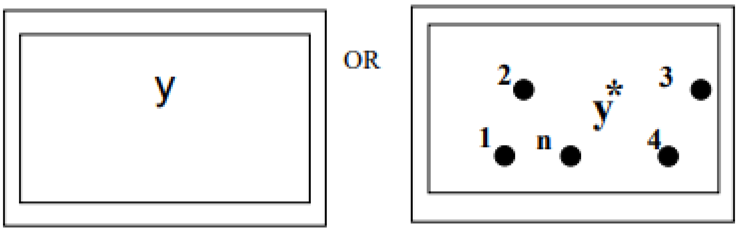
2 Mathematical Formulation

Consider a discrete mixture of sand(x_1) and cement(x_2) on a complex plane $z = x_1 + ix_2$, with the principal component of the mixture x_1 containing 0 or n number of residues. On transformation [$w = f(z) = u(x_1, x_2) + iv(x_1, x_2)$], x_1 *and* x_2 gives either y *or* $y^\star$; namely, an entire or a meromorphic function. If x_1 and x_2 transformed into y *or* $y^\star$. We are to ascertain the time (t) impact on y *and* $y^\star$, in order to determine the effect of residues (n) on the transformation(w). As shown in Figures 1, 2



$Z = x_1 + i\,x_2$

Figure 1: Discrete state of the mixture**.**



$w = f(z) = u(x_1, x_2) + iv(x_1, x_2)$

Figure 2: Transformed continuous state of the mixture.

3. SOLUTION METHOD

Using the equation as formulated by [12].

$$a\Delta u + c\Delta u + b\ grad\ div\ u + d\ grad\ div\ u = w \qquad (1)$$

where Δ is two-dimensional Laplacian, grad. and div. are principal operators of field theory, u is displacement vector and a, b, c, and d are combination of constitutive constants characterizing the physical properties of the mixture, while w is the transformed state of the mixture; defined as, $w = u + iv$ [12].

3.1 Theory of Complex Variable

Solving equation (1), using complex variable as follows:

$$z = x_1 + ix_2 \qquad (2)$$

And

$$\bar{z} = x_1 - ix_2 \qquad (3)$$

Adding equation (2) and (3)

$$2x_1 = z + \bar{z} \qquad (4)$$

Introducing partial differential operator to equation (4) [18]

$$2\frac{\partial}{\partial x_1} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} \qquad (5)$$

subtracting equation (3) from (2)

$$2ix_2 = z - \bar{z} \qquad (6)$$

expressing equation (6) in partial differential equation

$$2i\frac{\partial}{\partial x_2} = \frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}} \qquad (7)$$

adding equation (5) and (7), we have

$$2\frac{\partial}{\partial x_1} + 2i\frac{\partial}{\partial x_2} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} + \frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}$$

$$2\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right) = 2\frac{\partial}{\partial z} \qquad (8)$$

subtracting equation (5) from (7); that is,

$$2\frac{\partial}{\partial x_1} + 2i\frac{\partial}{\partial x_2} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} - \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}$$

$$2\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right) = 2\frac{\partial}{\partial \bar{z}} \qquad (9)$$

multiplying equation (8) and (9); we have,

$$2\frac{\partial}{\partial z}.2\frac{\partial}{\partial \bar{z}} = 2\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right).2\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)$$

$$4\frac{\partial^2}{\partial z\partial \bar{z}} = 4\left(\frac{\partial^2}{\partial x_1^2} - i\frac{\partial^2}{\partial x_1\partial x_2} + i\frac{\partial^2}{\partial x_1\partial x_2} + \frac{\partial^2}{\partial x_2^2}\right)$$

$$4\frac{\partial^2}{\partial z\partial \bar{z}} = 4\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + 4i\left(\frac{\partial^2}{\partial x_1\partial x_2} - \frac{\partial^2}{\partial x_1\partial x_2}\right) \qquad (10)$$

Equating like terms of equation (10) to the real and imaginary parts of equation (5) and (7) respectively; we have,

$$4\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) = 4\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}\right) \qquad (11)$$

$$4i\left(\frac{\partial^2}{\partial x_1\partial x_2} - \frac{\partial^2}{\partial x_1\partial x_2}\right) = -4i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right) \qquad (12)$$

Replacing the two right hand terms of equation (10) with the right hand terms of equation (11) and (12); to have,

$$4\frac{\partial^2}{\partial z\partial \bar{z}} = 4\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}\right) - 4i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right) \qquad (13)$$

Let the displacement vector component u in equation (1), be represented in its complex form as:

$$u_1 + iu_2 = \varphi \tag{14}$$

and

$$u_1 - iu = \bar{\varphi} \tag{15}$$

Substituting equation (14) and (15) into equation (13) as components of the dependent variable of the partial differential equation; to have,

$$4\frac{\partial^2\varphi}{\partial z\partial\bar{z}} = 4\left(\frac{\partial\varphi}{\partial z} + \frac{\partial\bar{\varphi}}{\partial\bar{z}}\right) - 4i\left(\frac{\partial\varphi}{\partial z} - \frac{\partial\bar{\varphi}}{\partial\bar{z}}\right) \tag{16}$$

To make equation (1) solvable, we shall as in [12]. Let

$$\Delta u = \frac{\partial^2\varphi}{\partial z\partial\bar{z}} \tag{17}$$

and

$$\frac{\partial\varphi}{\partial z} + \frac{\partial\bar{\varphi}}{\partial\bar{z}} = 2\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}\right) = 2\nabla . u = 2\theta \tag{18}$$

Substituting equation (17) and (18), for $\Delta u \ and \ div.u$ in equation (1); we have,

$$4a\frac{\partial^2\varphi}{\partial z\partial\bar{z}} + 4c\frac{\partial^2\varphi}{\partial z\partial\bar{z}} + 2b\nabla\theta + 2d\nabla\theta = \mathrm{w} \tag{19}$$

Note: Here, our Laplacian is defined as

$$\Delta = \nabla.\nabla = \frac{\partial}{\partial z}.\frac{\partial}{\partial\bar{z}}$$

Such that

$$\nabla = grad = \frac{\partial}{\partial z}$$

Substituting for gradient (grad) in equation (19) with $\frac{\partial}{\partial z}$; we have,

$$4a\frac{\partial^2\varphi}{\partial z\partial\bar{z}} + 4c\frac{\partial^2\varphi}{\partial z\partial\bar{z}} + 2b\frac{\partial\theta}{\partial z} + 2d\frac{\partial\theta}{\partial z} = \mathrm{w} \tag{20}$$

$$\frac{\partial}{\partial z}\left(4a\frac{\partial\varphi}{\partial\bar{z}} + 4c\frac{\partial\varphi}{\partial\bar{z}} + 2b\theta + 2d\theta\right) = \mathrm{w} \tag{21}$$

$$\int d\left(4a\frac{\partial\varphi}{\partial\bar{z}} + 4c\frac{\partial\varphi}{\partial\bar{z}} + 2b\theta + 2d\theta\right) = \int w\, dz \tag{22}$$

$$4a\frac{\partial\varphi}{\partial\bar{z}} + 4c\frac{\partial\varphi}{\partial\bar{z}} + 2b\theta + 2d\theta = \oint w\, dz \tag{23}$$

3.2 Analytic Analysis of the Non-homogeneous Part

$$\oint w\, dz \tag{24}$$

Where

$$w = u + iv \ and \ z = x_1 + ix_2$$

So that equation (24); become,

$$\int (u + iv)d(x_1 + ix_2)$$

$$\int (u + iv)(dx_1 + idx_2) \tag{25}$$

Expanding equation (25) ; we have,

$$\int (udx_1 + ivdx_1 + iudx_2 - vdx_2) \tag{26}$$

$$\int [(udx_1 - vdx_2) + (ivdx_1 + iudx_2)]$$

$$\int (udx_1 - vdx_2) + i\int (vdx_1 + udx_2) \tag{27}$$

From Green's Theorem, the line integral in equation (28) become an area integral [13] ; such as,

$$\iint \left(-\frac{\partial v}{\partial x_1} - \frac{\partial u}{\partial x_2}\right)dx_1dx_2 + i\iint \left(\frac{\partial u}{\partial x_1} - \frac{\partial v}{\partial x_2}\right)dx_1dx_2 \tag{28}$$

equating like terms in equation (27) and (28); that is,

$$\iint \left(-\frac{\partial v}{\partial x_1} - \frac{\partial u}{\partial x_2}\right)dx_1dx_2 = \int (udx_1 - vdx_2) \tag{29}$$

and

$$\iint \left(\frac{\partial u}{\partial x_1} - \frac{\partial v}{\partial x_2}\right)dx_1dx_2 = \int (vdx_1 + udx_2) \tag{30}$$

From Cauchy Morare Theorem [13], the line integrals in equation (29) and (30) is equal to zero; leaving,

$$\iint\left(-\frac{\partial v}{\partial x_1}-\frac{\partial u}{\partial x_2}\right)dx_1dx_2 = 0 \Rightarrow \frac{\partial v}{\partial x_1} = -\frac{\partial u}{\partial x_2} \tag{31}$$

and

$$\iint\left(\frac{\partial u}{\partial x_1}-\frac{\partial v}{\partial x_2}\right)dx_1dx_2 = 0 \Rightarrow \frac{\partial u}{\partial x_1} = \frac{\partial v}{\partial x_2} \tag{32}$$

Equation (31) and (32) satisfy Cauchy Riemman condition, the necessary condition for the transformed mixture (w) of equation (23) to be analytic [13].

4. TIME IMPACT ON RESIDUE

The analytic state of the transformed mixture (w) is categorized into two forms; name, (i) an entire function (y) and (ii) a meromorphic function ($y^{\star}$).

(i) A transformed mixture [$w = u(x_1,x_2) + iv(x_1,x_2)$] is an entire function (y) when it is analytic at every point in a given domain; that is, it contains no pole (residue), represented by Taylor series, which gives an infinite radius of convergence [13]. To see this, we formulate Cauchy-Euler's equation of the entire function (y) as the dependent variable and time (t) as the independent variable as:

$$t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} + y = 0 \tag{33}$$

With the transformer $x = e^t$, equation (33) becomes a second order linear differential equation with constant coefficient [14]; that is,

$t = \ln x$

$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx} = \left(\frac{1}{x}\frac{dy}{dt}\right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{d^2y}{dt^2}\times\frac{dy}{dt}\right) - \frac{1}{x^2}\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0 \tag{34}$$

Whose auxiliary equation is,

$$m^2 - 2m + 1 = 0$$

with roots, m = 1 twice.

Giving a complementary solution

$$y = (c_1t + c_2)e^t = (c_1t + c_2)(2.718)^t \text{ [14]}$$

Assigning our arbitrary constants, $c_1 = c_2 = 1$; we have,

$$y = (t+1)(2.718)^t$$

Hence, the entire function (y) can be simulated against time (t) with the table generated from the solution above as:

Table 1.1

t	0	1	2	3	4
y	1	5	22	80	273

(ii)A transformed mixture [$w = u(x_1,x_2) + iv(x_{1,}x_2)$], is meromorphic ($y^{\star}$) if it is analytic with finite number of poles (residues). It is represented by Laurent series. Laurent series is made of an analytic part (represent by Taylor series), and the principal part [represented by Residual Theorem (n)], which gives a finite radius of convergence on simulation [13].To show this, we formulate Bessel's equation of the mixture, taking the meromorphic function ($y^{\star}$) as the dependent variable, and time (t) as the independent variable.

That is,

$$t^2\frac{d^2y^{\star}}{dt^2} + t\frac{dy^{\star}}{dt} + (t^2 - n^2)y^{\star} = 0 \tag{35}$$

Where (-n) is the residual term of the mixture.

Simplifying equation (35) using power series extension [15]; we have,

$$y^{\star} = a_0t^n[1 - \frac{t^2}{2.2(n+1)} + \frac{t^4}{2.4.2^2(n+1)(n+2)} - \cdots]$$

where

$$a_o = 2^n n!$$

so that

$$y^{\star} = J_n(t) = \frac{t^n}{2^n n!}[1 - \frac{t^2}{2.2(n+1)} + \frac{t^4}{2.4.2^2(n+1)(n+2)} - \cdots]$$

when the residue, n = 1;

$$y^{\star} = J_1(t) = \frac{t}{2} - \frac{t^3}{2.2.2} + \frac{t^5}{2.4.2^2.6} - \dots$$

The meromorphic function ($y^{\star}$), can be simulated against time (t) with the table generated from the solution above.

Table 1.2

t	1	2	3	4	5
$y^{\star}$	0.4	0.5	-0.2	-2.0	-5.3

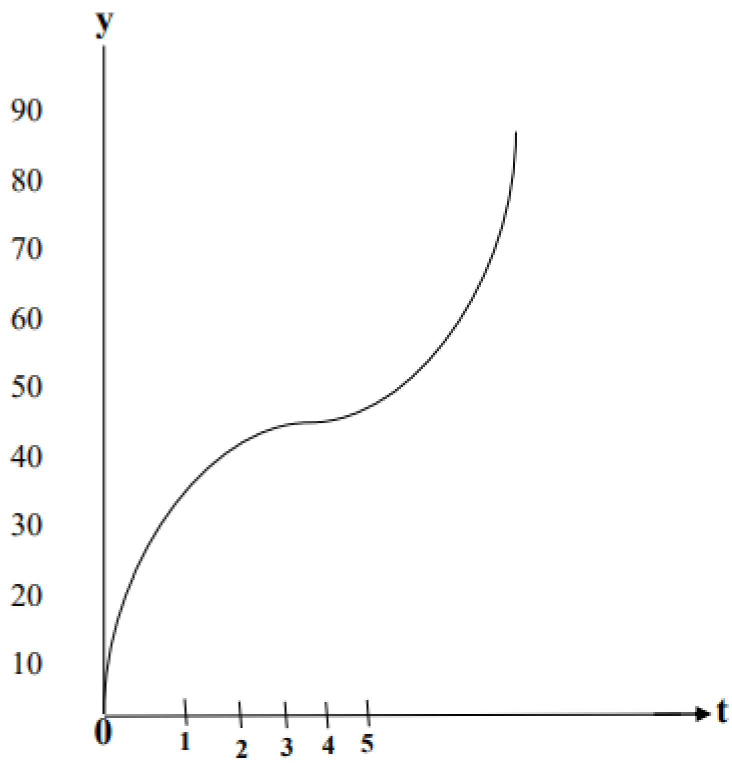


Figure 3: Graph of the Entire Function (y) against time (t)

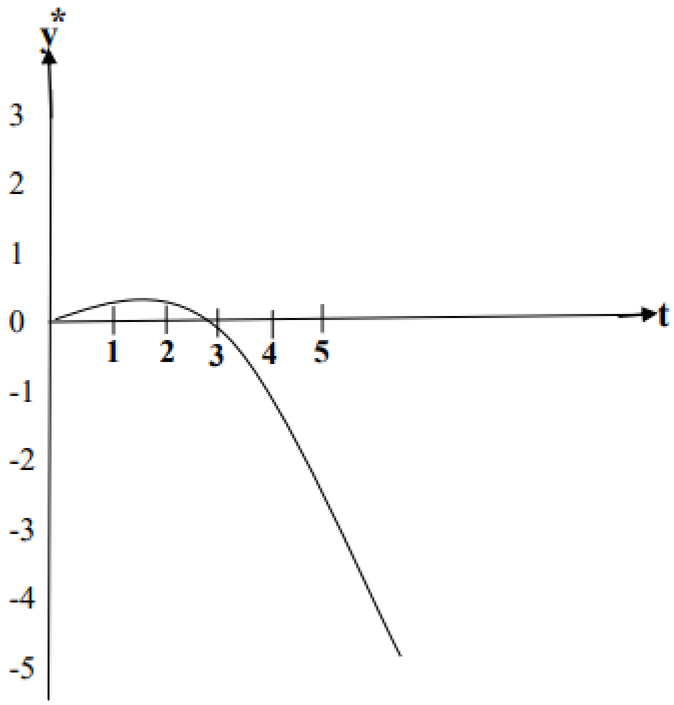


Figure 4: Graph of the Meromorphic Function (y*) against time (t)

5. DISCUSSION OF RESULT

The graphs in Figure 3, and 4 show the reaction of the continuum to time. Figure 3, shows an infinite radius of convergence; implying that, the entire function is not bounded by time; that is, continuum at this state is a constant. While, Figure 4, gives a finite radius of convergence; meaning, the continuum in the meromorphic state is limited by time.

6. CONCLUSION

Convergence in mathematics generally implies the existence of limit of any analytic system [11]. The continuum in the meromorphic state differs from the entire state, due to the existence of residues that are inherent in the mixture. The continuum in this state undergoes dehydration as time progresses in order to solidify, as it is usual with all concrete. This give rise to the contraction of the residues; thereby, creating voids at the spaces they occupy. This voids, alters the internal molecular matrix of the solid, making the solid prone to implosion (an inward explosion; that is, so sudden and complete)] with the smallest external pressure. [16] [17].

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