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Effects of Temperature Dependent Viscosity and Thermal Conductivity of A Micropolar Fluid Over A Stretching Surface With Radiation

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ABSTRACT

The effects of temperature dependent viscosity and thermal conductivity of a micropolar fluid over a continuous moving stretching surface with radiation is examined. The micropolar model due to Eringen is used to describe the working fluid. The partial differentia equations governing problem under consideration have been transformed into a system of non linear differential equations by the similarity transformations and then solved numerically using shooting technique. Numerical results are carried out for various dimensionless parameters of the problem especially variable viscosity parameter, thermal conductivity parameter, micro-rotation parameter along with the Prandtl number. The results are presented graphically for velocity distribution, temperature distribution and micropolar distributions for various values of non-dimensional parameters. It is found that the effects of the parameters representing variable property of viscosity and thermal conductivity are significant.

Key Words: Micropolar fluid, stretching surface, porous media, radiation.

1. INTRODUCTION:

The theory of micro polar fluids was originally formulated by Eringen ([3], [4]). In essence, the theory introduces new material parameters, an additional independent vector field, the micro rotation and new constitutive equations, which must be solved simultaneously with the usual equations for Newtonian flow. The desire to model the non-Newtonian flow of fluid containing rotating micro-constituents provided initial motivation for the development of the theory, but subsequent studies have successfully applied the model to a wide range of applications including blood flow, lubricants, porous media, turbulent shear flows and flowing capillaries and micro channels by Lukaszewiesz [7].

The theory of thermo-micropolar fluids has been developed by Eringen taking into account the effect of micro-elements of fluids on both the kinematics and conduction of heat. Later Ariman et al. [2] describe some of the various applications which have been explored. Boundary layer on continuous surface is an important type of flow occurring in a number of technical problems. The boundary layer flow of a micropolar fluid past a semi-infinite plate has been studied by Peddieson and McNitt [8] where as a similarity solution for boundary layer flow near stagnation point was presented by Ebert [5]. The boundary layer flow of micropolar fluids past a semi infinite plate was studied by Ahmadi[1] taking into account the gyration vector normal to the xy- plane and the micro-inertia effects. By drawing the continuous strips through a quiescent electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. Kelson and Farrell [6] studied micropolar flow over a stretching sheet with strong suction and injection.

Flow and heat transfer through porous media have several practical engineering applications such as transpiration cooling, packed bed chemical reactors, geothermal systems, that the radiation effect is important under many non isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristic. Raptis [9] studied the boundary layer flow of a micropolar fluid through non-Darcian porous medium. The problem of micropolar fluid flow over a continuously moving stretching surface through a fluid saturated porous medium with radiation is therefore an important one. It is now to study effects of variable viscosity and thermal conductivity of flow and heat transfer of an electrically conducting micropolar fluid on a continuously moving plate embedded in a non-Darcian porous medium in the presence of a radiation.

2. GOVERNING EQUATIONS :

The equation of motion for incompressible viscous micropolar fluid is given by

$$\rho\left\{\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}.\nabla\right)\vec{V}\right\}=-\nabla p+\nabla\left(\mu\nabla.\vec{V}\right)+\kappa\nabla^{2}\vec{V}+\kappa\left(\nabla\times\vec{N}\right)+\vec{F}\,, \tag{2.1}$$

where ρ is the mass density of the fluid, p is the pressure, μ is the viscosity, $\vec{N}$ is the angular velocity, κ is the material constant and t denotes time. $\vec{F}$ is the body force per unit volume due to flow through porous media given by

$$\vec{F}=\frac{\nu}{\lambda^{*}}\vec{V}\,, \tag{2.2}$$

where ν is the kinematic viscosity of the fluid and λ^* is the coefficient of permeability of the porous media.

The equation of angular momentum for incompressible viscous micropolar fluid is given by

$$\rho j\left\{\frac{\partial\vec{N}}{\partial t}+\left(\vec{V}.\nabla\right)\vec{N}\right\}=-2\kappa\vec{N}+\kappa\left(\nabla\times\vec{V}\right)-\gamma\left\{\nabla\times\left(\nabla\times\vec{N}\right)\right\}, \tag{2.3}$$

where j is the micro-inertia per unit mass, γ is the material constants. The equation of heat transfer is given by

$$\rho C_p\left\{\frac{\partial T}{\partial t}+\left(\vec{V}.\nabla\right)T\right\}=\nabla.\left(\lambda\nabla T\right)+\left(\mu+\kappa\right)\phi\,, \tag{2.4}$$

where C_p is specific heat at constant pressure, T is the temperature of the fluid, λ is the coefficient of thermal conductivity of the fluid, ϕ is the viscous dissipation function and is given by

$$\phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2, \tag{2.5}$$

3. MATHEMATICAL FORMULATION OF THE PROBLEM:

Consider a steady, two-dimensional laminar flow of an incompressible, electrically conducting micropolar fluid over a continuously moving stretching surface embedded in a non-Darcian porous medium which issues from a thin slit. The x-axis is taken along the stretching surface in the direction of the motion and y-axis is perpendicular to it. We assume that the velocity is proportional to its distance from the slit. A uniform magnetic field B_0 is imposed along y-axis. Under the usual boundary layer approximations, the flow and heat transfer of a micropolar fluid in porous medium with the non-Darcian effects included are governed by the following equations.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.1}$$

The equation of momentum is

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho k_1\frac{\partial N}{\partial y}-\rho u\phi\left(\frac{v}{k}+u\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{3.2}$$

equation of angular momentum is

$$G_1\frac{\partial^2 N}{\partial y^2}-2N-\frac{\partial u}{\partial y}=0 \tag{3.3}$$

the equation of energy is

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3.4}$$

where γ is the apparent kinematic viscosity, μ is the coefficient of dynamic viscosity, S is a constant characteristic of the fluid, Nis the microrotation component, $k_1=\frac{S}{\rho}(>0)$ is the coupling constant, $G_1(>0)$ is the microrotation constant, ρ is the fluid density, u and v are the components of velocity along x and y directions respectively, ϕ is the porosity, k is the permeability of the porous medium, T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the fluid far away from the plate, T_w is the temperature of the plat, λ is the thermal conductivity, C_p is the specific heat at constant pressure, and q_r is the radiative heat flux.

The appropriate physical boundary conditions of equations are

$$y=0: \quad u=ax, \quad v=0, \quad T=T_w, \quad N=0 \tag{3.5}$$

$$y\to\infty : u\to 0, T\to T_\infty, \; N\to 0 \tag{3.6}$$

The governing equations subject to the boundary conditions can be expressed in a simpler form by introducing the following transformations:

$$\eta=\sqrt{\frac{a}{\nu}}y, \; \psi=\sqrt{a\nu}xf, \; N=\sqrt{\frac{a^3}{\nu}}xg \tag{3.7}$$

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}$$

using the Rosselant approximation , we have

$$q_r=\left(-\frac{4\sigma_0}{3k_0}\right)\frac{\partial T^4}{\partial y} \tag{3.8}$$

where σ_0 is the Stefan-Boltzmann constant and k_0 is the mean absorption coefficient. The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki [10]) as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\alpha\left(T-T_\infty\right)\right], \frac{1}{\mu}=a\left(T-T_r\right), \; a=\frac{\alpha}{\mu_\infty} \text{ and } T_r=T_\infty-\frac{1}{\alpha} \tag{3.9}$$

where a and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general a>0 for liquids and a<0 for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T=T_\infty$ similarly, consider the variation of thermal conductivity as,

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\xi\left(T-T_\infty\right)\right],\frac{1}{\lambda}=b\left(T-T_k\right),\, b=\frac{\xi}{\lambda_\infty} \text{ and } \quad T_k=T_\infty-\frac{1}{\xi} \tag{3.10}$$

where b and T_k are constants and their values depends on the reference state and thermal property of the fluid ξ is constant based on thermal property and λ_∞ is the thermal conductivity at $T=T_\infty$. Using equation (3.7), it can be easily verified that the continuity equation is satisfied automatically and using equation (3.7) - (3.8) in the equation (3.2) - (3.4) become,

$$f'''=D_a^{-1}f'+\left(1+\alpha\right)\frac{\theta_r-\theta}{\theta}f'^2-\frac{\theta_r-\theta}{\theta}Cg'-\frac{1}{\theta_r-\theta}f''\theta'-\frac{\theta_r-\theta}{\theta}ff'' \tag{3.11}$$

$$Gg''-\left(2g+f''\right)=0 \tag{3.12}$$

$$\left[3F\frac{\theta_k}{\left(\theta_k-\theta\right)}+4P_r\left(1+r\theta\right)^3\right]\theta''+\left[3RP_rf+3F\frac{\theta_k}{\left(\theta_k-\theta\right)^2}\right]\theta'+4P_r\left[3r\left(1+r\theta\right)^2\right]\theta'^2=0 \tag{3.13}$$

The transform boundary conditions are

$$\eta=0,\quad f'=1,\quad f=0,\quad g=0,\quad \theta=1 \tag{3.14}$$

$$\eta=\infty,\quad f'=0,\quad g=0,\quad \theta=0 \tag{3.15}$$

where

$C=\dfrac{k_1}{\nu}$ denotes the coupling constant parameter

$D_a^{-1}=\dfrac{\varphi\nu}{ka}$ denotes the inverse Darcy number

$\alpha=\varphi x$ denotes the inertia coefficient parameter

$P_r=\dfrac{\nu\rho C_p}{k}$ denotes the Prandtl number

$R=\dfrac{\rho C_p k_0 \nu}{4\sigma_0 T_\infty^3}$ denotes the radiation parameter

4. RESULTS AND DISCUSSION:

The system of coupled nonlinear ordinary differential equation (3.11-3.13) together with the boundary conditions(3.14-3.15) is solved numerically by using the fourth order Runge-Kutta method along with the shooting technique. We have considered in some detail the influence of the physical parameters D_a^{-1}, R , θ_k , θ_r on the velocity, micro rotation and temperature distributions which shown in figures

(1-4). Figures (1) and (2) show the velocity and micro rotation profiles for various values of D_a^{-1} and θ_r . From (1) ,It is seen that the velocity distribution increases with the increasing values of D_a^{-1}. Application of a transverse magnetic field normal to the flow direction gives rise to a resistive drag-like force acting in a direction opposite to that of flow. This has a tendency to reduce both the fluid velocity and angular velocity .This indicates that the velocity distribution and microrotation distribution decreases then increases with the increasing values of *R*. Figures (3) and (4) depict the influence of the thermal conductivity parameter θ_k and the radiation parameter *R* on the temperature distributions respectively. From figure (3), it is observed that the temperature distributions increases with the increasing values of thermal conductivity parameter θ_k . From (4), it is seen that the temperature distributions decreases as *R* increases.

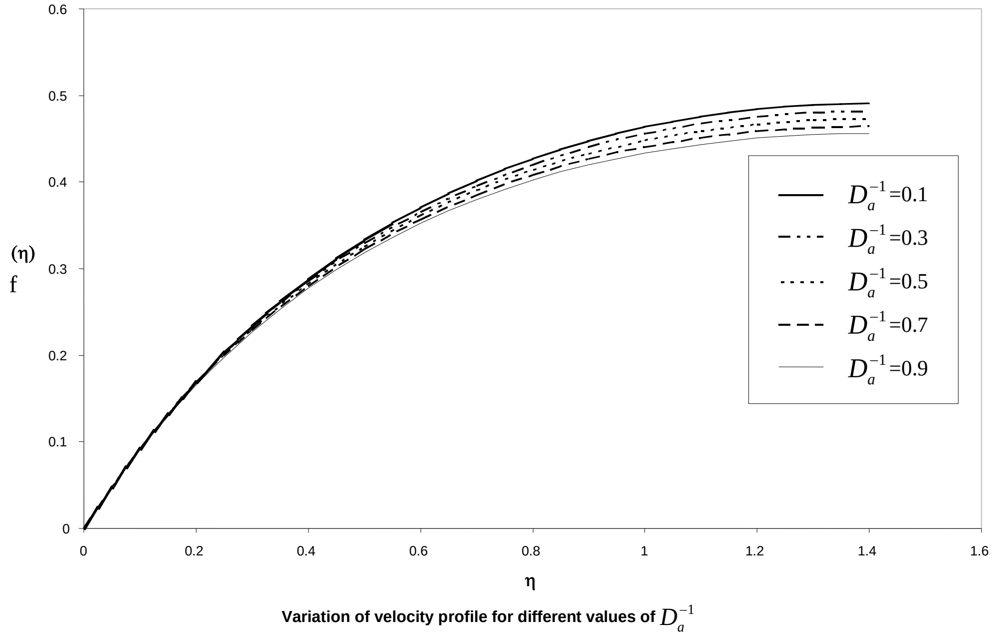

Fig. 1. Velocity distribution profiles along against η for various values of parameter D_a^{-1} taking

Pr=0.70 G=0.50 lm1=0.50 r=0.30 θr =-10.00 θ_k =-10.00

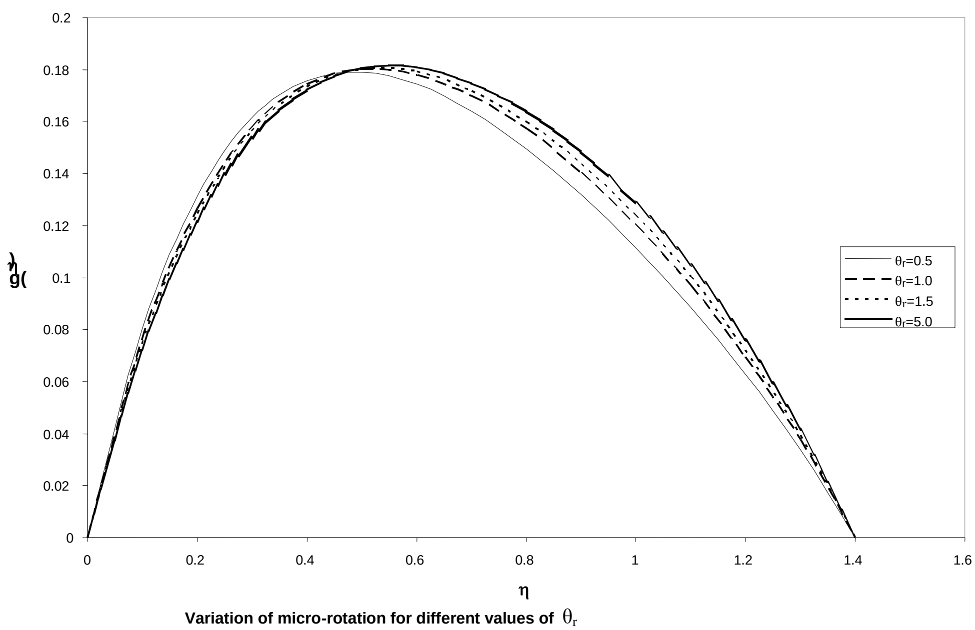

Fig. 2. microrotation distribution profiles along against η for various values of parameter θ_r

taking Pr=0.70 R=0.80 G=0.50 lm1=0.50 r=0.30 θ_k =-10.00

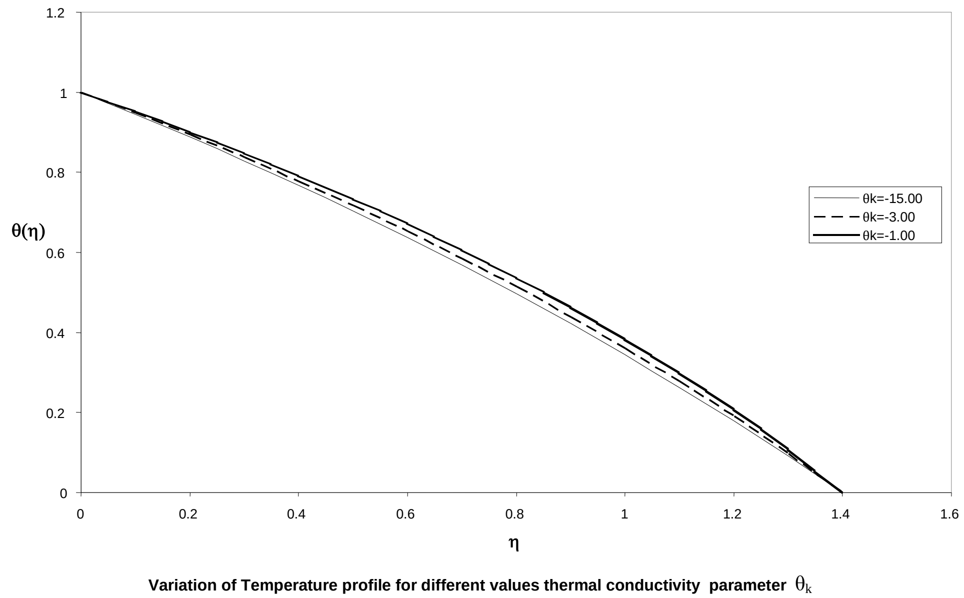

Fig. 3. Temperature distribution profiles along against η for various values of parameter θ_k taking Pr=0.70 R=0.80 G=0.50 lm1=0.50 D_a^{-1}=0.5 r=0.30 θ_r=-10.00

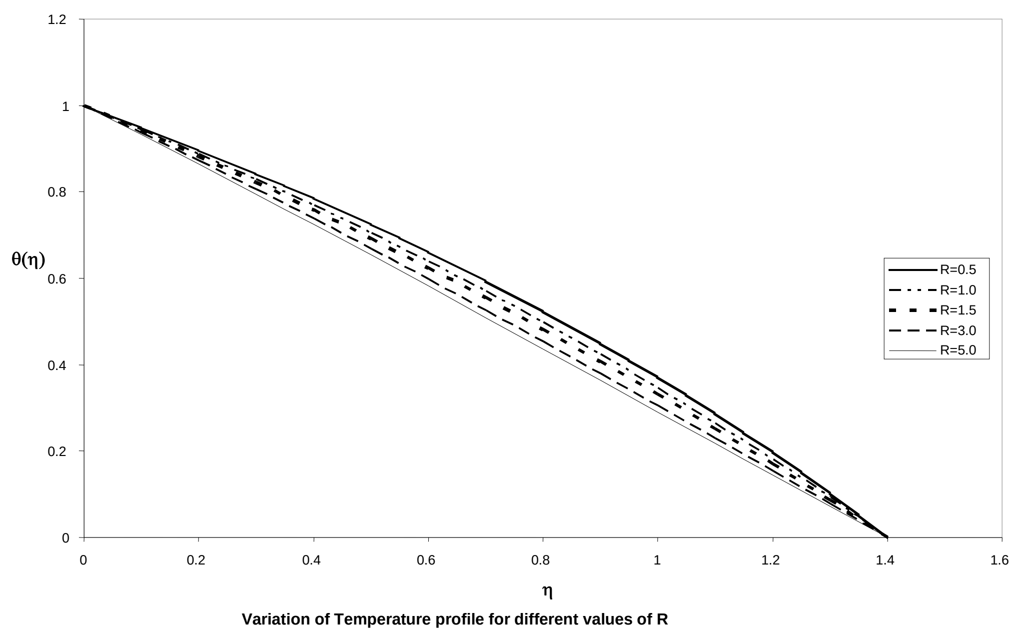

Fig. 4. Temperature distribution profiles along against η for various values of parameter R taking Pr=0.70 G=0.50 lm1=0.50 D_a^{-1}=0.5 r=0.30 θ_r =-10.00 θ_k =-10.00

5. CONCLUSION:

In this study the effects of temperature dependent viscosity and thermal conductivity of a micropolar fluid over a continuous moving stretching surface with radiation is examined. The resulting partial differential equations, which describe the problem, are transformed into ordinary differential equations by using similarity transformations. Numerical evaluations are performed and graphical results are obtained. The results presented demonstrate clearly that the viscosity and thermal conductivity parameters have a substantial effect on velocity distribution, micropolar distribution and temperature distribution. The effect of inverse Darcy number D_a^{-1}, viscosity parameter, thermal conductivity parameter, Prandtl number P_r radiation parameter R are quite significant.

6. REFERENCES:

[1] Ahmadi, G., Self-similar solution of imcompressible micropolar boundary layer flow over a semi-infinite plate, international Journal of Engineering Science, 14,(1976),no7,639-646.

[2] Ariman T., Turk M.A. and Sylvester N.D Microcontinum fluid mechanics ,Int. J.Engng. Sci., 11,(1973) ,905-930.

[3] Eringen, A.C., Theory of micropolar fluids, Journal of Mathematics and Mechanics, 16,(1966),1-18.

[4] Eringen, A. C., J. Math. Anal. Appl., Vol.38, (1972), 480

[5] Ebert F. A similarity solution for the boundary layer flow of a polar fluid. The Chem. Eng. J.,5,(1973),85-92.

[6] Kelson N.A. and Farrell T.W Micropolar flow over a porous stretching sheet with strong suction or injection. Int. Comm. Heat Mass Transfer, 28, .(2001), 479-488.

[7] Lukaszewiez,G. Micropolar fluids, theory and applications, Boston, (1999).

[8] Peddieson J. and McNitt R.P, Boundary layer theory for a micropolar fluid , (1970).

[9] Raptis ABoundary layer flow of a micropolar fluid through a porous medium. Journal of Porous Media, 3(1),(2000) , 95-97.

[10] Lai, F. C. and Kulacki, F. A., The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous media, Int. J. Heat Mass Transfer ,Vol.33,No.5, (1990), 1028-1031.
