International Journal of Mathematics and Computer Research
ISSN: 2320-7167
Volume 09 Issue 07 July 2021, Page no. – 2344-2347
Index Copernicus ICV: 57.55, Impact Factor: 7.184
DOI: 10.47191/ijmcr/v9i7.03

Analytical Solution of Two-Phase Incompressible Flow Equation in a Homogeneous Porous Medium

Zuonaki Ongodiebi[1], Orukari, M. A.[2]
[1,2] Department of Mathematics and Computer Science, Niger Delta University, P.M.B 071 Amassoma, Bayelsa State.

ARTICLE INFO
Published Online: 12 July 2021
Corresponding Author: **Zuonaki Ongodiebi**

ABSTRACT
In this research, we present analytical solution of two phase incompressible flow through a homogeneous porous medium. Water was injected at one end of the porous medium to stimulate oil recovery at the other end. From the modelled equations, we are able to determine pressure variation at different depth profiles. The results revealed increase in pressure as depth increases. This is in line with what is obtainable in practical scenarios.

KEYWORDS: Incompressible flow, oil recovery, pressure, Porous media

1. INTRODUCTION

The study of fluid flow through porous media consists of solving the conservation of mass and the balance of momentum on a representative elementary volume (REV). This is essential to numerous environmental, biological and industrial systems; such as the movement of contaminants in the subsurface and their remediation, geologic nuclear waste disposal, medical application such as brain and liver cancer treatment and most notably in oil recovery from petroleum reservoirs Arezou *et al.* (2019) are some examples of porous media transport. In petroleum reservoirs, the inherent heterogeneity of subsurface porous media, as well as the complexity involved in the multiphase physics, highlights some of the most important technological challenges of our time (Knut-Andreas, 2015; Pan and Miller 2003; Nagi, 2009). One of such complexities is that there are no definite flow paths in porous media thereby making porous media flow capacity as a function of pressure difficult to estimate. Due to the complex nature of multiphase flow, nonlinearity of their governing equations and reservoir intricacies, finding analytical solutions to practical fluid flow problems is extremely difficult and rarely reported in literature; thereby, motivating the present study. In this study, we analytically solve and analyzed one dimensional case of two phase flow scenario where water is injected at one end of a cylindrical porous slab in form of a pipe to stimulate oil production at the other end as shown in figure 2.1.

2. ANALYTICAL SOLUTION OF ONE-DIMENSIONAL INCOMPRESSIBLE POROUS MEDIA FLOW

In order to determine the analytical solution of one dimensional incompressible flow equation through porous media, the following assumptions are made (i) the flow is along a single spatial dimension x (ii) the reservoir model has a constant cross-sectional area A at all locations x with length x_L and (iii) the rock and fluids are incompressible and there are no sources and sinks. Furthermore, water is injected at a specified volumetric flow rate Q at location $x=0$ which stimulates fluid production at $x=x_L$. The effect of capillary pressure is neglected as is often done in two-phase reservoir studies; Jiseng and Shuyu (2010)

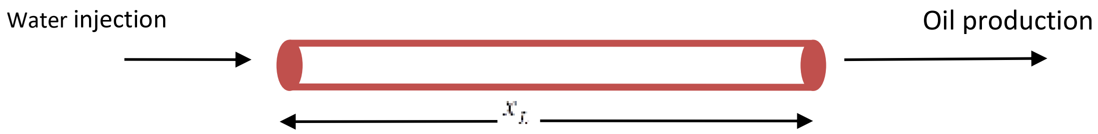

Figure 2.1: Schematic Reservoir Model

The equations describing our system are as follows:

$$\phi\frac{\partial S_w}{\partial t}=-\frac{\partial}{\partial x}u_w \tag{2.1}$$

$$\phi\frac{\partial S_o}{\partial t}=-\frac{\partial}{\partial x}u_o \tag{2.2}$$

$$u_w=-\frac{Kk_{rw}}{\mu_w}\frac{\partial P}{\partial x} \tag{2.3}$$

$$u_o=-\frac{Kk_{ro}}{\mu_o}\frac{\partial P}{\partial x} \tag{2.4}$$

$$S_o+S_w=1 \tag{2.5}$$

Where $s_w, s_o, u_w, u_o, \mu_w, \mu_o, k_{rw}, k_{ro}, K, P$ refer to as water saturation, oil saturation, velocity of the water phase, velocity of the oil phase, viscosity of water, viscosity of oil, relative permeability of water, relative permeability of oil, absolute permeability and pressure respectively Ahmed and Mckinney (2005), Aziz and Settari (1979), Cheng (2012), Zhengxin et, al (2006) and Darcy (1856).

adding equations (2.1) and (2.2) results to

$$\frac{\partial}{\partial x}(\mathrm{u}_w+u_o)=0 \tag{2.6}$$

equation (2.6) implies that the total superficial velocity is independent of location (that is, it is constant throughout the domain). In this study, the total superficial velocity u_t is known at $x=0$ ($u_t=\frac{Q}{A}$), where Q is the volumetric flow rate injected. Note that u_t in this context does not imply change in u with respect to time t but a symbol to represent the combine velocity of the two fluids. Thus the solution of equation (2.6) is

$$u_t=u_w+u_o \tag{2.7}$$

Fractional flow of water f_w by Tore and Eyvind (2008) is defined as:

$$f_w=\frac{u_w}{u_t} \tag{2.8}$$

Now, using (2.8); equation (2.1) can be written as:

$$\frac{\partial S_w}{\partial t}=-\frac{u_t}{\phi}\frac{\partial f_w}{\partial x} \tag{2.9}$$

Eliminating the pressure gradients in equations (2.3) and (2.4) we have:

$$u_o=u_w\frac{k_{ro}}{\mu_o}\frac{\mu_w}{k_{rw}} \tag{2.10}$$

With equation (2.10) the following relation for f_w is obtained using equations (2.7) – (2.9):

$$f_w=\left(1+\frac{\mu_w}{\mu_o}\frac{k_{ro}}{k_{rw}}\right)^{-1} \tag{2.11}$$

Since the relative permeabilities are functions of saturation, equation (2.9) can be expressed as:

$$\frac{\partial S_w}{\partial t}=-\frac{u_t}{\phi}\frac{df_w}{dS_w}\frac{\partial S_w}{\partial x} \tag{2.12}$$

Given appropriate initial and boundary conditions, we can solve equation (2.12) for saturation distribution $S_w(x,t)$. For brevity, let S, f and f'represent water saturation, fractional flow of water and derivative of fractional flow of water with respect to saturation respectively. With these notations, equation (2.12) alongside its initial and boundary conditions become:

$$\frac{\partial S}{\partial t}=-\frac{u_t}{\phi}f'\frac{\partial S}{\partial x} \tag{2.13}$$

$$S(x,0)=S_{in}(x) \tag{2.14}$$

$$S(L,t)=1-S_{ro} \tag{2.15}$$

where S_{ro} is the residual oil saturation. Now, from equation (2.3), we can solve for the pressure $P(x,t)$. From the definition of fractional flow of water f_w in equation (2.8); equation (2.3) can be written as

$$u_t f=-\frac{Kk_{rw}}{\mu_w}\frac{\partial P}{\partial x} \tag{2.16}$$

by specifying the pressure at $x=0$, the appropriate boundary condition on pressure is given as:

$$P(0,t)=P_0(t) \tag{2.17}$$

The solution to equations (2.16) and (2.17) is

$$P(x,t)-\mathrm{P}_0(\mathrm{t})=u_t\mu_w\int_0^{x_L}\frac{f}{Kk_{rw}}dx \tag{2.18}$$

In this study, we are particularly interested in the pressure drop across the reservoir model; thus equation (2.18) is written as:

$$\Delta P(t)=P(x_L,t)-\mathrm{P}_0(\mathrm{t})=u_t\mu_w\int_0^{x_L}\frac{f}{Kk_{rw}}dx \tag{2.19}$$

3. PRESSURE GRADIENT DETERMINATION

This research also help to determine the analytical solution of one dimensional incompressible porous media flow as given in equation (2.19). Here we present the numerical values of change in pressure with reservoir depth. The parameter data presented in table 3.1 are adopted from Tore and Eyvind (2008) to determine the pressure gradient at different depths of the reservoir.

Table 3.1: Test case Parameter values

S/N	Parameter	Value
1	Viscosity of water μ_w $[kg/ms]$	0.001
2	Viscosity of oil μ_o $[kg/ms]$	0.02
3	Total superficial velocity u_t $[m/s]$	0.02
4	Phase permeability of water kk_{rw} $[m^2$	0.003
5	Fractional flow of water f $[m/s]$	0.0105
6	Initial pressure $[Psi]$	10000
7	Reservoir depths $[m]$	100, 200, 300, 400, 500, 600, 700, 800, 900, 1000

The parameter values from table 3.1 when applied to equation (2.19), gave the following values for $P(x_L,t)$ as displayed in table 3.2.

Table 3.2: Determination of total pressure and pressure gradient

S/N	x_L (m)	$P(x_L,t)$ [Psi]	$\Delta P\,[Psi]$
1	100	10002.1	2.1
2	200	10004.2	4.2
3	300	10006.3	6.3
4	400	10008.4	8.4
5	500	10010.5	10.5
6	600	10012.6	12.6
7	700	10014.7	14.7
8	800	10016.8	16.8
9	900	10018.9	18.9
10	1000	10021	21

Table 3.2 clearly shows that the pressure $P(x_L,t)$ has a linear relationship with depth.

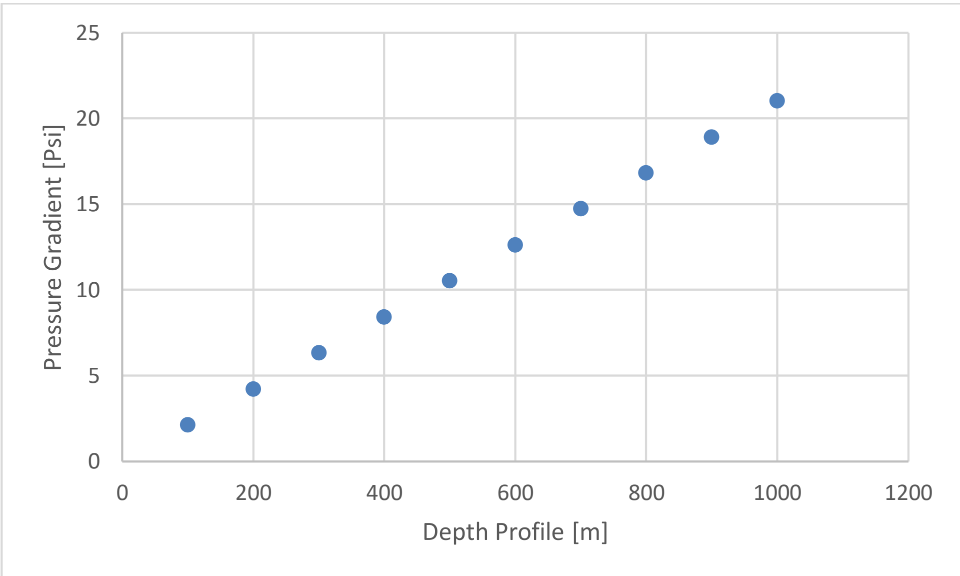

Figure 3.3: Pressure versus depth line graph

Figure 3.3 is a line graph plotted with the results presented in table 3.2. The graph reveals that as the depth increases, the pressure gradient also increases. This result is in line with physical reality; that is pressure increases with depth.

4. RESULTS AND DISCUSSION

4.1 CONCLUSION

In this research, we present analytical solution of two phase incompressible flow through a homogeneous porous medium.

Water was injected at one end of the porous medium to stimulate oil recovery at the other end. From the modelled equation, we are able to determine pressure variation at different depth profile. The results revealed increase in pressure as depth increases. This is in line with what is obtainable in practical scenarios. Sufficient understanding of pressure distribution within and the vicinity of production wells would be of great asset to oil exploration practitioners. This will aid in the establishment of effective reservoir monitoring and pressure maintenance plans in order to improve ultimate recovery from the target reservoir and other reservoir systems alike.

REFERENCES

1. Ahmed, T., McKinney, P. D. (2005). *Advanced Reservoir Engineering*, Gulf Professional Publishing, USA
2. Arezou, J., Mohammadreza, H., Mostafa, H. and Reza G. (2019). Application of CFD technique to simulate enhanced oil recovery processes: current status and future opportunities. *Springer* 1-23
3. Aziz, K. and Settari, A. (1979). *Petroleum Reservoir Simulation*, 2nd ed. Applied Science Publishers New York.
4. Cheng, Y. (2012). *Reservoir Simulation*. Petroleum Engineering-Upstream. COMSOL Multiphysics Users Guide.
5. Darcy, H. P. G. (1856). Les Fountaines Publiques de la Villa de Dijon, *Victor Dalamont*, Paris.
6. Jiseng, K and Shuyu, S. (2010) On iterative formulation for two phase flow with capillarity in heterogeneous porous media. International Journal of Numerical Analysis and Modeling. 1(1): 20-40
7. Knut-Andreas Lie (2015). *An introduction to reservoir simulation using MATLAB*. User Guide for the Matlab Reservoir Simulation Toolbox (MRST)
8. Nagi, A. A. (2009). Flow and Transport Problems in Porous Media Using CFD M.Sc thesis, Alexandria University.
9. Pan, M. H. and Miller, C. T. (2003). Lattice-Boltzmann Simulation of Two-phase Flow in Porous Media. *Journal of Water Resources Research*. 29(3), 40-53
10. Zhangxin, C., Huan, G. and Yuanle, M. (2006). Computational Methods for Multiphase Flows in Porous Media. *Society for Industrial and Applied Mathematics*, Philadelphia.