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Co-Homological Dimension of Random Graph Groups

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ARTICLE INFO	ABSTRACT
Published Online: 19 January 2022 Corresponding Author: **Abdel Radi Abdel Rahman Abdel Gadir Abdel Rahman**	Introduced the homological algebra and presented some interesting basic properties of the notion.In this paper we extend the above notion to homology groups and tried to proof the some similar basic properties of the topological homolog groups. We also studied more about the random graph groups of the homology order to find necessary and sufficient conditions for which the hematology is discrete. We followed the analytical induction mathematical method and we found that studying homology groups may be more important than cohomology groups.

KEYWORDS: Homology - Cohomological - Dimension –Graph –Random- Graph Groups -Sub Graph -Maximal – Complex - Homology Groups -Shine – Image

1. INTRODUCTION

Algebra homology is twentieth century field of mathematics that can trace its origins and connection and homology is one of the main idea of algebraic topology. Algebra's topology is one of the most important creation in mathematics which uses algebra tools to study homological groups.The most important of this invariants are homology groups, cohomology groups. The goal of this paper is to acquire uses study of some classes of algebraic homology (some underline geometry notation). Homology groups, graph and topology theory and cohomological diminution of random graph groups.

2. SIMPLICIAL COMPLEXES AND HOMOLOGY GROUPS

For now we shall deal mainly with dimensions 2 and 3. Animpticit exercise throughout this is to generalize all the definitions and proof to higher dimensions.

Definition (2.1):

An oriented 0 – simplex is apoint P: Anoriented 1- simplex is a directed line segment $P_1 P_2$. An oriented 2 = simplex is triangle $P_1 P_2 P_3$ with a prescribed order.

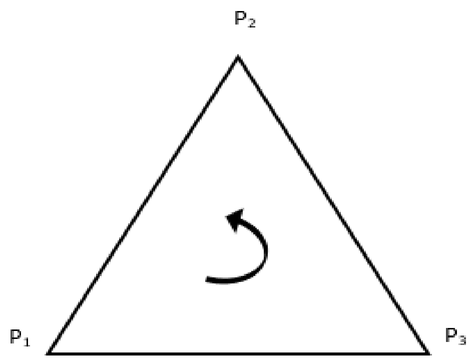

Figure No.2.1 An Oriented 2-Simplex

Notice that the simplex of opposite orientations is (defined to be) the negative of the simplex:

1. $P_1 P_2 = -P_2 P_1 \neq P_2 P_1$ (1)
2. $P_1 P_2 P_3 = P_2 P_3 P_1 = P_3 P_1 P_2 = -P_1 P_2 P_3 = -P_2 P_3 P_1 = -P_2 P_1 P_3$ (2)

An oriented 3 – simplex is a tetrahedron . $P_1 P_2 P_3 P_4$ with a prescribed orientation.

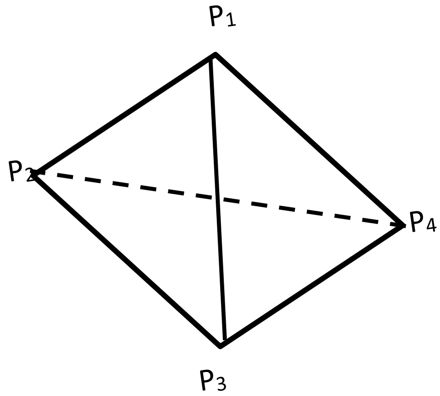

Figure No. 2:2 An Oriented 3-Simplex

Notice that 2- Simplexes:

$$P_i P_j P_k = sign \begin{bmatrix} 1 & 2 & 3 \\ i & j & k \end{bmatrix} P_1 P_2 P_3 \quad (3)$$

And this extend to higher dimensions.

$$P_i P_j P_k P_e = sign \begin{bmatrix} 1 & 2 & 3 & 4 \\ i & j & k & e \end{bmatrix} P_1 P_2 P_3 P_4 \quad (4)$$

The equation $\partial^2 = 0$. [1]

Theorem 2.2 : Let K be a simpelical complex. Then the homomorphism

$$\partial n_1 = \partial n : G_n(k) \rightarrow G_n - 2(k)$$

Is trivial that is, $\partial^2 = 0$

Proof: first we show that is enough to check this on simplexes, since these generate C_n

Then simply calculate: for example in dimension 2 we have

$$\partial_1(\partial_2(P_1 P_2 P_3)) = \partial_1(P_2 P_3 - P_1 P_3 + P_1 P_2) = (P_3 - P_2) - (P_3 - P_1) + (P_2 - P_1) = 0$$

Corollary 2.3: B_n (k) = $\partial_{n+1}(n+1\ (k))$ is a sub group of Zn (k) = ker (∂_n).[5]

3. HOMOLOGICAL ALGEBRA

We compute homology groups, and to construct the homology groups of pair of spaces. These turn out to be stronger invariants from chain groups.

Definition 3.1:

A chain complex < A, ∂> is a doubly infinite sequence $A = \{\dots, A_2, A_1, A_0, A_{-1}, A_{-2}, \dots\}$

Of Belgian groups A_k, together with A collection $\partial = \{\partial_k | \ k \in z\}$ Of homeomorphisms such that $\partial_k : A_k \rightarrow A_{k-1}$

And $\partial_k \partial_{k-1} = 0$

For braving we shall sometimes denote the chain complex by< A, ∂> A. in a chain complex, it is clear that the image of ∂_k is a subgroups of the kernel of ∂_{k-1} [3]

Definition 3.2:

If A is a chain complex, then the kernel $z_k(A)$of ∂_k is the group of K- cycles, and the image $B_k(A) = \partial_{k+1}(A_{k+1})$

Is the group of K- boundaries, the factor group

$H_k(A) = Z_k(A)/B_k(A)$

Is the K the homology group of A. [11]

Example 3.3:

Let X be the one – dimensional simplicial complex shown in figure 3.3

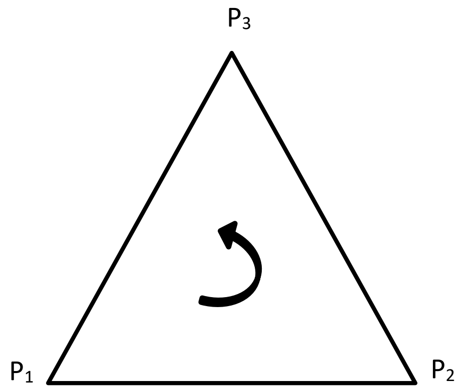

Figure No. 3:3: The Simplicial Complex X

And let y be the sub complex consisting of the edge $P_1\,P_3$
We know that

$$H_1(x) \cong Z \qquad [2]$$

Geometrically, shrinking $P_2\,P_3$ to a point collapses the rim of the triangle, as shown in figure 3.4

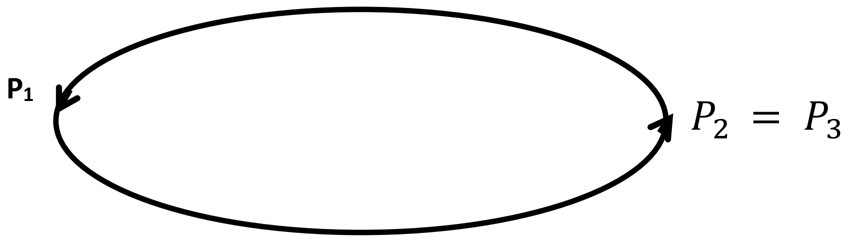

Figure No. 3.4 The Simplicial Complex X with Edge $P_2\,P_3$ Shrunk to a Point

Theorem 3.4The Chain Complex(3.1) is Exact:

The exact sequence is called the exact homology sequence of the pair (A, A*). If the chain complexes arise from a pair of topological spaces. In this case , we obtain for subcomplex L of the simplicial complex K,

$$\xrightarrow{\partial * k+1} \to H_k(L) \xrightarrow{i*k} \to H_k(k) \xrightarrow{j*k} \to H_k(k, L)$$
$$\xrightarrow{\partial * k} \to H_{k-1}(L) \xrightarrow{i*k-1} \to H_{k-1}(k) \xrightarrow{i*k-1} \to H_{k-1}(k, L) \xrightarrow{i*k-1}$$
$$\xrightarrow{j*1} \to H_1(KL) \xrightarrow{j*1} \to H_0(k) \xrightarrow{j*0} H_0(k, L) \to 0$$.[9]

4. Graph:

Example 4.1 Two important families graphs are the complete graphs and the complete bipartite graphs. The complete graphs on n vertices is the graph K_n that has n vertices and a collection of edges such that each pair of distinct vertices is joined by a single edge (see figure 4.1)

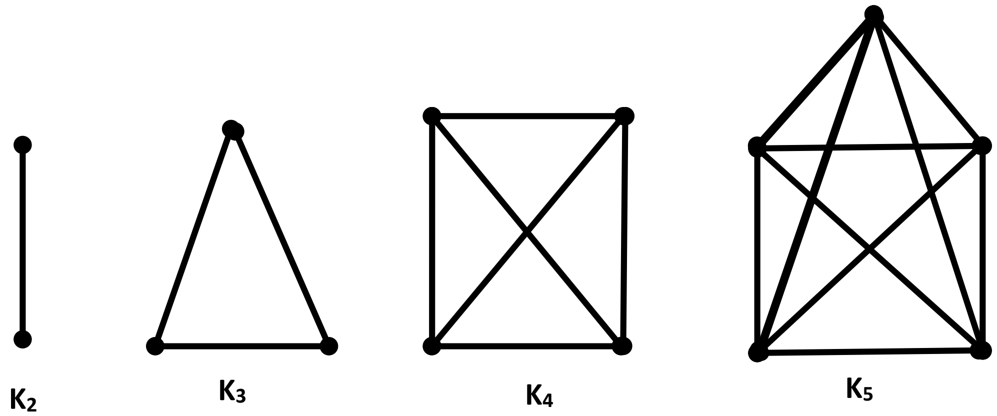

Figure N0. 4.1: The Complete Graphs K_2 , K_3 , K_4 , K_5 [8]

Definition 4.2:
The complete bipartite graph K_{mn} is a graph having mtn vertices, respectively, such that:
i. Each edge joined a vertex in $V_m$to a vertex in V_n,
ii. Each pair of vertices $V \in V_m$ and $V \in V_n$ is joined by a single edge.

Examples 4.3 Of Complete Bipartite Graphs are Shown in Figure 4.3

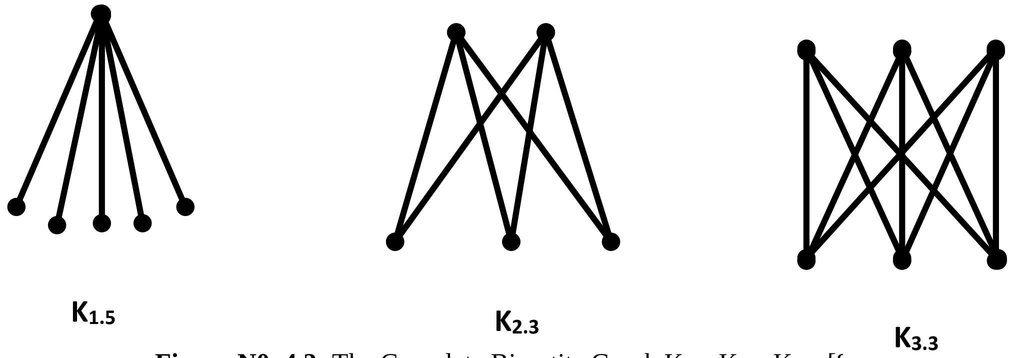

Figure N0. 4.3: The Complete Bipartite Graph K1.5, K2.3, K3.3 [8

Theorem 4.4: Let G and G^y be graphs with vertex sets $V_G and$ V_C respectively if there is a homeomorphism $h: G \rightarrow G^r$ that's maps V_G bigectvely to $V_G$then G and G are isomorphic and the function $h_v V_G \rightarrow V_{Gr}$defined by hv (V) is a graph isomorphism.

Proof:
If we drop the assumption in this theorem that h maps the vertex set of G bigectvely to the vertex set of G, then it does not necessarily follow that G and G^- are homeomorphic but not isomorphic.

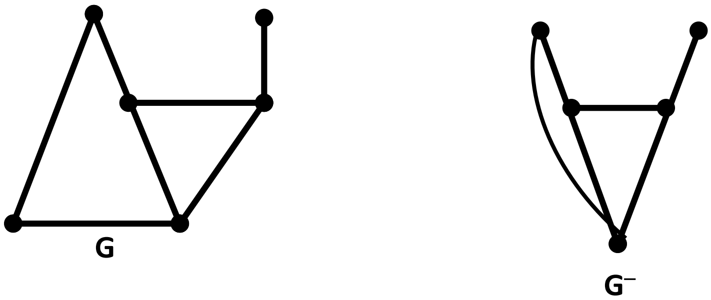

Figure No. 4.4 Graphs G and G^- are Hometmorphic but not Isomorphic [9]

5. L^2- COHAMOLOGY

Definition 5.1 We define the cellular

$$L^2\text{- cohamology by } H^p_{(2)}(\bar{X}.\Gamma) = \ker(d^p)/\overline{\imath m}(d^{p-1})$$

We have affodge decomposition

$$G^p_{(2)}(\bar{M}.\Gamma) = Ker\left(\Delta_p\right)\oplus\overline{\imath md} + \oplus\overline{\imath md^*} \ [5]$$

Theorem 5.2 L^2- cohomology and in particular L^2- Bett numbers have the following basic properties. Here, let $\bar{X}$ be a normal covering of a finite GW- complex X with covering group Γ.

1. Assume $\Gamma$is finite, then $\bar{X}$ is itself a finite GW- complex and it (ordinary) Bett number $b^p_{(2)}(\bar{X}.\Gamma) = \frac{1}{|\Gamma|}b^p(\bar{X})$

Proof:
If $\Gamma$is finite. all Hebert spaces in question are finite dimensional.
Consequently, im (d) is antomatically closed, and there is no difference between L^2- cohamology and ordinary cohomology with complex coefficients of $\bar{X}$ in particular.

$$b^p(\bar{X}) = dim_c H^p_{(2)}(\bar{X},\Gamma) = dim_\Gamma\left(H^p_{(2)}(\bar{X}.\Gamma)\right) = |\Gamma|.b^p_{(2)}(\bar{X}.\Gamma) \ [8]$$

Proposition 5.3: For G afinite group, the homology$H*(G,m).*> 0$, is totsion of exponent $|G|$ for any G- module M.
If M= Z with travial G- action, we write $H*(G) = H*(G,Z)$ here are some useful properties:
1. $H_1(G) = G/|G,G|$.
2.$If\ G = F/R$ were F is a free group and R is normal subgroup of F then $H_2(G) = R \cap [F,F]/[F,R](Hopf.sformular)$
3.If $G = B_1 * B_2$ is the amalgamated free product of B_1 and B_2 over a common subgroup A. then there is an exact sequence

$$\ldots\ldots \rightarrow H_i(B_1) \oplus H_i(B_2) \rightarrow H_i(G) \rightarrow H_{i-1}(A) \rightarrow n$$
$$4. H_i(GXZ) = H_i(G) \oplus H_{i-1}(G) \text{ [12]}$$

6. COHOMOLOGY

Definition 6.1:

Given a cohomology filtration F= F" of an R-module A the associated graded module is the graded, R-module denoted by Gr (A,F) and defined by

$$Gr(A,F)^p \frac{F^p}{F^{p+1}} \text{ [10]}$$

Definition 6.2:

A straight forward verification shows that b2 = 0 Thus, the Hoch child differential defines the two complexes :

$$0 \rightarrow C_\lambda^0(A) \xrightarrow{b} C_\lambda^1(A) \xrightarrow{b} C_\lambda^1(A) \xrightarrow{b} \ldots . (C.10)$$
$$0 \rightarrow C_\lambda^0(A) \xrightarrow{b} C_\lambda^1(A) \xrightarrow{b} C_\lambda^1(A) \xrightarrow{b} \ldots . (C.11)$$

The cohomology group of the complexes (C.10) and (C.11) are called the cyclic cohomology and the Hochschild home age groups of the complexescc-101 and can cohomology. respectively, of the algebra A and are denoted by

$$Hc^z(A) = Z_\lambda^n(A)/B_\lambda^n(A)$$
$$Hc^z(A) = Z_\lambda^n(A)/B_\lambda^n(A) \text{ [7]}$$

Definition 6.3:

A cohomology theory for Paws of Spaces with values in the category of R-modules consists of a family (h"Inez) of contravariantfunctors

$$h^n : ToP(2) \rightarrow R - MoD$$

and a family δ^n|nez of natural transformations

$$\delta^n : h^{n-1}ok \rightarrow h^n \text{[5]}$$

Definition 6.4:

Let $\bar{T} = \Omega^n \rightarrow c$ be acontinuous Linear functional possessing the cyclic invariance property:

$$\bar{T}w_1w_2 = (-1)^{ki}\bar{T}w_2w_1, \quad w_1 \in \Omega^k, \bar{T} = w_2 \in \Omega^j, k+i=n$$

Each functional of this form will be called a graded track of degree non Ω. [5]

Definition 6.5:

Given a brigaded cohomology spectral sequence E_r^{pq}, dr and aggraded R-module A^* we say the spectral Sequence converges to A^* and write

$$E_2^{pq} \Rightarrow A^{p+q}$$

If:

1. For each (p, q) there exists an r_0 so that $dr: E_r^{p-r,q+r-1} \rightarrow E_r^{pq}$is 0 for all $r \geq r_0$, in particular there is an injection $E_{r+1}^{p,q} \rightarrow E_{r,q}^{p}$for all $r \geq r_0$.
2. there is a convergent filtration of A^*, so that for each n, thelimit $E_\infty^{p,q} = \cap_{rp \geq r0} E_r^{p,q}$ is isomorphic to the associated graded $\Rightarrow Gr(A^*)$. [6]

Definition 6.6:

If M is an A-module and p ≥ 0, then the path exterior power of M, denoted by $\Lambda^p M$, is the abelian group with the following Presentation.

$Generators: A\, xMx, \ldots, xM$ (P factorsm).

$$Relations: For\ all\ a, a' \in A\ and\ m_i, m_i \in M,$$
$$(a, m_i, \ldots, m_i + m_i, \ldots, m_p) = (a, m_i, \ldots, m_i, \ldots, m_p) + (a, m_i, \ldots, m'_i, \ldots, m_p)$$

For all i,

$$(a + a', m_i, \ldots, m_p) = (a, m_i, \ldots, m_p)(aa', m_i, \ldots, m_i, \ldots, m_p) = (a, m_i, \ldots, a'^{m_i}, \ldots, m_p)$$

For all i

$(a, m_i, \ldots, m_p) = 0\ if\ m_i = m_j\ for\ some\ i \neq j$

$$if\ p = 0, then\ \Lambda^0 M = A, andif\ p = 1, then$$

$\Lambda^1 M \cong M$ [11].

Definition 6.7:

For a based space x, we define the reduced cohomology of x to be

$$E^{-q}(x) = E^{q}(x,*)$$

These results a direct Sussi decomposition:

$$E^{*}(x) \cong E^{*}(x) \oplus E^{*}(*)$$

That is natural with respect to based maps for $* \in Acx$, the Summand $E^{*}(A)$ maps isomorphic ally under the maps $E^{*}(x) \to E^{*}(A)$, and the exactness axiom implies that there is a reduced long exact sequence

$$\dots \to E^{-q-1}(A) \xrightarrow{\delta} E^{q}(x,A) \to E^{-2}(x) \to E^{-2}(A) \to \cdots$$

The unreduced cohomology groups are recovered as the special cases.

$$E^{*}(x) = E^{*}(x_1) \text{ [10].}$$

Definition 6.8:

A cochain complex G is a z-graded R-module with a graded homogeneous homomorphism of degree = 1. It is customary to denote the components of a cochain complex by Superscripts c2 and call the graded homeomorphisms
$d^{q}: c^{q} \to c^{q+1}$ [6].

RESULTS

we showed that the discuss of the constant mean Co-homological dimension of random graph groups and we found the following results:

1. We showed that the homology group gives accurate and high-speed results.
2. The ability of the graph or the diagram to any group through the homology and Possibility of calculating the constant mean cohomological dimension of random graph a group.

CONCLUSION

we calculated the constant mean co-Homological dimension of random graph groups, we also discuss many concepts of Simplicial complexes and Homology groups Homological algebra, basic theories of graph and Homology.

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