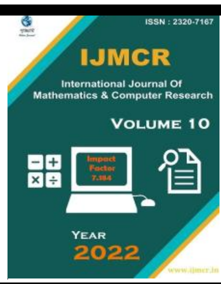
International Journal of Mathematics and Computer Research
ISSN: 2320-7167
Volume 10 Issue 03 March 2022, Page no. – 2613-2616
Index Copernicus ICV: 57.55, Impact Factor: 7.184
DOI: 10.47191/ijmcr/v10i3.03



Finding the Roots of Non-linear Equations Numerically using Newton's Raphson Method by A New Mathematical Technique

Abdel Radi Abdel Rahman Abdel Gadir Abdel Rahman[1], Subhi Abdalazim Aljily Osman[2], Hassan Abdelrhman Mohammed Elnaeem[3], Abdelhakam Hassan Mohammed Tahir[4], Musa Adam Abdullah[5], Wafaa Elsanousi Mukhtar Elsanousi [6]

[1]Department of Mathematics, Faculty of Education, Omdurman Islamic University, Omdurman, Sudan.
[2]Department of Mathematics, Faculty of Computer Science and Information Technology, University of ALBUTANA, Sudan.
[3]Department of Information Security, College of Computer Science and Information Technology, Karary University Khartoum, Sudan
[4]Department of Mathematics, Faculty of Education, Alsalam University, Alfula, Sudan
[5]Department of Mathematics, College of Computer Science and Information Technology, University of the Holy Quran and Tassel of Science, Sudan
[6]Department of Mathematics, College of Computer Science and Information Technology, University of the Holy Quran and Tassel of Science, Sudan

ARTICLE INFO	ABSTRACT
Published Online: 08 March 2022 Corresponding Author: **Abdel Radi Abdel Rahman Abdel Gadir Abdel Rahman**	Numerical methods are used to approximate solutions of equations when exact solutions can not be determined via algebraic methods. They construct successive approximations that converge to the exact solution of an equation or system of equations. The aim of this paper is to find the roots of Non-linear Equations Numerically using Newton's Raphson Method by A New Mathematical Technique**.** We followed the applied mathematical method using a new mathematical technique and we found the following some results: The New Mathematical Technique facilitates the process of finding the roots of non-linear equations of different degrees, the possibility of drawing these roots graphically in addition to the accuracy, speed and logicality of the solution and reduce errors compared to the numerical analytical solution manually.

KEYWORDS: Non-linear Equations, Mathematical Technique

1. INTRODUCTION

Numerical methods are used to approximate solutions of equations when exact solutions cannot be determined via algebraic methods. They construct successive approximations that converge to the exact solution of an equation or system of equations **[1].** Many of the complex problems in science and engineering contain the functions of nonlinear and transcendental nature in the equation of the form $f(x) = 0$ in single variable. The boundary value problems appearing in kinetic theory of gases, elasticity and other areas are reduced to solve these equations**[5].** Perhaps the most celebrated of all one-dimensional root-finding routines is Newton's method, also called the Newton-Raphson method. Joseph Raphson was a contemporary of Newton who independently invented the method in1690, some 20 years after Newton did, but some 20 years before Newton actually published it. This method is distinguished from the methods of previous sections by the fact that it requires the evaluation of both the function $f(x)$andthe derivative$f'(x)$, at arbitrary points x**[9].**

Newton's (or Newton-Raphson) method can be used to approximate the roots of any linear or nonlinear equation of any degree**[6].**

2. FUNCTIONS OF ONE VARIABLE:

Newton's method for finding the root of a function of one variable is very simple to appreciate. Given some point, say, x^k, we may estimate the root of a function, say $f(x)$, by constructing the tangent to the curve of $f(x)$ at xk and noting where that linear function is zero. Clearly for Newton's method to be defined we need $f(x)$to be differentiable. Algebraically the method is that of approximating the nonlinear function at the current iterate by a linear one and using the location of the zero of the linear approximation as the next iterate**[8].**

3. NEWTON'S METHOD

Assume that an initial estimate x_0 is known for the desired root a of $f(x) = 0$. Newton's method will produce a sequence of iterates $\{x_n : n \geq 1\}$which we hope will converge to α. Since x_0 is assumed close to α, approximate the graph of $y = f(x)$ in the vicinity of its root α by constructing its tangent line at $(x_0, f(x_0))$ Then use the root of this tangent line to approximate α; call this new approximation x_1• Repeat this process, ad infinitum, to obtain a sequence of iterates x_n. this leads to the iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n \geq 0 \qquad (1)$$

The process is illustrated in Figure 1, for the iterates x_1 and x_2 •

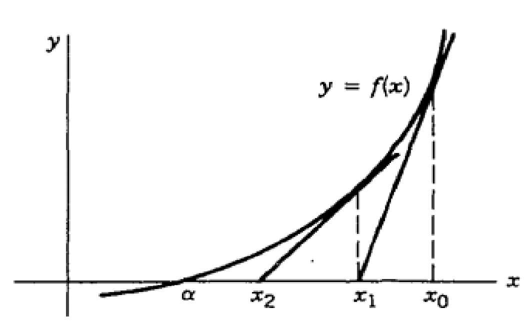


Figure 1. Newton's Method.

Newton's method is the best known procedure for finding the roots of an equation. It has been generalized in many ways for the solution of other, more difficult nonlinear problems, for example, systems of nonlinear equations and nonlinear integral and differential equations. It is not always the best method for a given problem but its formal simplicity and its great speed often lead it to be the first method that people use in attempting to solve a nonlinear problem.

$$f(x) = f(x_n) + (x - x_n)f'(x_n) + \frac{(x - x_n)^2}{2} f''(\xi)$$

withξ between x and x_n. *Letting* $x = \alpha$and using $f(a) = 0$, we solve for a to obtain

$$\alpha = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{(\alpha - x_n)^2}{2} \frac{f''(\xi)}{f'(x_n)}$$

withξ_n between x_n and α we can drop the error term (the last term) to obtain a better approximation to a than x_n and we recognize this approximation as x_{n+1} from (1). Then

$$\alpha - x_{n+1} = -(\alpha - x_n)^2 \frac{f''(\xi)}{f''(x_n)} \qquad n \geq 0 \qquad (2)\textbf{[4]}$$

Example 1 In this example we compute, approximately the square root of non-linear equation from second degree by applying Newton's method for the equation

$$f(x) = x^2 - 2 = 0$$

Since $f(x) = 2x$, Newton's method says that we should generate approximate solutions by iteratively applying

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{{x_n}^2 - 2}{2x_n} = \frac{1}{2}x_n + \frac{1}{x_n}$$

Since $1^2 = 1 < 2$ *and* $2^2 = 4 > 2$ the square root of two must be between 1 *and* 2, so let's start Newton's method with the initial guess $x_1 = 1.5$ Here goes:

$$x_1 = 1.5$$

$$x_2 = \frac{1}{2}x_1 + \frac{1}{x_1} = \frac{1}{2}(1.5) + \frac{1}{1.5} = 1.416666667$$

$$x_3 = \frac{1}{2}x_2 + \frac{1}{x_2} = \frac{1}{2}(1.416666667) + \frac{1}{1.416666667} = 1.414215686$$

$$x_4 = \frac{1}{2}x_3 + \frac{1}{x_3} = \frac{1}{2}(1.414215686) + \frac{1}{1.414215686} = 1.414213562$$

$$x_5 = \frac{1}{2}x_4 + \frac{1}{x_4} = \frac{1}{2}(1.414213562) + \frac{1}{1.414213562} = 1.414213562$$

Since $f\,(1.4142135615) = -2.5 \times 10^{-9} < 0$ and $f\,(1.4142135625) = 3.6 \times 10^{-9} > 0$ the square root of two must be between 1.4142135615 and 1.4142135625 **[2].**

Solution of example 1by A New Mathematical Technique:

```
% Example1
clc
clearall
symsxf(x)xnnyx1z
f(x)=x^2-2
Y=diff(f(x))
x=1.5
for s=1:5
 x=(1/2)*x + 1/x;
disp(x)
end
x=[1.5 1.4167 1.4142 1.4142 1.4142]
y=[1 2 3 4 5]
plot(x,y)
```

Solution:

```
f(x) =
x^2 - 2
Y =
2*x
x =
   1.5000
   1.4167
   1.4142
   1.4142
   1.4142
   1.4142
x =
   1.5000    1.4167    1.4142    1.4142    1.4142
y = 1     2     3     4     5
```

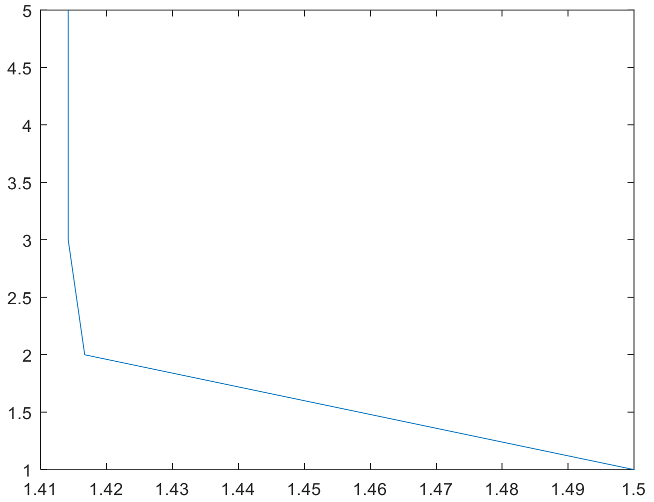


Figure 2. Newton's Method

Example 2: Find a real root of the equation $x\sin x + \cos x = 0$ using the Newton's Raphson method .

We have

$$f(x) = x\sin x + \cos x \quad and \quad f'(x) = x\cos x$$

The iteration formula is therefore

$$x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n}$$

With $x_0 = \pi$ the successive iterates are given below **[7]**

n	x_n	$f(x_n)$	x_{n+1}
0	3.1416	-1.0	2.8233
1	2.8233	-0.0662	2.7986
2	2.7986	-0.0006	2.7984
3	2.7984	0.0	2.7984

Solution of example 2 by A New Mathematical Technique:

```
% Example2
clearall
clc
symsxf(x)xnnyxn1
f(x)=x*(sin(x))+cos(x)
Y=diff(f(x))
x=pi
for s=1:5
 x=x-(x*(sin(x))+cos(x)/x*cos(x));
%  disp(xn)
disp(x)
end
x=2.8233
for s=1:4
 x=x-(x*(sin(x))+cos(x)/x*cos(x));
%  disp(xn+1)
disp(x)
end
xn=[3.1416 2.8233 1.6202 4.7156 -2.1206]
y=[0 1 2 3 4]
plot(xn,y)

f(x) =
cos(x) + x*sin(x)
Y =
x*cos(x)
x =
    3.1416
    2.8233
    1.6202
   4.7156e-04
  -2.1206e+03
  -2.0722e+03
x =
    2.8233
    1.6203
   4.7276e-04
  -2.1152e+03
 -428.4982
xn =
    3.1416    2.8233    1.6202    4.7156   -2.1206
y =
     0     1     2     3     4
```

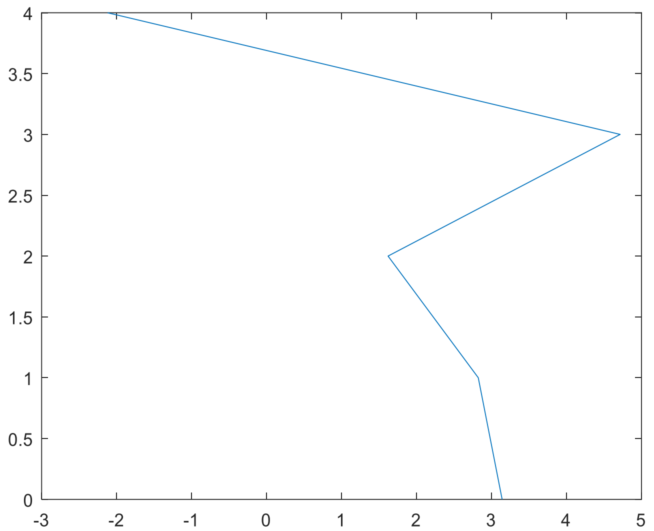


Figure 3. Newton's Method

4. ALGORITHM OF NEWTON'S METHOD

Input: Initial approximation p_0; tolerance TOL; maximum number of iterations N_0

Output: Approximate solution p, or failure message.

1. Set $i = 1$
2. While $i \le N_0$ do 3—6
3. Set $p = p_0 - {f(p_0)}/{f'(p_0)}$
4. If $|p - p_0| < \text{TOL}$ then

4a. output p

4b. stop program

5. Set $i = i + 1$

6. Set $p_0 = p$.
7. Output: “Failure after N_0iterations**[3].**

RESULTS

The importance of using aA Mathematical Technique to find the roots of non-linear equations because it reduces time and errors and helps in representing solutions graphically.

CONCLUSION

The Newton-Raphson method is used when you have some function $f(x)$ and you want to find the value of the independent variable x when the function equals zero. If you have an initial guess at some points x_i the tangent can be extended to some points that crosses 0 at an easily calculable point x_{i+1}. This point gives an improved estimation of the root so it’s a best to use Newton’s Raphson method by A New Mathematical Technique to reach improved , accurate and scientific solutions in a short time.

REFERENCE

1. Courtney Remani, Numerical Methods for Solving Systems of Nonlinear Equations, 201
2. Joel Feldman, Newton’s Method, October, 2012.
3. Joe Mahaffy, Numerical Analysis and Computing, 2010.
4. Kendall E. Atkinson, John Wiley &Sons , an introduction to numerical analysis, Second Edition , 1989.
5. ManojKuma,AkhileshKumarSingh and Akanksha Srivastava, Various Newton-type iterative methods for solving nonlinear equations, 2013.
6. Steven T. Karris, Numerical Analysis, Second Edition, .ISBN 0-9744239-1-2, .2019.
7. S.S. Sastry, Introductory Method s of Numerical Analysis, Fifth Edition, New Delhi- 110001, 2012.
8. Walter Murray, Newton-type Methods, July 5, 2010.
9. William H. Press, Saul A. Teukolsky, William T. Vetterling, Brian P. Flannery, NUMERICAL RECIPES, The Art of Scientific Computing Third Edition, 2007.