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Nonlocal Problem for a Third Order Equation

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ABSTRACT
This paper examines the nonlocal problem for the third-order equation.

KEYWORDS: fundamental solutions, technique, nonlocal problem, multiple characteristics.

1. INTRODUCTION

It is known that in the work of E.Del Vecchio, a technique was given for constructing fundamental solutions to an equation with multiple characteristics and, as an application, a fundamental solution of equations was constructed (see [1])

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial t} = 0, \quad (1)$$

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial t^2} = 0. \quad (2)$$

Further, L. Cattabriga, developing the work of E. Del Vecchio in 1961, studied the properties of the potentials of fundamental solutions of equation (1), i.e. constructed the theory of potentials of fundamental solutions of the equation (see [2]). Further researchers considered a number of boundary value problems for equation (1) with local and nonlocal boundary conditions, for example, (see [2]-[5]).

In this paper, the following problem is considered:

It is required to find the function $u(x,t) \in K_u$, which is a regular solution to the equation.

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial t} = 0. \quad (3)$$

in the area of $\Omega = \{(x,t): 0 < x < 1,\ 0 < t \le T\}$ and satisfies the conditions

$$u(x,0) = \mu u(x,T), \qquad \mu = const, \quad (4)$$

$$u_{xx}(0,t) = \varphi_1(t), \qquad u_x(0,t) = \varphi_2(t), \qquad u_x(1,t) = \psi(t). \quad (5)$$

Here $K_u = \{u(x,t):\ u(x,t) \in C_{x,t}^{4,1}(\Omega) \cap C_{x,t}^{2,0}(\overline{\Omega}),\ \ u_{xt} \in C(\overline{\Omega})\}$.

It is known that the fundamental solutions of Eq. (2) have the form (see [2]).

$$U(x-\xi; t-\tau) = (t-\tau)^{-1/3} f\left(\frac{x-\xi}{(t-\tau)^{1/3}}\right), \quad x \ne \xi, \quad t > \tau; \quad (6)$$

$$V(x-\xi; t-\tau) = (t-\tau)^{-1/3} \varphi\left(\frac{x-\xi}{(t-\tau)^{1/3}}\right), \quad x > \xi, \quad t > \tau. \quad (7)$$

Here

$$f(z)=\int_0^{\infty}\cos\left(\lambda^3-\lambda z\right)d\lambda,\quad -\infty<z<\infty,$$

$$\varphi(z)=\int_0^{\infty}(\exp\left(-\lambda^3-\lambda z\right)+\sin\left(\lambda^3-\lambda z\right))d\lambda,\quad z>0,$$

$$z=(x-\xi)(t-\tau)^{-1/3}.$$

For the function $U(x-\xi;t-\tau),\quad V(x-\xi;t-\tau),\quad f(z),\quad \varphi(z)$, the following relations are true

$$f''(z)+\frac{1}{3}zf(z)=0,\quad \varphi''(z)+\frac{1}{3}z\varphi(z)=0, \tag{8}$$

$$\int_{-\infty}^{\infty}f(z)=\pi,\quad \int_{-\infty}^{0}f(z)=\frac{\pi}{3},\quad \int_0^{\infty}f(z)=\frac{2\pi}{3},\quad \int_0^{\infty}\varphi(z)=0, \tag{9}$$

$$\lim_{(x,t)\to(a-0,t)}\int_{\tau}^{t}U_{\xi\xi}(x-a;t-\tau)\alpha(\xi,\tau)d\tau=\frac{\pi}{3}\alpha(t), \tag{10}$$

$$\lim_{(x,t)\to(a+0,t)}\int_{\tau}^{t}U_{\xi\xi}(x-a;t-\tau)\alpha(\xi,\tau)d\tau=-\frac{2\pi}{3}\alpha(t), \tag{11}$$

$$\lim_{(x,t)\to(a+0,t)}\int_{\tau}^{t}V_{\xi\xi}(x-a;t-\tau)\alpha(\xi,\tau)d\tau=0, \tag{12}$$

$$f^{n}(z):\ c_n^{+}z^{\frac{2n-1}{4}}\sin\left(\frac{2}{3}z^{3/2}\right),\quad z\to\infty, \tag{13}$$

$$\varphi^{n}(z):\ c_n^{+}z^{\frac{2n-1}{4}}\sin\left(\frac{2}{3}z^{3/2}\right),\quad z\to\infty, \tag{14}$$

$$f^{n}(z):\ c_n^{-}|z|^{\frac{2n-1}{4}}\exp\left(-\frac{2}{3}|z|^{3/2}\right),\quad z\to-\infty, \tag{15}$$

2. MAIN RESULTS

Theorem 1. *Let* $\mu^2\le\exp\{-T\}$. *Then problem (3)-(5) does not have more than one solution.*

Proof. Let problem (3)-(5) have two solutions: $u_1(x,t),\ \ u_2(x,t)$. Then sloping $v(x,t)=u_1(x,t)-u_2(x,t)$ we get the following problem regarding the function $v(x,t)$

$$Lv\equiv\frac{\partial^3 v}{\partial x^3}-\frac{\partial v}{\partial t}=0. \tag{16}$$

$$v(x,0)=\mu v(x,T), \tag{17}$$

$$v_{xx}(0,t)=0,\quad v_x(0,t)=0,\quad v_x(1,t)=0. \tag{18}$$

Now we differentiate equation (16) with respect to x and introduce the notation $w(x,t)=v_x(x,t)$. Then, with respect to the function $w(x,t)$, we obtain the following problem

$$Lw\equiv\frac{\partial^3 w}{\partial x^3}-\frac{\partial w}{\partial t}=0. \tag{19}$$

$$w(x,0)=\mu w(x,T), \quad (20)$$

$$w_x(0,t)=0, \quad w(0,t)=0, \quad w(1,t)=0. \quad (21)$$

Consider the identity

$$\int_0^1\int_0^T L(w)w\exp\{-t\}dxdt=0. \quad (22)$$

Integrating by parts, taking into account the homogeneous boundary conditions (20), (21), we have

$$-\frac{1}{2}\int_0^1\int_0^T w^2(x,t)\exp\{-t\}dxdt-\frac{1}{2}\int_0^T w_x^2(1,t)\exp\{-t\}dt-$$

$$-\frac{1}{2}\int_0^1 w^2(x,T)\{\exp\{-T\}-\mu^2\}dx=0$$

From here, $w(x,t)=0$ в Ω. Due to continuity $w(x,t)=0$ in $\overline{\Omega}$. Then $v_x(x,t)=0 \Rightarrow v(x,t)=p(t)$. Because $v(0,t)=0$, то $v(0,t)=p(t)=0$ at $\forall t\in[0,T]$. Therefore, $v(x,t)=0$ in $\overline{\Omega}$.

Теорема 2. *Let* $\psi(t)\in C^1([0,T]), \quad \varphi_2(t)\in C^1([0,T]), \quad \varphi_1(t)\in C^1([0,T])$. *Then there is a solution to problem (3)-(5).*

Proof. Consider the auxiliary problem:

Find the function $u(x,t)\in K_u$ which is a regular solution to the equation

$$Lu\equiv\frac{\partial^3 u}{\partial x^3}-\frac{\partial u}{\partial t}=0. \quad (23)$$

in the area of $\Omega=\{(x,t):0<x<1,\ 0<t\le T\}$ and satisfies the conditions

$$u(x,0)=\tau(x), \quad (24)$$

$$u_{xx}(0,t)=\varphi_1(t), \quad u_x(0,t)=\varphi_2(t), \quad u_x(1,t)=\psi(t). \quad (25)$$

By virtue of [3], the solution of problem (23)–(25) will be in the following form

$$\pi u(x,t)=-\int_0^t G_\xi(x-1;t-\tau)\psi(\tau)d\tau-$$

$$-\int_0^t G(x-0;t-\tau)\varphi_1(\tau)d\tau+\int_0^T G_\xi(x-0;t-\tau)\varphi_2(\tau)d\tau+$$

$$+\int_0^1 G(x-\xi;t-0)\tau(\xi)d\xi, \quad (26)$$

where

$$G(x-\xi;t-\tau)=U(x-\xi;t-\tau)-W(x-\xi;t-\tau),$$

function $W(x-\xi;t-\tau)$ is a solution to the following problem

$$M(W)\equiv-\frac{\partial^3 W}{\partial x^3}-\frac{\partial W}{\partial t}=0,$$

$$U_{\xi\xi}|_{\xi=1}=W_{\xi\xi}|_{\xi=1}, \quad U|_{\xi=1}=W|_{\xi=1}, \quad U_{\xi\xi}|_{\xi=0}=W_{\xi\xi}|_{\xi=0}, \quad W|_{\tau=t}=0.$$

Denote $u(x,T)=\alpha(x)$. Then passing to the limit $t\to T$ from (26) we get

$$\pi\alpha(x)=-\int_0^t G_\xi(x-1;T-\tau)\psi(\tau)d\tau-$$

$$-\int_0^t G(x-0;T-\tau)\varphi_1(\tau)d\tau + \int_0^T G_\xi(x-0;T-\tau)\varphi_2(\tau)d\tau +$$

$$+\mu\int_0^1 \{G(x-\xi;T-0)\alpha(\xi)d\xi, \qquad (27)$$

Thus, we have obtained an integral equation of the Fredholm type with respect to the function $\alpha(x)$

$$\alpha(x) = \int_0^1 K(x,\xi)\alpha(\xi)d\xi + F(x), \qquad (28)$$

where

$$\mu G(x-\xi;T-0) \equiv |K(x,\xi)| < \frac{C}{|x-\xi|^{1/4}},$$

$$-\int_0^t G_\xi(x-1;T-\tau)\psi(\tau)d\tau - \int_0^t G(x-0;T-\tau)\varphi_1(\tau)d\tau +$$

$$+\int_0^T G_\xi(x-0;T-\tau)\varphi_2(\tau)d\tau \equiv F(x) \in C^3([0,1]).$$

Due to the uniqueness of the solution of problem (3)-(5), the integral equation (28) has a unique solution.

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