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Unreliable Inventory Model with Price Varying Demand Rate under the Partially Backlogged Situation

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ARTICLE INFO	ABSTRACT
Published Online: 12 March 2022 Corresponding Author: **Preeti Sharma**	The model is proposed for the unreliable supply system and an amount of defective products are found during the screening process. Demand rate for the system is assumed to be price dependent. Shortages are unavoidable hence partially backlogged. Theoretical results have been generated via mathematical modeling to get the optimum order quantity, optimum cycle length and maximum profit. These theoretical results are illustrated by numerical verification and sensitivity analysis has been reported to check the effect of variation of different parameters on optimum results.

KEYWORDS: Inventory, imperfect items, price varying demand, partial backlogging

I. INTRODUCTION

In today's era, inventory management is a passion of decision-makers. In order to obtain competitive advantages, decision-makers cannot afford to undermine the inventory policy. Moreover, the learning effect is a human phenomenon that can be used to overcome the deficiency in production or handling facilities in the line of inventory management. It is a well known fact that the organizations engaged in a repetitive process to improve their performance through learning by time.

Most decision makers used to assume that all of their products are of perfect quality, but that was never the case in real life. Natural disasters, transit damage, or breakage of items in the lot, are some reasons for the presence of defective items. As a consequence, there may be some defects in the lot produced/received. Inventory policies usually developed based on real scenarios, while developing a policy in inventory management, one needs to consider the fact that not all lots contain 100% perfect items. Rosenblatt and Lee (1986) concluded that defective items could be reworked instantly at a cost, which would result in smaller lots of products when defective products are present. They estimate the time between in-control and out-of-control based on exponential distributions. Kim and Hong (1999) improved upon Rosenblatt and Lee's model by reversing the distributions between the in-control and out-of-control states. Salameh and Jaber (2000) discussed how faulty items could be combined to form one batch before the next delivery is made at a discounted price. Economic lot size quantity increases as the average number of defective items decreases. After Salameh and Jaber (2000) published their study, several researchers extended it in different directions. In Goyal and Cárdenas-Barron (2002), a simple approach to calculating the optimal lot size is presented using the model formulated by Salameh and Jaber (2000). Chang (2004) determined the order lot size that maximizes profitability for lots containing imperfect quality items. The research of Salameh and Jaber (2000) was extended by Chang (2004) in an integrated production and shipping context, while Papachristos and Konstantaras (2006) proposed provisions that made the model proposed by Salameh and Jaber (2000) operational. Maddah and Jaber (2008) determined the expected profit per unit time using Salameh and Jaber's (2000) model using renewal theory. Using an economic order quantity model, Roy et al., (2011) modeled how some items in the order are of imperfect quality. A study by Yadav et al. (2012) showed how learning affects the holding costs, ordering costs, and the percentage of defective items in each lot by dealing with the fuzziness aspect of demand. Based on this model, items received are assumed to be of imperfect quality, and after 100% screening, the incomplete items are withdrawn from inventory and sold at a discount. Their analysis of the holding costs, ordering costs, and defective items in each lot, Yadav et al. (2012), utilized algebraic method to determine optimal values. Recently lots of research papers have been developed along the same line of research with different assumptions such as; Taheri-Tolgari et al. (2019), Sinha et al. (2020), Dwicahyani et al. (2020), De-la-Cruz-Márquez et al. (2021), Mashud et al. (2020) and Sepehri et al. (2021).

A model of economic order quantity is published by Goyal et al. (2015) in which the demand for items varies with the frequency of advertisements. Models for calculating the probability of lost sales and backorders were formulated by including learning effects on the number of defects present in each lot. Generally, the above literature indicates that most researchers believe imperfect items are fixed, while the other researchers assume that imperfect quantity is a random variable with known distribution. In most real life situations where repetitive processes are in play, this assumption is not true because defective items remain the same in each run/lot for a long time. This chapter introduces the learning phenomenon in order to increase the applicability of the model.

With each lot, the quality of the product is increasing based on the experiences of the worker, on the tooling, on the instructions, on the blueprints, on the layout of the workplace, and the conditions of the process. Despite the fact that every item can create its own niche within the mind of the customer over time, thus increasing its demand with time, it was evident that over time everyone's items can be categorized into niches. Choosing the best item for use is largely determined by the selling price of a product in the present highly competitive market. The sales price of a product is linked to the demand, whereas the sales price of a product is linked to the supply. It still remains largely unknown how selling price affects demand among researchers and inventory practitioners. A price-dependent demand model was first developed by Whitin (1955). The demand for finished goods tends to be of this type. According to Burwell et al. (1997), price-dependent demand with discounting can be modelled by an economic lot size model. A price-dependent demand rate improvement system was presented by Mondal et al. (2003). Based on a price- and time-dependent demand model, you developed an inventory model (2005). A model for the demand rate based on inventory was developed by Teng et al. (2005). The inventory model of Maiti et al. (2009) investigated items with price-dependent demand in stochastic environments over a finite period of time. According to Chang et al. (2010), the optimal sold price and order quantity for deteriorating items are calculated simultaneously using an EOQ model. Demand is assumed to vary not only according to the quantity and quality of the merchandise on display but also according to the selling price per unit as well as the amount of shelf or display space available. Shastri et al. (2014) developed a supply chain inventory model to help determine the proper order policies for deteriorating inventory items in a retailer under 2 levels of trade credit in order to represent the SCM scenario. The researchers also reasoned that price determines the demand rate for the product.

A shortage of quantities may also result in backorders. Because of the higher carrying cost of the product, overproduction or over carrying is not a solution. When developing an inventory model, it's impossible to ignore shortages because each lot contains imperfect items. Assuming that only a few customers can wait for their delivery during stock out periods while others go somewhere else, that seems more reasonable. Taking all of this into consideration, it could be defined as a partial backorder if there is a shortage during the stock out period. This chapter examines how learning can be used to solve an inventory problem in the optimal way. This study takes place within a context of inflation. The chapter develops an inventory model by taking demand into account when making selling price estimates. It is also examined how learning affects the amount of imperfect items presented in each lot. In this model, the items received are assumed to be of imperfect quality and they are sold at discounted prices after 100% screening and removal from inventory. In industries like automotive and stabilizers, learning by doing is especially useful. Due to the learning effect, the number of imperfect items in these industries decreases with each shipment. Numerical analysis is used to show the model's feasibility and applicability. Different inventory parameters are also subjected to sensitivity analysis.

II. ASSUMPTIONS AND NOTATIONS

Assumptions:

During this chapter, the following assumptions are made to develop a mathematical model.

1. A product's price determines the demand rate.

2. During screening, demand and screening take place simultaneously, but screening takes precedence over demand.

3. A single batch of defective items sells at a discount once the screening process is completed.

4. A partial backlog of shortages is allowed.

Notations:

In this section, the mathematical model is developed using the following notations:

$D(p)$:The rate of demand is measured by the number of units per unit of time depending on the price

q_n : Order quantity in units for the n^{th} shipment, where $n \geq 1$

r : a constant that represents the difference between the discount rate (k_1) and the inflation rate (k_2)

c : purchasing cost per item

O : ordering cost cycle

h : holding cost per item

p : selling price per item

c_l : lost sale cost per item

c_b : backordering cost per item

S_c : screening cost per item

α : percentage of defective items in each shipment follows the learning curve $\left(= a / \left(g + e^{bn}\right)\right)$, a,b,g>0

T_n : cycle time per shipment

x : screening rate measured in units per unit of time

B : maximum backordering quantity in units

δ : rate of partial backordering

III. THE MATHEMATICAL MODEL

At the beginning of each cycle, q_n items are acquired. When there is a positive inventory level, this quantity will be used as a backordering quantity. A lot can contain both good and defective items, so each cycle begins with the Inspection process. Inspections are presumed to be higher than demand. t_n is the time when the screening process is completed. As part of the screening process, items are requested simultaneously with the screening. The screening process verified that the items in demand were of perfect quality during this time period. A salvage value is calculated for items of poor quality upon which a discount price is applied. Here t_{1n} is the time to screen q_n units, where $t_{1n}=q_n/x<T_n$, while T_{1n} is the time where inventory level reaches to zero. Moreover, α satisfies the following restrictions within the screening time $t_{1n}(= q_n/x)$ to avoid shortages of good items:

$$(1-\alpha)q_n \geq D(p)t_{1n} \text{i.e.,} \quad \alpha \leq 1-\frac{D}{x} \tag{1}$$

The following differential equation can be used to describe the inventory level at any point in time, t during the positive inventory level:

$$\frac{dQ_{1n}(t)}{dt} = -D(p), \quad 0 \leq t \leq T_{1n} \tag{2}$$

By using boundary conditions, one can solve the above differential equation

$Q_{1n}(0)=q_n-\delta B$, we get

$$Q_{1n}(t) - q_n - \delta B - D(p)t, \quad 0 \leq t \leq t_{1n} \tag{3}$$

Inventory level at any time t_{1n} is

$$Q_n(t) = q_n - \delta B - D(p)t_{1n}, \tag{4}$$

Number of defective items = αq_n

Therefore, the inventory level of good items during $t_{1n} \leq t \leq T_{1n}$ is

$$Q_n(t) = (1-\alpha)q_n - \delta B - D(p)t, \quad t_{1n} \leq t \leq T_{1n} \tag{5}$$

We have $Q_{1n}(T_{1n}) = 0$ which gives

$$T_{1n} = \frac{(1-\alpha)q_n - \delta B}{D(p)} \tag{6}$$

A differential equation can be used to represent inventory levels in shortage periods:

$$\frac{dQ_{2n}(t)}{dt} = -\delta D(p), \quad T_{1n} \leq t \leq T_n \tag{7}$$

A boundary condition is used in solving the differential equation above $Q_{2n}(T_{1n})=0$, we get inventory level at time 't' is

$$Q_{2n}(t) = -\delta D(p)(t - T_{1n}), \quad T_{1n} \leq t \leq T_n \tag{8}$$

Inventory level at time T_n is $Q_{2n}(T_n) = -\delta B$, therefore we get

$$B = D(p)(T_n - T_{1n}) \tag{9}$$

From equation (6) and (9), we get

$$B = \frac{D(p)T_n - (1-\alpha)q_n}{(1-\delta)} \tag{10}$$

The following steps will allow us to evaluate the different costs associated with inventory.

Holding cost

$$=\sum_{n=0}^{\infty} h(nT)\left[\int_0^{t_{1n}} Q_n(t)dt + \int_{t_{1n}}^{T_{1n}} Q_n(t)dt\right]$$

$$= \sum_{n=0}^{\infty} h(nT)\left[\int_0^{t_{1n}}(q_n - \delta B - D(p)t)\,dt + \int_{t_{1n}}^{T_{1n}}((1-\alpha)q_n - \delta B - D(p)t)dt\right]$$

$$= \frac{h}{(1-e^{-kT_n})}\left[\alpha q_n t_{1n} + ((1-\alpha)q_n - \delta B)T_{1n} - D(p)\frac{T_{1n}^2}{2}\right]$$

Ordering cost $= \frac{K}{(1-e^{-rT_n})}$

Purchase cost $= \frac{cq_n}{(1-e^{-rT_n})}$

Screening cost $= \frac{S_c q_n}{(1-e^{-rT_n})}$

Selling price of good items $= \frac{sc_n(1-\alpha)}{(1-e^{-rT_n})}$

Selling price of defective items $= \frac{vq_n\,\alpha}{(1-e^{-rT_n})}$

Backlogging cost $= \sum_{n=0}^{\infty} c_b(nT)\int_{T_{1n}}^{T_n}(-I_{2n}(t))dt$

$$= \frac{c_b\delta D(p)}{2(1-e^{-rT_n})}(T_n \quad T_{1n})^2$$

Lost sale cost $=$

$$\sum_{n=0}^{\infty} c_l(nT)\int_0^{T_n-T_{1n}}(1-\delta)D(p)Ddt - \frac{c_l(1-\delta)D(p)(T_n-T_{1n})}{(1-e^{-rT_n})}$$

The total revenue per cycle, $TR(q_n)$, is the sum of total sales volume of good quality and imperfect quality items.

Therefore,

$$TR(q_n) = \frac{(s(1-\alpha)+v\alpha)q_n}{(1-e^{-rT_n})} \qquad (11)$$

The total cost per cycle $TC(q_n)$ is the sum of ordering cost, purchasing cost, screening cost and holding cost.

TC(q)=

$$TC(qN) = \frac{1}{(1-e^{-kT_n})}\left\{O + cq_n + S_c q_n + h\left[\alpha q_n t_{1n} + ((1-\alpha)q_n - \delta B)T_{1n} - D(p)\frac{T_{1n}^2}{2}\right] + \frac{c_b\delta D(p)}{2}(T_n - T_{1n})^2 + c_l(1-\delta)D(p)(T_n - T_{1n})\right\} \qquad (12)$$

Total profit of the retailer is

$TP = TR(q_n) - TC(q_n)$

$$TP = \frac{1}{(1-e^{-kT_n})}\left[\{(s(1-\alpha)+v\alpha)q_n\} - \left\{K + cq_n + S_c q_n + h\left(\alpha q_n t_{1n} + ((1-\alpha)q_n - \delta B)T_{1n} - D(p)\frac{T_{1n}^2}{2}\right) + \frac{c_b\delta D(p)}{2}(T_n - T_{1n})^2 + c_l(1-\delta)D(p)(T_n - T_{1n})\right\}\right]$$

or one can say that

$$TP(q_n, T_n) = \frac{1}{(1-e^{-kT_n})}\left[\{(s(1-\alpha) + v\alpha)q_n\} - \left\{K + cq_n + S_c q_n + h\left(\frac{\alpha q_n^2}{x} + \frac{1}{2D(p)}\left((1-\alpha)q_n - \alpha\left(\frac{D(p)T_n - (1-\alpha)q_n}{(1-\delta)}\right)\right)^2\right) + \frac{c_b\delta}{2D(p)}\left(\frac{D(p)T_n - (1-\alpha)q_n}{(1-\delta)}\right)^2 + c_l(1-\delta)\left(\frac{D(p)T_n - (1-\alpha)q_n}{(1-\delta)}\right)\right\}\right] \qquad (13)$$

Our objective is to find the value of q_n and T_n in order that the profit of the retailer is maximum.

For this, we set $\frac{\partial TP(q_n, T_n)}{\partial q_n} = 0, \quad \frac{\partial TP(q_n, T_n)}{\partial T_n} = 0$

$$\frac{1}{(1-e^{-kT_n})}\left[(s(1-\alpha)+v\alpha)-\left\{c+S_c+h\left(\frac{2\alpha q_n}{x}+\frac{1}{D(p)}\left((1-\alpha)q_n-\delta\left(\frac{D(p)T_n-(1-\alpha)q_n}{(1-\delta)}\right)\right)\left((1-\alpha)+\frac{\delta(1-\alpha)}{(1-\delta)}\right)\right)-\frac{c_b\delta}{D(p)}\left(\frac{D(p)T_n-(1-\alpha)q_n}{(1-\delta)}\right)\frac{(1-\alpha)}{(1-\delta)}+-c_l(1-\alpha)\right\}\right]=0 \tag{14}$$

$$-\frac{ke^{-kT_n}}{(1-e^{-kT_n})^2}\left[\{(s(1-\alpha)+v\alpha)q_n\}-\left\{K+cq_n+S_cq_n+h\left(\frac{\alpha q_n^2}{x}+\frac{1}{2D(p)}\left((1-\alpha)q_n-\delta\left(\frac{D(p)T_n-(1-\alpha)q_n}{(1-\delta)}\right)\right)^2\right)+\frac{c_b\alpha}{2D(p)}\left(\frac{D(p)T_n-(1-\alpha)q_n}{(1-\delta)}\right)^2+c_l(1-\delta)\left(\frac{D(p)T_n-(1-\alpha)q_n}{(1-\delta)}\right)\right\}\right]+\frac{1}{(1-e^{-kT_n})}\left[\frac{h}{D(p)}\left((1-\alpha)q_n-\delta\left(\frac{D(p)T_n-(1-\alpha)q_n}{(1-\delta)}\right)\right)\left(\frac{\delta D(p)}{(1-\delta)}\right)-\frac{c_b\delta}{(1-\delta)}\left(\frac{D(p)T_n-(1-\alpha)q_n}{(1-\delta)}\right)-c_lD(p)\right]=0 \tag{15}$$

From equation (14), we get

$$q_n=\frac{(s(1-\alpha)+v\alpha)-c-S_c+\frac{\delta hT_n(1-\alpha)}{(1-\delta)^2}+\frac{c_b\delta T_n(1-\alpha)}{(1-\delta)^2}+c_l(1-\alpha)}{\left(\frac{2\alpha h}{x}+\frac{(1-\alpha)^2}{D(p)(1-\delta)}+\frac{1}{D(p)}\left(\frac{1-\alpha}{1-\delta}\right)^2+\frac{c_b\delta}{D(p)}\left(\frac{1-\alpha}{1-\delta}\right)^2\right)} \tag{16}$$

On putting the value of q_n from equation (16) in equation (15), we get the value of T_n and hence from equation (14) we get q_n.

IV. NUMERICAL ANALYSIS

In order to demonstrate the overall result of the model, an inventory system is assumed in order to illustrate the proposed models. The number of defective items present in each lot is assumed to be deterministic, and is calculated based on a learning curve.

S_c = \$0.75/unit	s = \$40/unit
c = \$20/unit	h = \$1.5/unit/year
K = \$85/cycle	e = 42
v = \$15/unit	f = 1000
d = 5000 units/year	x = 6350 units/year
n=4	

Based on the above parametric values following optimal results have been computed by using the theoretical results

Optimum order quantity q_n=1325 unit, optimum replenishment cycle T_n=0.652 and maximum profit =\$ 8,51,205

V. SENSITIVITY ANALYSIS

Whenever the parameters of a decision-making process are changed, it is often because of uncertainty. A sensitivity analysis is therefore an excellent way to examine the implications of the changes. Here sensitivity analysis with respect to different parameters is determined based on the numerical data presented in the numerical section.

Sensitivity Analysis with to selling price is shown in figure 1. A selling price variation of -40% to 40% was taken to study how it affects the optimal respect solution. It is evident that the optimal decision is highly sensitive to total profit and order quantity of the system, and to the supply chain supply time, as the selling price of the perfect items increases.

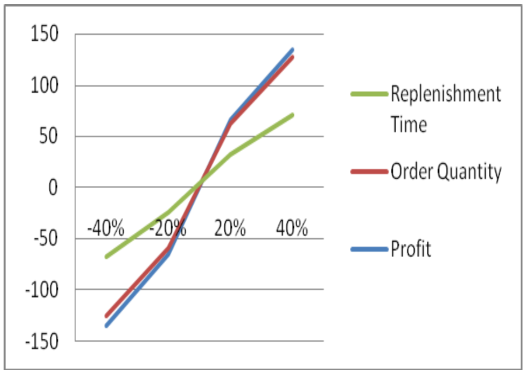

Figure 1

Sensitivity analysis with respect to the selling price

The effect of improved selling price of imperfect goods on the optimal value of the decision variable can be seen in Figure 2 where we change the discount factor from -40% to 40%. We can see that optimal decisions are not sensitive to changes in selling prices. The optimal decision is relatively insensitive to changes in the sale price of imperfect items.

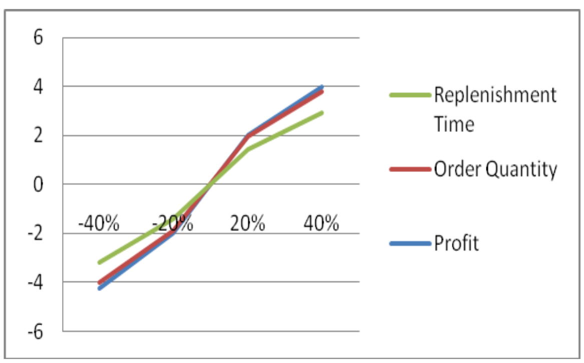

Figure 2

Sensitivity analysis with respect to the no. of shipment

Demand plays a major role in making the decision about inventory, so it is important to consider its effect on optimal values within the context of sensitivity analysis. Figure 3 shows its effect on optimal values within the context of sensitivity analysis.

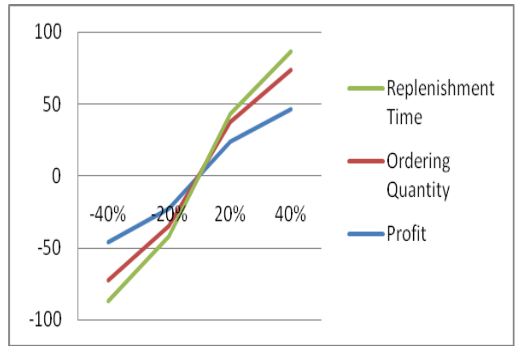

Figure 3

Sensitivity analysis with respect to demand

Based on Figure 3, it appears that as the demand of items increases, the profits generated by the system increase. For this reason, the ordering quantities and replenishment times increase as well. Therefore, demand increases profitability and increases decision variables.

Figure 4 shows how holding costs affect the optimal value of the decision variables in the sensitivity analysis presented here. According to Figure 4, the holding cost has a negative impact on the optimal solution.

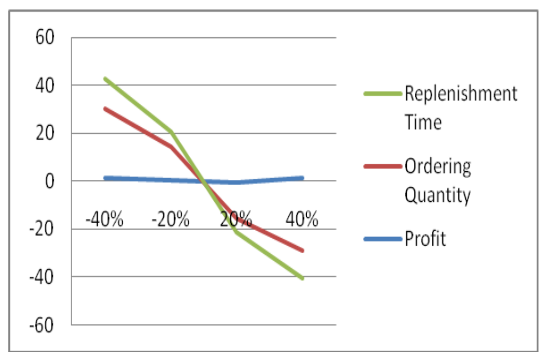

Figure 4

Sensitivity analysis with respect to holding cost

Figure 4 depicts that all decision variables have significant effects when holding cost is decreasing. As holding cost drops, decision makers automatically stock up on more inventories, increasing order quantity and thus increasing return time. A system's profit is hardly affected by its holding costs.

Inflation describes the fact that prices are continually rising; therefore, rates of inflation change the optimal decisions, as you might expect. As a consequence, it would be interesting to carry out the sensitivity analysis in relation to inflation. This is illustrated in Figure 5.

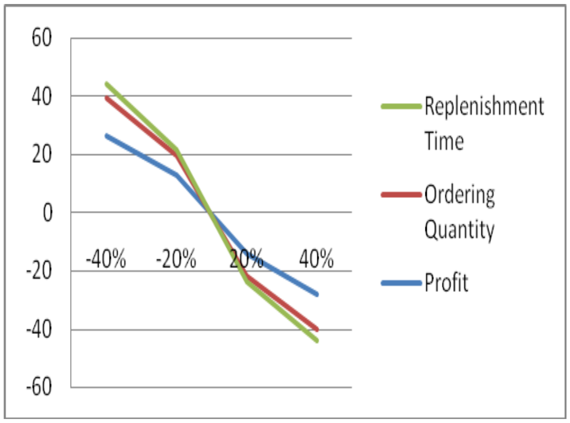

Figure 5

Sensitivity analysis with respect to the inflation

A dramatic increase in inflation decreases the total profit of the system, since the system will undergo such a noticeable change. Consequently, decision-makers can buy fewer inventories, which ultimately finish off sooner as inflation lowers the purchasing power.

Here, a sensitive analysis in relation to the proposed inventory model is performed in order to study its effect on the optimal solution. Figure 6 presents effects of higher and lower backlogging rates on the optimal solution based on -40% to 40% variation in the backlog rate.

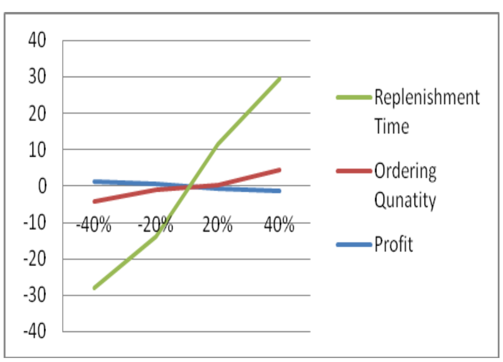

Figure 6

Sensitivity analysis with respect to the backlogging rates

Figure 6 represents that, as the value of δ increases, total quantities ordered and replenishment time increase, while profits decrease due to the increase in backorder costs. A planned partial back order would be preferable to a full back order and loss of sale.

Figure 7 demonstrates the impact of screening rate on optimal solution based on sensitive analysis.

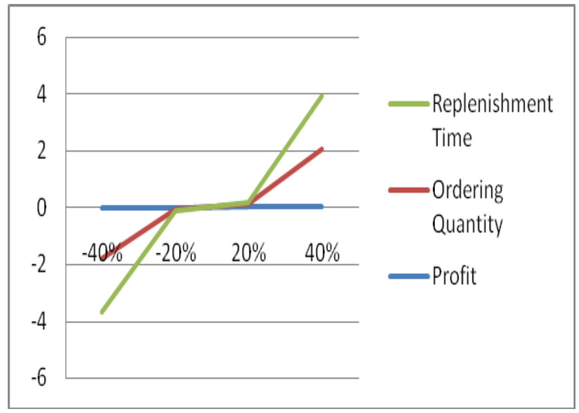

Figure 7

Sensitivity analysis with respect to the screening rates

The Figure 7 indicates that the profit of the system increases as the screening rate increases due to the lower inventory cost of the system. In turn, this increases the ordering and refill costs of the system. The optimal solution is found to be affected less by the screening rate than by other parameters. The next step in this process is to analyze the consequences of learning the number of imperfect items in each lot by analyzing the profit of the system.

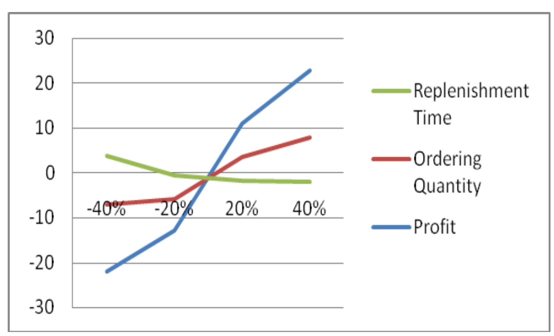

Figure 8

Sensitivity analysis with respect to the ordering **quantities**

As shown in Figure 8, there is an increase in ordering quantity when there is an increase in the number of imperfect items in each lot as learning takes effect. This scenario calls for managers to order smaller lots to decrease the number of imperfect items in the organization's lot by learning from their mistakes.

VI. CONCLUSION

Due to the stiff competition in the market, customers have plenty of choices and there is very little preventing them from moving on to the next shop if they do not receive adequate service from their current provider. Inventory practitioners, therefore, strive to maximize the service to customers and their profits by exploring all opportunities. Inventory practitioners may consider partial backordering, learning effect, and inflation effect when developing inventory policies to reduce inventory costs and improve profitability. This chapter developed a model that can be applied to a variety of industries, including chemicals, food, nuclear energy, and stabilizers. The aim of this chapter is to extend economic order quantity modeling to specify explicitly the degree of imperfection and partial backlog in inflationary environments brought about by the learning phenomenon. It results in an increase of net profit with an increase of learning as the number of imperfect units and ordering quantity decrease. It is recommended to reduce the frequency of ordering as learning speeds up. So a manager should make sure that he keeps learning new things in all directions to make the organization more profitable. Partial backlogs are better than total backorders, as shown by a numerical example. By means of this model, we can say that it offers the inventor quick remedies for changes in market performance and thus, can save his life by providing quick solutions. This chapter presents models that have been numerically proven to be feasible and have also been proven to be financially feasible. Stochastic learning curves can be incorporated into the above models in future research. If n equal cycles are taken into account, then we can extend the model.

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