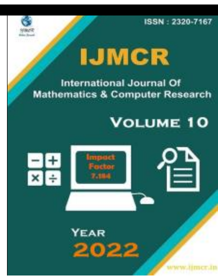
International Journal of Mathematics and Computer Research
ISSN: 2320-7167
Volume 10 Issue 04 April 2022, Page no. – 2646-2649
Index Copernicus ICV: 57.55, Impact Factor: 7.184
DOI: 10.47191/ijmcr/v10i4.02



Study of Fuzzy Conformable Fractional Differential Equations

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ARTICLE INFO
Published Online:
18 April 2022
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ABSTRACT
It is essential to use fuzzy differential equations in order to simulate a wide range of physical, applied, and engineering phenomena and uncertainties. For some fuzzy differential equations, obtaining precise solutions is quite challenging, hence reliable and effective analytical methods are essential. These fuzzy fractional differential equations will be studied in this research. Conformable fractional differentiability is examined in this paper, with the goal of developing an existence and uniqueness thesis for a fuzzy fractional differential equation using the idea of conformable differentiability, which is based on expanding a fuzzy mapping's class of differentiable fuzzy mappings. For this, we consider lateral Hukuhara derivatives of order $q \in (0, 1)$.

KEYWORDS: Fuzzy conformable, fractional, differential equations

1. INTRODUCTION

Many mathematical structures may be found in the theory of fuzzy fractional calculus, both theoretical and practical. Derivatives in this area theoretically rely heavily on Riemann-Liouville or Caputo-Liouville problems. They share a weakness in the kernel functions that are detected in the integral operator side-by-side with the normalising function, which occurs alongside the integral ticks: non-locality and singularity. Real-world core simulations of dynamic fractional systems must undoubtedly be the source of a more realistic and fruitful definition, which cannot be rejected.[1] A new fuzzy fractional derivative construct, ABC, is introduced and utilized to construct and describe new concrete fuzzy mathematical ideas in this direction. Because of the reliance on exponential decay in the kernel of this novel fuzzy fractional ABC derivative, it appears to be freeing singularity with local kernel function.[2]

As an appealing mathematical tool for describing numerous complicated nonlinear processes in many domains of research, fractional differential equations have recently garnered much interest, including plasma oscillation, theorems on control and quantum mechanics as well as fluid flow and bimolecular dynamics. Fractional notions in the literature include Grünwald, Caputo-Katugampola conformable, Riesz, Caputo, Riemann-Liouville, Caputo-Fabrizio, and Riemann-Liouville-Caputo. Since the majority of physical applications rely on historical and nonlocal features, nonlocal derivatives are more intriguing. Riemann-Liouville and Caputo singular kernels have been used to propose some of these operators. Fractional derivatives based on nonsingular kernels, such as those presented by Caputo-Fabrizio and Atangana-Baleanu, have recently been proposed to better represent physical dissipative processes and decrease numerical result collusion.[3]

Fuzzy differential equations rely on the properties of heredity, nonlocality, and memory aspects of many uncertain processes, which may be naturally described by fractional operators in this study. Fuzzy fractional derivatives and fuzzy fractional integrals are being developed to regulate and enhance the performance of uncertain functions and parameters in fuzzy ABC fractional calculus theory for representation and knowledge acquisition. Fuzzy SGD technique, in fact, allows us to solve fuzzy FDEs without losing potential fuzzy solutions. The uniqueness, existence, and characterisation of theorem will be argued under the fuzzy SGD and fuzzy ABC fractional calculus. The solution is described in terms of an analogous system of crisp ABC FDEs in order to attain this goal.[4]

It has been more common in recent years to use fuzzy differential equations in a wide range of applications in applied science and engineering fields, such as signal processing and porous media as well as thermal systems and electrical circuits. There are a variety of techniques to fuzzy logic. Using Hukuhara derivative or Seikkala derivative, one relies on crisp differential equations being extended to a fuzzy sense, whilst the other relies on Zadeh extension concept.

SGD, on the other hand, provides unique solutions to fuzzy differential equations and fixes the Hukuhara fuzzy differentiation flaw. Both practical and theoretical applications of SGD are promising in terms of quality. For more realistic mathematical modelling and a greater understanding of actual processes, fuzzy equations can be used with fractional operators. [5]

2. SECTION SNIPPETS

2.1 Fuzzy ABC fractional integral

The ABC fractional integral is explained in detail here. There is a strong connection between fuzzy RL fractional integral and several fuzzy fractional integral characterizations.[6]

2.2 Fuzzy ABC fractional derivative

Based on the fuzzy SGD concept, the fuzzy ABC fractional derivative is described. Some important hypotheses, facts and relationships are also examined and confirmed. The fuzzy SGD technique helps to ensure that fuzzy FDEs are not lost in the shuffle when resolving them. The fuzzy ABC fractional notion is used to realise a novel form of fuzzy fractional derivative using the fuzzy SGD features, which generalises the Seikkala and Hukuhara results a little.

2.3 Fuzzy ABC fractional differential equations

For numerical analysis and its applications, the study of fuzzy ABC fractional solutions to fuzzy ABC FDEs is critical. As a result of this approach, we may extract theoretical conclusions on (1)- and (2)-fuzzy ABC fractional solutions and their characterisation.

2.4 Algorithm and applications

In order to build the framework for the solution approaches, a computational methodology is provided here. After that, several real-world examples are provided to illustrate the theoretical analysis offered. There are three examples of fuzzy ABC fractional derivatives, fuzzy ABC fractional integrals and fuzzy ABC fractional FDEs.[7]

3. FUZZY CONFORMABLE FRACTIONAL DIFFERENTIABILITY AND FUZZY FRACTIONAL INTEGRAL

3.1. Fuzzy Conformable Fractional Differentiability

in this case the fractional derivative of order $q \in (0, 1)$ is defined in the most natural and efficient way possible.[8]

Definition. Let F: $(0, a) \longrightarrow$ RF be a fuzzy function, and q^{th} order fuzzy conformable fractional derivative of F is defined by

$$T_q(F)(t) = \lim_{\varepsilon \longrightarrow 0^+} \frac{F\left(t + \varepsilon t^{1-q}\right) \ominus F(t)}{\varepsilon}$$

$$= \lim_{\varepsilon \longrightarrow 0^+} \frac{F(t) \ominus F\left(t - \varepsilon t^{1-q}\right)}{\varepsilon},$$

for all $t > 0$, $q \in (0, 1)$. Let $F(q)(t)$ stands for $T_q(F)(t)$.

Hence,

$$F^{(q)}(t) = \lim_{\varepsilon \longrightarrow 0^+} \frac{F\left(t + \varepsilon t^{1-q}\right) \ominus F(t)}{\varepsilon}$$

$$= \lim_{\varepsilon \longrightarrow 0^+} \frac{F(t) \ominus F\left(t - \varepsilon t^{1-q}\right)}{\varepsilon}.$$

If F is q-differentiable in some $(0, a)$ and $\lim_{t \to 0+} F(q)(t)$ exists, then

$$F^{(q)}(0) = \lim_{t \longrightarrow 0^+} F^{(q)}(t),$$

and the limits (in the metric d).

From the definition, it directly follows that if F is q-differentiable, then the multivalued mapping F_α is q-differentiable for all $\alpha \in [0, 1]$ and

$$T_q F_\alpha = \left[F^{(q)}(t)\right]^\alpha.$$

where $T_q F_\alpha$ is denoted from the conformable fractional derivative of F_α of order q.

Theorem 1: *Let* F: $(0, a) \longrightarrow R_F$ *be q-differentiable. Denote* $F_\alpha(t) = [f_1^\alpha(t), f_2^\alpha(t)], \alpha \in [0, 1]$.

Then $f_1^\alpha(t)$ *and* $f_2^\alpha(t)$ *are q-differentiable and*

$$\left[F^{(q)}(t)\right]^\alpha = \left[\left(f_1^\alpha\right)^{(q)}(t), \left(f_2^\alpha\right)^{(q)}(t)\right].$$

Theorem 2: *Let F*: (0, *a*)⟶R_F *is q-differentiable on* (0, *a*). *If* $t_1, t_2 \in (0, a)$ *with* $t_1 \leq t_2$*, then there exists* $\lambda \in R_F$ *such that* $F(t_2) = F(t_1) + \lambda$.

Proof. For each $s \in [t_1, t_2]$, there exists $\delta(s) > 0$ such that the H-differences $F(s + \varepsilon s^{1-q}) \ominus F(s)$ and $F(s) \ominus F(s - \varepsilon s^{1-q})$ exist for all $0 \leq \varepsilon < \delta(s)$. Then, we can find a finite sequence $t_1 = s_1 < s_2 < \cdots < s_n = t_2$ such that the family $\{I_{s_i} = (s_i - \delta(s_i), s_i + \delta(s_i)) | i = 1, 2, \ldots, n\}$ covers $[t_1, t_2]$ and $I_{s_i} \cap I_{s_{i+1}} \neq \varnothing$. Pick $x_i \in I_{s_i} \cap I_{s_{i+1}}, i = 1, 2, \ldots, n - 1$, such that $s_i < x_i < s_{i+1}$. Then,

$$F(s_{i+1}) = F(x_i) + A_1 = F(s_i) + A_2 + A_1$$
$$= F(s_i) + \lambda_i, \quad i = 1, 2, \ldots, n - 1,$$

where ε is so small that the H-difference $F(t + t_1 - q\varepsilon) \ominus F(t)$ exists. By the differentiability, the right-hand side goes to zero as ε⟶0+, and hence, *F* is right continuous. The left continuity is proved similarly.

3.2. Fuzzy Fractional Integral

Let $q \in (0, 1)$ and *F*: (0, *a*)⟶RF be such that[9]

$$[F(t)]^\alpha = [f_1^\alpha(t), f_2^\alpha(t)]$$

For all $t \in (0, a)$ and $\alpha \in [0, 1)$. Suppose that

$$f_1^\alpha, f_2^\alpha \in C((0,a),\mathbb{R}) \cap L^1((0,a),\mathbb{R})$$

for all $\alpha \in [0, 1]$ and let

$$A_\alpha =: \left[\int_0^t \frac{f_1^\alpha}{x^{1-q}}(x)\mathrm{d}x, \int_0^t \frac{f_2^\alpha}{x^{1-q}}(x)\mathrm{d}x\right], \quad t \in (0, a).$$

4. FUZZY COMFORMABLE FRACTIONAL DIFFERENTIAL EQUATIONS

We study the fuzzy initial value problem [10]

$$T_q x(t) = F(t, x(t)), \quad q \in (0, 1],$$
$$x(0) = x_0,$$

Where *F*: (0, *a*) × R_F⟶R_F is the continuous fuzzy mapping, and x_0 is the fuzzy number. From 'eorems 5, 8, and 9, it immediately follows.

Theorem 3: *mapping x*: (0, *a*)⟶R_F *is a solution to problem if and only if it is continuous and satisfies the integral equation:*

$$x(t) = x_0 + I_q F(t, x(t)),$$

for all $t \in (0, a)$ *and* $q \in (0, 1)$.

5. CONCLUSION

The generalised conformable fractional derivatives notion is used to understand fuzzy fractional issues for order $q \in (0, 1)$ FFDEs, Establish and verify numerous conclusions for fuzzy conformable differentiability and fuzzy fractional integration of such functions. There will be a class of dynamical systems with linear differential equations that can be solved using the new method.

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