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Acyclic graph coloring of the star and circular chorametic number

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ABSTRACT:
A. Vince introduced a natural generalization of graph coloring and proved same basic facts, revealing it to be a concept of interest. The coloring theory for digraphs is similar to the coloring theory for undirected graphs when independent sets of vertices are replaced by acyclic sets. Since the directed K-cycle has circular chromatic umber $k/k-1$ for $k \geq 2$ values of χ_c between 1 and 2 are possible. In fact, χ_c takes on all rational values greater than 1. Now $\chi_c < \chi$ if and only if a particular digraph is acyclic and the decision problem associated with this question is probably not in NP through it is both NP hard and NP easy.

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Keywords: star chromatic number, Circular chromatic number, acyclic graph

INTRODUCTION:
For graphs, the circular chromatic number is a refinement of the usual chromatic number. [2-5] These papers introduces the chromatic and circular chromatic number of digraphs and define the first of these invariants by replacing the requirement that color classes are independent sets by the weaker condition that they are acyclic [7]. Vince [1] introduced a natural generalization of the chromatic number, the star chromatic number χ^*. He proved among the other things, that
$\chi(G)-1 < \chi^*(G) \leq \chi(G)$ that $\chi^*(C_{2n+1}) = 2+\frac{1}{n}$ and $\chi^*(K_n) = n$. In some sense, a graph G for which $\chi^*(G)$ is easier to clour that one for which these two numbers are the same. Vince closed his paper with four questions, the most general being " What determines whether $\chi^* = \chi$? Here we provide a usual and interesting characterization of $\chi^*(G) < q$, where $q \epsilon Q$.

Definition 1: For any real number x and positive integer k the circular norm $|x|_k$ is the distance from x to the nearest multiple of k.

Definition 2: Let k and d be positive integers. A (k,d) – coloring of a graphs G is a functurc: $V(G) \rightarrow Z_k$ such that for any adjacent vertices u and $\left|C(u)-C(v)\right|_k \geq d$. This (k,d) colouring and k/d coloring.

Definition 3: The star chromatic number $\chi^*(G)$ is $inf\{k/d, G$ has (k,d) colouring$\}$. i.e., G is a graph of n vertices than

$$\chi^* = min\{k/d, G \text{ has } (k,d) \text{ coloring and } k \geq n.\}$$

Definition 4: Let C be a Circle in R^2 of length 1 and let $r \geq 1$ be any real numbers. Denote $C^{(r)}$ the set of all open intervals of C of length $1/r$. An r-circular coloring of a graph G is a mapping C from $V(G)$ to $C^{(r)}$ such that $C(x) \cap C(y) = \emptyset$ whenever $(x,y) \epsilon E(G)$. if such an r-circle colouring exists, we say that G is r- circle-colorable.

The circular chromatic number of G is $\chi^c(G) = \inf\{r: G$ is r- circle colorable$\}$ and for any graph G we have $\chi^*(G) = \chi^c(G)$.

Definition 5: The chromatic number $\chi(D)$ of D to be the minimum integer K such that $V(D)$ can be partitioned into K acyclic subset. We shall call such partition a k-colouring of D.

We establish a condition that is equivalent to $\chi^*(G) \leq q$ for graph G then prove that the question $\chi^* = \chi$?

Now we write $H(C)$ when k and d are clear from the context.

In this paper, I found the acyclic graph coloring of star and circular chromatic number.

Theorem 1: For $\chi^*(G) < k/d$ if and only if $H(C)$ is acyclic for same (k, d)-coloring C of G.

Proof: This first part of the proof appears in Vince [1] in a different context. Suppose $H(C)$ is acyclic. Let $C'(v) = C(v)/k$ and number of vertices $v_1, v_2, v_3, \dots, v_n$ so that if (v_i, v_j) is an edge in $H(C)$ then $i > j$. For rational $\epsilon > 0$ we define $f(u_i) = C'(v_i) + \epsilon/i$ and note that for ϵ sufficiently small $f(u_i) \in [0,1)$ $|f(v_i) - f(v_j)|_1 > |C'(v_i) - C'(v_j)|_1 > d/k$

If (v_i, v_j) is an edge of H and likewise $|f(v_i) - f(v_j)|_1 > d/k$ if (v_i, v_j) is not an edge. Hence, $\chi^*(G) < k/d$.

Now suppose $\chi^*(G) < k/d$ and $C_1 : V(G) \to I$ be a colouring of G such that $d(C_1) > d/k$.

Define $C_0(v) = \lfloor kC_1(v) \rfloor / k$ where $kC_0(v)$ is a (k, d) coloring of G. Let $\epsilon(v) = C_1(v) - C_0(v) \geq 0$ note that $\epsilon(v) < 1/k$. Number the vertices $v_1, v_2, v_3, \dots, v_n$ so that $\epsilon(v_i) \geq \epsilon(v_{i+1})$ suppose there is an edge in $H(kC_0)$ from v_i to v_j with $i > j$. Then

$$C_1(v_j) - C_1(v_i) = C_0(v_j) + \epsilon(v_j) - C_0(v_i) + \epsilon(v_i) \leq 0$$

if $C_0(v_j) = C_0(v_i) + d/k$ then $0 < C_1(v_j) - C_1(v_i) \leq C_o(v_j) - C_0(v_i) = d/k$

Which is contradiction, Hence $C_0(v_j) = 0$ and $C_0(v_i) = (k-d)/k$ so $-1 < C_1(v_j) - C_1(v_i) \leq -(k-d)/k$ and again $d(C_1) \leq d/k$ a contradiction. Thus all edges (v_i, v_j) have $i \geq j$ so $H(kC_0)$ is acyclic. Thus k- coloring c of G an acyclic coloring $H(kC_0)$ is acyclic.

Corollary 1: For $\chi^*(G) < k$ if and only if G has an acyclic k-colouring.

Now $\chi^*(G) < \chi$ if and only if a particular digraph is acyclic and the decision problem associated with this question is probably not in NP through it is both NP hard and NP easy.

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