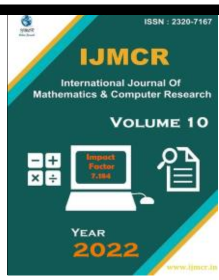
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 10 Issue 04 April 2022, Page no. – 2650-2654

Index Copernicus ICV: 57.55, Impact Factor: 7.184

DOI: 10.47191/ijmcr/v10i4.03



Using the New Mathematical Method to Find the Third Fundamental Form of Surface

Abdel Radi Abdel Rahman Abdel Gadir Abdel Rahman[1], Tasabeh Awadallah Abdel Majid Ali[2], Sulima Ahmed Mohammed Zubir[3], Hassan Abdelrhman Mohammed Elnaeem[4]

[1,2] Department of Mathematics, Faculty of Eduction , Omdurman IsIamicUniversity , Omdurman, Sudan

[3]Department of Mathematics, College of Science and Arts, Qassim University, Ar Rass , Saudi Arabia

[4]Department of Information Security, College of Computer Science and Information Technology, Karary University Khartoum, Sudan

ARTICLE INFO

Published Online:
19 April 2022

Corresponding Author:
Abdel Radi Abdel Rahman Abdel Gadir Abdel Rahman

ABSTRACT

The Third Fundamental Form of Surface which consider simple form is a differential geometry through which it is possible to know the measurement of surface symbolize it III. The third fundamental form is the intrinsic Property of surface. The program MATLAB is on of the most famous Mathematical program helps us to verify the validity of any calculation performed manually The main MATLAB stands for matrix laboratory. MATLAB has advantages compared to conventional computer language for solving technical Problems. This study aims to find the Third Fundamental Form of Surface using the new mathematical method. We followed the applied mathematical method using the new mathematical method and we found the following some results: To find the Third Fundamental Form of Surface we must be calculated the First and Second Natural Curvature. Solving the Third Fundamental Form of Surface using the new mathematical method is more accurate and fast but we cant find the solution graph.

KEYWORDS: New mathematical method, Surface, Third Fundamental Form, MATLAB.

1. INTRODUCTION

In daily life, we see many surfaces such as balloons, tubes, tea cups and thin sheets such as soapbubble, which represent physical models. To study these surfaces, we need coordinates for the work of the necessary calculations. These surfaces exist in the triple space, but we cannot think of them as three dimensional. For example, it we cut a cylinder longitudinal section, it can be individual or spread to become a flat ona desktop. This shows that these surfaces are two dimensional inheritance and this should be described by the coordinates. This gives us the first impression of how the geometric description of the surface .The regular surface can be obtained by distorting the pieces of flat paper and arranging them in such away that the resulting shape is free of sharp dots or pointed letters or intersections (the surface cutsitse If) and thus can be talked about the tangent level at the shape points .The surface is said to be in the triple vacuumR3 is subgroup fromR3 (Le, a special pool of points) of course, not all particle groups are surfaces and certainly mean smooth and two dimensional surfaces[12,pp216]

MATLAB is a software package for computation in engineering, science, and applied mathematics. It offers a powerful programming language excellent graphics and a wide range of expert knowledge. MATLAB is published by a trademark of the math works.The focus in MATLAB is on computation, not mathematics: Symbolic expressions and manipulations are not possible (except through the optional symbolic tool box, a clever interface to Maple). All results are not only numerical but inexact The limitation to numerical computation can be seen as

a drawback, but it is a source of strength too: MATLAB is much preferred to Maple, Mathematic, and the like when it comes to numeric. On the other hand compared to other numerically oriented languages like C++ and Fortran, MATLAB is much easier to use and comes with a huge standard library. The unfavorable comparison here is a gap in execution speed. This gap is not always as dramatic as popular lore has it, and it can often be narrowed or closed with good MATLAB programming. Moreover,one can link other codes into MATLAB, or vice versaand MATLAB [3]

2. THE CONCEPT OF SURFACE

Definition (2.1):

Let the components of the position vector r be single valued and continuously differentiable function of two variable parameters u1and u2, where

r(u1,u2) = { x1u1,u1,x2u1,u2,x3(u1,u2)} (2,1)

if in some region the variable u1and u2

r1r2≠0 where ri=∂rui (i=1,2)

Then the end point of r defines a point set which is called a continuously differentiable surface.

Note: In what follows the notation ri(i=1,2), or

ru=∂r∂u ,rv=∂r∂v , where r=r(u ,v) ,will be used

The parametricfor the surface (2,1)is

xi=xi(u1,u2) (i=1,2,3) (2,2)

Eliminating the parametersu1 and u2 form (2,2) one obtaions the implicit form

F(x1,x2,x3)

=0 (2,3)

Which is satisfied for all points {x1,x2,x3} of the surfaces (2,2).

[5,pp61]

Examples (2.2):

Consider the local coordinate chart:

x(u,v)=(sin u cos u ,sin u sin v ,cos v)

The vector equation is equivalent to three scalar functions in two variables:

x=sin u cos u , y=sin u sin v , z=cos v (2,4)

Clearly, the surface represented by this chart is part of the sphere

x2+y2+z2=1. The chart cannot possibly represent the whole sphere because although a sphere is locally Euclidean. There is certainly a topological difference between a sphere and a plane. Indeed, if one analyzes the coordinate chart carefully, one will note that at the North pole(u=0,z=1) the coordinates become singular. This happen because u=0 implies that x=y=0 regardless of the value of v, so that the North pole has an infinite number of labels. The fact that it is required to have two parameters to describe a patch on a surface in R3is a manifestation of the 2-dimensional nature of the surfaces. If one holds one of the parameters constant while varying the other, then the resulting 1- parameter equations describe a curve on the surfaceThus, for example, letting u=0 constant in equation (2.4),we get the equation of a meridian great circle.

Notation (2.3):*Given a parameterization of a surface in local chart*

x(u,v)=x*(*u1,u2*)*=x*(*u*), we denote the partial derivatives by any of the following notations :*

xu=x1=∂x∂uxuu=x11=2xu2

xv=x2=∂x∂vxvv=x22=2xv2

x=∂xuxαβ=2xuv

[6,pp44]

3. THEFIRST FUNDAMENTAL FORM

In the last unit you have studied about the metric. The metric of a surface determines the first fundamental form of the surface. Thus the quadratic differential from

Edu2+2Fdudv+Gdv2

is called the first fundamental form and the quantities E,F,G are calledthe first order fundamental magnitudes or first fundamentalcoefficients. Here it should be noted that since the quantities E,F,G

depend onU and therefore, in general, they vary from point to point on the surface.

Definition (3.1):

The differential of arc length ds of curve of a curve ui=uiti=1,2

On the surface r=r(u1,u2) is defined by

ds2=g11(du1)2+2g12du1du2+g22(du2)2

Where gik=ri.rk This expression for ds2 is called the first fundamental form the surface. Using the summation convention, this expression can be written in the form

ds2=gikduiduk (i,k=1,2)

Writing u,v instead of u1,u2and E,F,G, instead of g11,g12,g22, one obtains the classical expression for ds

ds2=Edu2+2Fdudv+Gdv2 [5,pp65]

Proposition (3.2):

The parametric lines are orthogonal if F=0 [1,pp45]

4. THE SECOND FUNDAMENTAL FORM

The next step in the development of the theory of surfaces is taken in studying the deviation of the surface from its tangent plane in the neighbor- hood of the point of tangency, and this in turn leads to various developments touching on the curvature of the surface. It is convenient to introduce the function ρ,u,v.=[x,u,v.−x(0,0)]·,x-3.(0,0) ,where ,x-3. is the unit normal vector Thus $\rho(u,v)$,is the perpendicular distance (with an appropriate sign) From the tangent plane to the point of the surface S fixed by x,u,v. Assuming (u,v) to have continuous third derivatives, it can be developed as follows :

xu,v=x0,0+x1u+x2v+12(x11u2+2x12uv+x22v2)

Here the vector functions xi=∂xui and xij=2xuiuj

are evaluated at (0,0) and the dots refer to terms that are cubic at least in u and v. Hence ρ(u,v) is given by

u,v=12(x11x3u2+2x12x3uv+x22x3v2)

Since x1x3=0 The quantities

L=x11x3=L11,M=x12x3=L12,N=x22x3=L22

are introduced, and the second fundamental form *II* is de- fined as the quadratic form

II=Lu2+2Muv+Nv2

For small values of u and v the function 2ρ(u,v) is approximated by the quadratic form II with errors of third or higher order in u *and*v;a study of the form II will therefore give information about the shape of the surface S near the point of tangency, as will be seen in the next and later sections.

The coefficientsLik=xikx3=xik·(x1x2)⁄EG-F2

of the second fundamental form II are invariant under coordinate transformations that preserve the orientation of the axes, but they change sign if the orientation is reversed. Like the coefficientsgik they are not invariant under para- meter transformations. However, the second fundamental form itself is an invariant under parameter transformations with a positive Jacobian, as will be seen later on.

[7,pp85]

5. THIRD FUNDAMENTAL FORM:

The third fundamental form of a space surface is defined by:

III=dN·dN=Cijduiduj (5.1)

Where N is the unit vector normal to the surface at a given point P,Cijare the coefficients of the third fundamental form at P and i, j =1 ,2

• The coefficients of the third fundamental form are given by:

cij= gijNi C (5.2)

Where gij is the space covariant metric tensor and the indexed N is the unit vector normal to the surface. The coefficients of the third fundamental form are also given by:

Cij= gkilikljі (5.3)

Where aklis the surface contravariant metric tensor and the indexed b are the coefficients Of the surface covariant curvature tensor.[15,pp81]

Definition (5.1):

It is possible to derive another interesting formula with the aid of the formulas (5.4) again assuming the coordinate vectors X1 and X2 to be in orthogonal principal directions. By combining these formulas the following relations result with respect to differentiation along an arbitrary curve:

Ẋ3+k1X=aX2 Ẋ3+k2X=bX1́´

With a and b certain scalars, the values of which play no role in what follows. Scalar multiplication of these equations with one another yields the equation

X3́X3́+k1+k2X3́Ẋ+k1k1ẊẊ= 0 (5.5)

Since X1.X2=0. The third fundamental form III is defined, rather naturally, by the formula

III=X3́X3́= (X3)́2 (5.6)

The form *III*evidently furnishes the line element of the spherical image of the surface. In a moment it will be shown thatX3.Ẋ́= -II, and since I=Ẋ.Ẋ (5.5) yields the following identity that holds for the three fundamental forms:

III-2H II+K I= 0 (5.7)

Since H = 12(k1+k2)and K= k1k2

It is still to be shown that II =-X3́Ẋ but this is done easily by the following calculation

II=ijXij.X3uíuj= -iXiuíjX3uj=́-X.X3́

It is of interest to observe that (5.7) is a relation that is invariant with respect to parameter transformations since the forms *II*, I, and the Gaussiancurvature K have that property, and although the form II changes its sign if the Jacobian of the transformation is negative, so also does H: thus while the derivation of the formula made use of special parameters the end result is seen to be invariant with respect to all parameter transformations. [7, p98]

Theorem (5.2):

The first, second and third fundamental forms are linked through the Gaussian curvature K and the mean curvature H, by the following relation:

KI-2HII+III=0 (5.8)

proof:

Let (L12=g12=0)

dN=Nudu+Nvdv=-k1udu-k2vdv

=- k1udu- k1vdv- k2vdv+k1vdv

=-k1udu+vdv+k1-k2vdv=- k1d σ+k1-k2vdv⟹dN+k1d=(k1-k2)udu

Similarly

d N+k2dσ=+k2-k1udu

since g12=uv=0 then

d N+k1dσd N+k2dσ=0 ⟹

d N.dN+k2d N. dσ+k1dN+k1k2dσ.dσ=0

d N.dN+k1+k2dN.dσ+k1k2dσ.dσ=0

III-2HII+KI=0 [15]

The coefficients of first, second and third fundamental forms are correlated, through the mean curvature H and the Gaussian curvature k, by the following relation

K gij-2Hlij+cij=0

By multiplying both sides with gij and contracting we obtain

Kgii-2Hlii+cii= 0

That is:

Trace (cij) =4H2 -2k [12, pp349]

Example (5.3):

If we have the following sphere

r=asin cos ∅ i+asin sin ∅ j+acos k

Find the third fundamental form of the surface

Solution:

r=acos cos ∅ i+acos sin ∅ j-asin k

r∅=-asin sin ∅ i+asin cos ∅ j

N= r∅r∅= a2 cos ∅ i+a2 sin ∅ j+a2sin cos ka2sin

N=sin cos ∅ i+sin sin ∅ j+cos k

N1=Nr= i j k sin cos sin sin cos acos cos acos sin -asin

N1=-asin -a sin i+a cos +a cos j+asin sin cos cos -asin sin cos cos k

=-asin + i+acos (+)j

N1=-asin i+acos j

A=N1.N2=-asin i+acos j-asin i+acos j

= a2 +a2 = a2 +

A= a2

N2=Nr= i j k sin cos sin sin cos -asin sin asin cos 0

N2= -asin cos cos i-asin sin cos j+a + k

N2=-asin cos cos i-asin sin cos j+a k

C=N2N2=-a sin cos cos i-asin sin cos j+a k . -asin cos cos i-asin sin cos j+a k
=a2 + + a2
C= a2 + =a2
B=N1.N2
B=(-sin i+acos j)-asin cos cos i-asin sin cos j+a k
B=a2sin sin cos cos -a2sin sin cos cos)
B = 0
III = Ad2+2Bdθdϕ+Cd2
III= a2d2+20+a2 d2
III= a2d2+a2 d2

6. METHODOLOGY AND DISCUSSION

Definition (6.1):

MATLAB is a high performance language for technical computing it integrates computation visualization and programming in easy to use environment where problems and solution are expressed in familiar mathematical notation.

Typical useinclude :

Math and computation ,Algorithm development , Date analysis exploration and visualization

And Application development [10]

Solve Part Differential Equation with MATLAB (6.2):

The MATLAB partial differential equation solver pdepe initial boundary value problems for systems of Parabolic and elliptic part differential equation in the one space variable x and time t. there must be at lead on parabolic equation in the system. The pdepe solver converts the PDF to ODE using a second order accurate spatial discrtizationbased on a set of nodes specified by the use. [11]

Part Differential Equation Solver Basic Syntax (6.3):

The basic syntax of the solver is:

sol=pdepe(m,pde fun,ic fun,bc fun,xmesh,tspan)

m is specifies the symmetry of problem.

pde fun is a function handle that computes m, f and s

]m u,F,s]:pde fun (m,t,u,ux)

ic fun is function handle that computes ∅

phi:ic fun(x)

bc fun: is a function handle that computes theBC

[pa,qq,pb,qb]=bc fun(a,uq,b,ub,t)

x mesh: is a vector of points in [a,b]

span: is a vector of time values [14]

Example(6.2):

Find the third fundamental From of the surface

r=asin cos ∅ i+asin sin ∅ j+acos k

using the new mathematical method if θ=2,ϕ=π

Solution:

```
clear all
clc
symsrtabpijkN1N2ABCrtrpNdtdpIII
% t=90
% p=180
r=
a*(sin(t)*(cos(p))*i)+(a*(sin(t)*(sin(p)))*j)+(a*(cos(t))*k)
% III=Adt2 + 2Bdtdp + Cdp2
% N1=N^rt first normal curvarte
% N2=N^rpsecand normal curvarte
rt=diff(r,t)
rp=diff(r,p)
N=(rt*rp)/(abs((rt*rp)));
N=simplify(N)
N1=[i j k;sin(t)*cos(p) sin(t)*sin(p) cos(t);a*cos(t)*cos(p)
acos(t)*sin(p) -a*sin(t)];
N1=det(N1)
A=N1*N1;
A=simplify(A)
N2=[i j k;sin(t)*cos(p) sin(t)*sin(p) cos(t);-a*sin(t)*sin(p)
a*sin(t)*cos(p) 0];
N2=det(N2)
B=N1*N2;
B=simplify(B)
C=N2*N2;
C=simplify(C)
III = A*dt^2 + 2*B*dt*dp + C*dp^2;
III=simplify(III)
```

Result

```
r =
a*k*cos(t) + a*i*cos(p)*sin(t) + a*j*sin(p)*sin(t)
rt =
a*i*cos(p)*cos(t) - a*k*sin(t) + a*j*cos(t)*sin(p)
rp =
a*j*cos(p)*sin(t) - a*i*sin(p)*sin(t)
N =
(a^2*sin(t)*(j*cos(p) - i*sin(p))*(i*cos(p)*cos(t) - k*sin(t) +
j*cos(t)*sin(p)))/(abs(sin(t)*(j*cos(p) -
i*sin(p))*(i*cos(p)*cos(t) - k*sin(t) +
j*cos(t)*sin(p)))*abs(a)^2)
N1 =
a*j*cos(p)*cos(t)^2 + a*j*cos(p)*sin(t)^2 -
a*i*sin(p)*sin(t)^2 - i*acos(t)*cos(t)*sin(p) +
k*cos(p)*acos(t)*sin(p)*sin(t) -
a*k*cos(p)*cos(t)*sin(p)*sin(t)
A =
(a*i*sin(p) - a*j*cos(p) - a*i*cos(t)^2*sin(p) +
i*acos(t)*cos(t)*sin(p) - k*cos(p)*acos(t)*sin(p)*sin(t) +
a*k*cos(p)*cos(t)*sin(p)*sin(t))^2
N2 =
a*k*cos(p)^2*sin(t)^2 + a*k*sin(p)^2*sin(t)^2 -
a*i*cos(p)*cos(t)*sin(t) - a*j*cos(t)*sin(p)*sin(t)
B =
a*(k*cos(t)^2 - k + i*cos(p)*cos(t)*sin(t) +
j*cos(t)*sin(p)*sin(t))*(a*i*sin(p) - a*j*cos(p) -
a*i*cos(t)^2*sin(p) + i*acos(t)*cos(t)*sin(p) -
k*cos(p)*acos(t)*sin(p)*sin(t) +
a*k*cos(p)*cos(t)*sin(p)*sin(t))
C =
a^2*(k*cos(t)^2 - k + i*cos(p)*cos(t)*sin(t) +
j*cos(t)*sin(p)*sin(t))^2
III =
```

(a*dt*i*sin(p) - a*dt*j*cos(p) - a*dp*k + a*dp*k*cos(t)^2 - a*dt*i*cos(t)^2*sin(p) + dt*i*acos(t)*cos(t)*sin(p) + a*dp*i*cos(p)*cos(t)*sin(t) + a*dp*j*cos(t)*sin(p)*sin(t) - dt*k*cos(p)*acos(t)*sin(p)*sin(t) + a*dt*k*cos(p)*cos(t)*sin(p)*sin(t))^2

Solution :

```
clear all
clc
symsrtabpijkN1N2ABCrtrpNdtdpIII
% t=90
% p=180
r=
a*(sin(t)*(cos(p))*i)+(a*(sin(t)*(sin(p)))*j)+(a*(cos(t))*k)
% III=Adt2 + 2Bdtdp + Cdp2
% N1=N^rt first normal curvarte
% N2=N^rpsecand normal curvarte
rt=diff(r,t);
rt=simplify(rt)
rp=diff(r,p);
rp=simplify(rp)
N=(rt*rp)/(abs((rt*rp)))
% t=90
% p=180
N1=[i j k;sin(t)*cos(p) sin(t)*sin(p) cos(t);a*cos(t)*cos(p) acos(t)*sin(p) -a*sin(t)];
N1=-a*j
A=N1*N1
N2=[i j k;sin(t)*cos(p) sin(t)*sin(p) cos(t);-a*sin(t)*sin(p) a*sin(t)*cos(p) 0];
N2=det(N2)
B=N1*N2;
B=simplify(B)
C=N2*N2;
C=simplify(C)
III = A*dt^2 + 2*B*dt*dp + C*dp^2;
III=simplify(III)
```

Result

```
r =
a*k*cos(t) + a*i*cos(p)*sin(t) + a*j*sin(p)*sin(t)
rt =
a*(i*cos(p)*cos(t) - k*sin(t) + j*cos(t)*sin(p))
rp =
a*sin(t)*(j*cos(p) - i*sin(p))
N =
(a^2*sin(t)*(j*cos(p) - i*sin(p))*(i*cos(p)*cos(t) - k*sin(t) + j*cos(t)*sin(p)))/(abs(sin(t)*(j*cos(p) - i*sin(p))*(i*cos(p)*cos(t) - k*sin(t) + j*cos(t)*sin(p)))*abs(a)^2)
N1 =
-a*j
A =
a^2*j^2
N2 =
a*k*cos(p)^2*sin(t)^2 + a*k*sin(p)^2*sin(t)^2 - a*i*cos(p)*cos(t)*sin(t) - a*j*cos(t)*sin(p)*sin(t)
B =
a^2*j*(k*cos(t)^2 - k + i*cos(p)*cos(t)*sin(t) + j*cos(t)*sin(p)*sin(t))
C =
a^2*(k*cos(t)^2 - k + i*cos(p)*cos(t)*sin(t) + j*cos(t)*sin(p)*sin(t))^2
III =
a^2*(dt*j - dp*k + dp*k*cos(t)^2 + dp*i*cos(p)*cos(t)*sin(t) + dp*j*cos(t)*sin(p)*sin(t))^2
```

CONCLUSION

At last we can find the Third Fundamental Form of Surface using the new mathematical method but we cant find t the solution graphically so we hope that to do our best in the future to find the solution of the Third Fundamental Form of Surface graphically.

REFERENCES

1. Alan wenstin and Joseph Oesterle, Analysis on CR Manifolds,My10013,united States of America.
2. Barret O’Neill, Elementary Differential Geometry,MA018303 ,Burlington ,USA,2006.
3. Brian D.Hahn and DarielT.valentine ,ESSENTIAL MATLAB for English and scientists,068417, Burlington ,Amsterdam.
4. Dietmar A. Salamon and Joel w.Robbin, introduction to differential geometry ,Newyork,5 September 2013.
5. D.S MitrinOvi and j UIcar, Differential Geometry ,tutorial text No5, Belegrade,1969, p61
6. Gabriel Lugo ,differential geometry in physics ,NC28403,Wilmington ,2006,pp44.
7. J. J. STOKER,DIFFERENTIA LGEOMETRY, Wiley ,1,inited state of America ,1976 ,p98.
8. JofferyM.lee, Differential analysis and physics , Boston , 2000.
9. Marian Fecko ,Deferential Geometry and Groups for Phycists, 84507 , camberidge university , Melbourne 2001.
10. Math work, Getting stated with MATLAB, second printing, may 1997.
11. math work, using MATLAB, versin, 6,2001.
12. Nasar solame, Differential Geometry, 2,Asuit, 2008,pp349.
13. Rao.v.Dukkipati, MATLAB on Introduction with Applications, new Age international (p),new delhi,2004.
14. Soren Bottcher, Solving ODE and PDE in MATLAB SCIE ,2009.
15. Taha Sochi, Principles of Differential Geometry, London, September 9, 2016,pp81.