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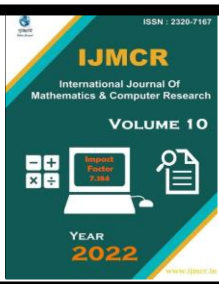

Extreme Value Charts and ANOM Based on Gumbel Distribution

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ARTICLE INFO	ABSTRACT
Published online: 29 June 2022	The quality is assessable by its feature is implicit with a probability model which follow Gumbel distribution. From each subgroups extreme values are used to construct extreme value charts and variable control charts. Probability model of the extreme order statistics and the size of each sub group are used in control chart constants. To find the decision lines of Gumbel distribution we implement a method of analysis of means (ANOM). The proposed ANOM decision lines are constructed for given number of subgroup within means category and between means category given by Ott (1967). These decision lines are illustrated by giving few examples.
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KEYWORDS: Extreme Value charts, In Control, Quantile-Quantile plot, Control Limits.

1. INTRODUCTION

Analysis and prediction of the business conditions is the top most trend of research especially in marketing field. Every business has interested to know the future market value of their products. To analyze this type of conditions there are common statistical techniques are available like weekly averages, monthly averages and moving averages etc., Averages gives a summary about the product and business. Statistical methods are important for solving the economic problems of industry. To asses quality common tool is Shewart control charts. These charts are used to diagnose the assignable causes when there is any adjustments made in the process. Otherwise expected assignable cause is treated as to be a quality diverse in character of the subgroup. In this situation analysis of means (ANOM) technique is an effective and appropriate. For example if statistic is a sample mean, which control diversity of the process mean showing away from the expected mean. To analyse the subgroups those are categorized within identical group of means and between the mean of diverse category is possible through ANOM, which is the alternative method of Analysis of Variance (ANOVA). A further most characteristic of ANOM include its interpretation and graphical presentation. An ANOM charts and control charts are theoretically having the same pattern among decision lines, thus magnitude differences and Statistical significance of the treatments may be evaluated at the same time. The ANOM technique was first developed by Prof. Ott (1967) to know the association of means subgroup and check whether there is any significance change from the over all mean. Statistical control charts are designed based on the normal distribution. Where as if the data follows skewed distribution these limits should be calculated according to the distribution based on the variable control limits.

In the present research paper we consider Gumbel Distribution is one of the skewed distribution and developed control limits for the distribution.

The probability density function (pdf) of a Gumbel distribution (GD) with scale parameter σ is given by

$$f(x) = \frac{1}{\sigma}\left[e^{-(z+e^{-z})}\right] \text{ where } z = \frac{x-\mu}{\sigma}; \quad -\infty<x<\infty,\ -\infty<\mu<\infty,\ \sigma>0 \qquad (1.0.1)$$

Its cumulative distribution function (cdf) is

$$\mathbf{F(x)=exp}[-e^{-(x-\mu)/\sigma}] \qquad (1.0.2)$$

To construct control charts by extreme elements drawn from the production process of subgroup which follows Gumbel Distribution. Let us suppose that $x_1, x_2, \ldots\ldots, x_n$ are the select sample from the production process plotted on control chart. Individual samples are plotted into the control chart without estimating any statistic out of them. Based on the plotted sample using x_1(sample minimum) and x_n(sample maximum) a right decision should be taken.

Let us suppose that $\bar{x}_1, \bar{x}_2 \ldots \ldots \bar{x}_n$ are arithmetic mean of k subgroups and n sample size drawn from the Gumbel distribution. From the population selected subgroup means are functioning with allowable quality or not will be assessed with the help of subgroup means. If we found any sample mean fall outside the decision lines then it's consider as difference is significant from the grand mean. Decision lines and significance of the samples can be assessed at a time by using the ANOM charts.

The main intention of developing Analysis of Means using extreme value control limits is that to represent the significance difference between groups in a visualization. We are not taken any developed ANOM tables or techniques. Many authors worked on Analysis of means for different distributions some references are quoted such as: Rao (2005)[15], Ott(1967) and Enrick (1976) [4],Schilling (1979) [6], Ohta (1981)[11], Ramig (1983)[13], Bakir (1994) [1], Bernardand Wludyka*et.al*. (2001)[2], Montgomery (2000)[8], Nelson and Dudewiczand Nelson (2002)[10], Farnum (2004)[5], Guirguis and Tobias (2004)[6],Srinivasa Rao B *et.al.*(2012) [17], Srinivasa Rao. B and Pratapa Reddy.J *et. al.* (2012)[18], B.Srinivasa Rao and P.Sricharani (2018)[19], R.Subba Rao, A. Naga Durgamamba *et. al.*(2018)[20], Kalanka P Jayalatha and Honkeung Tony Ng.(2020)[21], Kalanka P Jayalatha and Jacob Turner (2021)[22], B.Srinivara Rao, K.N.V.RLakshmi *et.al.*(2021)[23], Anil and B. Srinivasa Rao(2021)[24] all the references listed at the end of the article.

The research article is organized in the following way. Extreme value control chart concepts and procedure of is explained in part 2. Using GD model in ANOM and control charts using extreme values of GD is shown in part 3. Examples and related illustrative are given in the part 4. Summary and conclusions are discussed in part 5.

2. EXTREME VALUE CHARTS

If α is the level of significance and we should have the probability of all the subgroup means to fall within the control limits is (1-α). By considering subgroups the above probability statement be converted into nth power of the probability of a subgroup mean to fall within the limits should be equal to (1- α). i.e., the confidence interval for mean$\bar{x}$of the sampling distribution of $\bar{x}$ lie between two control limits should be equal to (1- α)$^{1/n}$. using the same concept for developing the ANOM charts for Gumbel Distribution.

Subgroups and samples are drawn from the GD model. Using the extreme order statistics concepts control lines are determined based on GD. By selecting the x_i of X=(x_1 , x_2 ,..., x_n) lies with probability $(1-\alpha)^{1/n}$ control lines are calculated and found to be fall within the limits. This is the probability inequality. $P(x_1 \leq L)=\alpha/2$ and $P(x_n \geq U)=\alpha/2$. By the procedures and concepts of order statistics, from any continues population the sample size n be the higher order statistics and the cumulative distribution function of the least are $1-[1-F(x)]^n$ and $[F(x)]^n$ respectively. Where $F(x)$ is the cumulative distribution function (cdf) of the population. If (1-α) is the required at 0.9973 then α would be 0.0027. Taking $F(x)$ as the CDF of a standard GD model (σ = 1), we can get solutions of the two equations $[F(x)]^n$= 0.99865 and$1-[1-F(x)]^n$= 0.00135, these equations are used to develop extreme value control limits. The results of the above two equations for n = 2 (1) 10 is given in Table 2.1 denoted as $Z_{(1)\,0.00135}$ and $Z_{(n)\,0.99865}$.

Table 2.1: Extreme value Control Limits

n	$Z_{(1)0.00135}$	$Z_{(n)\,0.99865}$
2	0.03733	4.49294
3	0.09505	3.55494
4	0.13846	3.12189
5	0.2064	2.83479
6	0.2426	2.68648
7	0.2759	2.46582
8	0.29866	2.36053
9	0.30828	2.28933
10	0.32988	2.23825

From the above Table 2.1 the following probability statements:

$P(Z_{(1)0.00135} \leq Z_i \leq Z_{(n)\,0.99865},$ □□ $i = 1,2,\ldots..n) = 0.9973$ (2.0.3)

$P(\sigma Z_{(1)0.00135} \leq Z_i \leq \sigma Z_{(n)\,0.99865},$ □□ $i = 1,2,\ldots..n) = 0.9973$ (2.0.4)

Taking $\overline{X}$ is 0.5772 as an unbiased estimate of σ, the above equation becomes

$\Rightarrow P(D_3^*\overline{X} < x_i < D_4^*\overline{X}, \forall i = 1,2,\ldots n) = 0.9973$

Where $D_3^* = \frac{Z(1)0.00135}{0.5772}$ and $D_4^* = \frac{Z(n)\,0.99865}{0.5772}$ Thus D_3^* and D_4^* would constitute the control chart constants for the extreme values charts. Results are given in table 2.2 for n=2(1) 10.

Table 2.2: Extreme value Constants

N	D_3^*	D_4^*
2	0.064674	7.784026
3	0.164674	6.15894
4	0.239882	5.40868
5	0.357588	4.911279
6	0.420305	4.654331
7	0.477997	4.272037
8	0.517429	4.089622
9	0.534096	3.966268
10	0.571518	3.877772

3. Analysis of Means (ANOM) - Gumbel distribution

From the Gumbel distribution model $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$ are arithmetic means(k) of size 'n' subgroups. Subgroup means drawn from the population are assess the functioning with the acceptable quality variations. Based on the selected population model, If all the subgroup means fall with in the limits then the process is in control. Otherwise we say the process is out of control. The above decisions can be converted into probability statements by assuming α is the level of significance.

$$P\{LCL < \bar{x}_i, \forall i = 1 \text{ to } k < UCL\} = 1-\alpha \quad (3.0.6)$$

Using the notion of independent subgroups (3.0.6) becomes

$$P\{LCL < \bar{x}_i, < UCL\} = (1-\alpha)^{1/k} \quad (3.0.7)$$

With equi–tailed probability for each subgroup mean, we can find two constants say L^*
and U^* such that

$$P\{\bar{x}_i < L^*\} = P\{\bar{x}_i, > U^*\} = \frac{1-(1-\alpha)^k}{2} \quad (3.0.8)$$

Usually in normal population L^* and U^* satisfy $U^* = -L^*$. If the distribution follows skewness like Gumbel Distribution $L^*$and U^* are calculated separately from the sampling distribution of $\bar{x}_i$. Based on the size 'n' subgroup and the number of subgroup means 'k'. We make us of the equations (3.07) and (3.08) for specified 'n' and 'k' toget L^* and U^* for α = 0.05 and α = 0.01. These are given in Tables 3.2 and 3.3.

Table 3.2: Gumbel Distribution Constants for Analysis of Means (1- α = 0.95)

Table 3.2: Gumble Distribution Constants for Analysis of Means (1- α = 0.95)

n	LIMITS	k=1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60
2	L	0.1578	0.1115	0.0916	0.0768	0.0701	0.0602	0.0572	0.0544	0.0529	0.0504	0.0396	0.0369	0.0336	0.0327	0.0325	0.0275
	U	2.7799	3.1496	3.3567	3.5481	3.6613	3.7286	3.8671	3.9978	4.0332	4.0579	4.1905	4.5026	4.7724	4.8167	4.9507	4.9840
3	L	0.2426	0.1848	0.1649	0.1481	0.1427	0.1361	0.1325	0.1303	0.1248	0.1204	0.1021	0.0868	0.0809	0.0766	0.0758	0.0696
	U	2.4326	2.7079	2.8699	2.9975	3.0838	3.1147	3.1562	3.2480	3.3076	3.3109	3.4753	3.5798	3.6686	3.6903	3.7674	3.8161
4	L	0.3166	0.2634	0.2425	0.2247	0.2126	0.2035	0.1955	0.1877	0.1823	0.1776	0.1513	0.1383	0.1331	0.1056	0.0959	0.0941
	U	2.1684	2.4070	2.5534	2.6263	2.7249	2.7618	2.8250	2.8691	2.9139	2.9439	3.0517	3.1519	3.2006	3.2088	3.3905	3.5417
5	L	0.3645	0.3040	0.2731	0.2625	0.2529	0.2436	0.2370	0.2343	0.2337	0.2305	0.2170	0.2047	0.1904	0.1737	0.1652	0.1619
	U	2.0251	2.2184	2.3270	2.4111	2.4829	2.5311	2.5551	2.5911	2.6163	2.6752	2.8098	2.8413	2.9225	3.0219	3.0227	3.0310
6	L	0.4135	0.3563	0.3255	0.3156	0.2934	0.2877	0.2744	0.2620	0.2590	0.2565	0.2498	0.2417	0.2241	0.1941	0.1936	0.1927
	U	1.9667	2.1480	2.2335	2.3173	2.3781	2.4005	2.4406	2.4853	2.5065	2.5232	2.5591	2.6876	2.8087	2.8672	2.9782	2.9812
7	L	0.4381	0.3975	0.3669	0.3499	0.3420	0.3333	0.3219	0.3149	0.3097	0.3057	0.2880	0.2748	0.2552	0.2445	0.2372	0.2318
	U	1.8771	2.0356	2.1541	2.2093	2.2493	2.2859	2.3189	2.3406	2.3544	2.3778	2.4355	2.4884	2.5821	2.6140	2.6190	2.6195
8	L	0.4694	0.4069	0.3798	0.3676	0.3551	0.3476	0.3381	0.3322	0.3261	0.3141	0.3014	0.2952	0.2811	0.2659	0.2583	0.2582
	U	1.7852	1.9251	2.0019	2.0624	2.0987	2.1563	2.1787	2.2286	2.2396	2.2602	2.3061	2.3894	2.4105	2.5134	2.5274	2.5539
9	L	0.4998	0.4510	0.4234	0.4061	0.3858	0.3770	0.3685	0.3566	0.3473	0.3419	0.3228	0.3034	0.2888	0.2764	0.2635	0.2231
	U	1.7322	1.8783	1.9304	1.9801	2.0119	2.0482	2.0683	2.1213	2.1305	2.1581	2.1935	2.2962	2.3264	2.3354	2.3751	2.3771
10	L	0.5119	0.4548	0.4268	0.4134	0.4014	0.3966	0.3829	0.3728	0.3641	0.3576	0.3367	0.3286	0.3117	0.3079	0.2912	0.2846
	U	1.7092	1.8344	1.9156	1.9689	2.0252	2.0654	2.0885	2.1037	2.1073	2.1154	2.1961	2.2388	2.3016	2.3373	2.3603	2.3979

Table 3.3: Gumbel Distribution Constants for Analysis of Means (1- α = 0.99)

Table 3.3: Gumble Distribution Constants for Analysis of Means (1- α = 0.99)

n	LIMITS	k=1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60
2	L	0.0685	0.0504	0.0391	0.0369	0.0359	0.0336	0.0334	0.0327	0.0325	0.0325	0.0223	0.0097	0.0097	0.0058	0.0058	0.0058
	U	3.6647	4.0579	4.1936	4.5026	4.5373	4.7724	4.8017	4.8167	4.9507	4.9840	5.0839	5.1209	5.3222	5.3222	5.6975	5.8811
3	L	0.1420	0.1204	0.1000	0.0868	0.0865	0.0809	0.0809	0.0766	0.0758	0.0758	0.0647	0.0635	0.0635	0.0545	0.0545	0.0545
	U	3.0910	3.3109	3.4848	3.5798	3.6131	3.6686	3.6759	3.6903	3.7674	3.8161	3.8275	3.8512	3.8555	3.8555	3.8945	3.9589
4	L	0.2122	0.1776	0.1469	0.1383	0.1343	0.1331	0.1211	0.1056	0.0959	0.0959	0.0940	0.0604	0.0604	0.0212	0.0212	0.0212
	U	2.7348	2.9439	3.0860	3.1519	3.1595	3.2006	3.2033	3.2088	3.3905	3.5417	3.5784	3.7032	4.0060	4.0060	4.1803	4.5836
5	L	0.2520	0.2305	0.2106	0.2047	0.1937	0.1904	0.1850	0.1737	0.1652	0.1652	0.1453	0.1448	0.1448	0.1293	0.1293	0.1293
	U	2.4932	2.6752	2.8212	2.8413	2.8670	2.9225	2.9663	3.0219	3.0227	3.0310	3.0328	3.0353	3.0693	3.0693	3.0797	3.2173
6	L	0.2924	0.2565	0.2474	0.2417	0.2316	0.2241	0.2219	0.1941	0.1936	0.1936	0.1902	0.1293	0.1293	0.1130	0.1130	0.1130
	U	2.3834	2.5232	2.5686	2.6876	2.7058	2.8087	2.8513	2.8672	2.9782	2.9812	3.0107	3.0111	3.1889	3.1889	3.2689	3.3749
7	L	0.3405	0.3057	0.2850	0.2748	0.2693	0.2552	0.2529	0.2445	0.2372	0.2372	0.2300	0.2055	0.2055	0.1938	0.1938	0.1938
	U	2.2549	2.3778	2.4395	2.4884	2.4940	2.5821	2.5873	2.6140	2.6190	2.6195	2.6728	2.7878	2.9143	2.9143	3.0613	3.6102
8	L	0.3519	0.3141	0.3007	0.2952	0.2837	0.2811	0.2765	0.2659	0.2583	0.2583	0.2344	0.1904	0.1904	0.1442	0.1442	0.1442
	U	2.1010	2.2602	2.3372	2.3894	2.4072	2.4105	2.4732	2.5134	2.5274	2.5539	2.5777	2.5924	2.7016	2.7016	2.7061	3.3559
9	L	0.3823	0.3419	0.3214	0.3034	0.2981	0.2888	0.2824	0.2764	0.2635	0.2635	0.1966	0.1845	0.1845	0.1775	0.1775	0.1775
	U	2.0138	2.1581	2.1989	2.2962	2.3021	2.3264	2.3299	2.3354	2.3751	2.3771	2.4031	2.4772	2.5265	2.5265	2.9165	2.9583
10	L	0.4010	0.3576	0.3346	0.3286	0.3191	0.3117	0.3090	0.3079	0.2912	0.2912	0.2716	0.2251	0.2251	0.1962	0.1962	0.1962
	U	2.0317	2.1154	2.2036	2.2388	2.2658	2.3016	2.3295	2.3373	2.3603	2.3979	2.3997	2.4648	2.5323	2.5323	2.5709	2.6815

All the subgroup means which are different themselves and homogeneous between groups fall with in indicates 'In Control'. Thus the estimated constants presented in Tables 3.2 and 3.3 can be used as Analysis of Mean technique. Earlier Ott(1967)[12] used normal population constants table available in every Statistical Quality Control (SQC) text book for any distribution. If the distribution follows Gumbel Model we recommend and suggest to use our constant values shown in the tables(3.2&3.3). To evaluate the goodness of fit of Gumbel Distribution model we consider few examples with quantile - quantile plot technique (it shows the strength of linearity between observed and theoretical quantiles of a model) and test the homogeneity of means in each group.

4. ILLUSTRATIVE EXAMPLES

Example 1: Wardsworth (1986): The following 25 observations on “Amanufactures of metal products and the raw material supplied by five suppliers which are identified variations in iron content. From each of the suppliers five ingots were randomly selected. The data is shown in the following table contains the iron determinations on each ingots in percent by weight.

Example-1				
Suppliers				
1	2	3	4	5
3.46	3.59	3.51	3.38	3.29
3.48	3.46	3.64	3.4	3.46
3.56	3.42	3.46	3.37	3.37
3.39	3.49	3.52	3.46	3.32
3.4	3.5	3.49	3.39	3.38

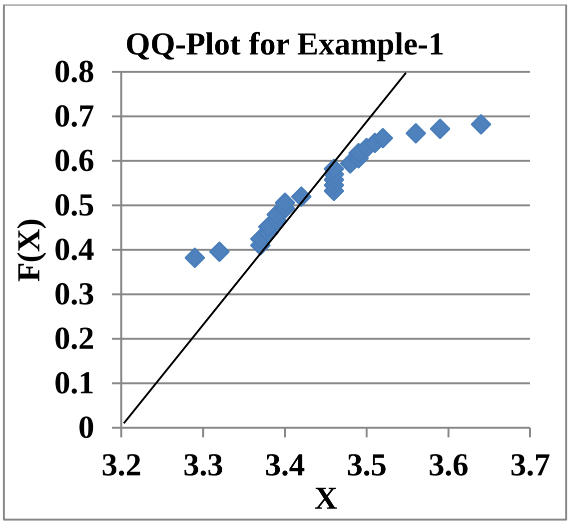
Example 2: a study is considered from a three brands of batteries. It is assumed that three brands life (in weeks) is different. A test is conducted by considering five batteries of each brand and the results are given in the following table. Check reliability of these brands of batteries are different at 5 %level of significance.

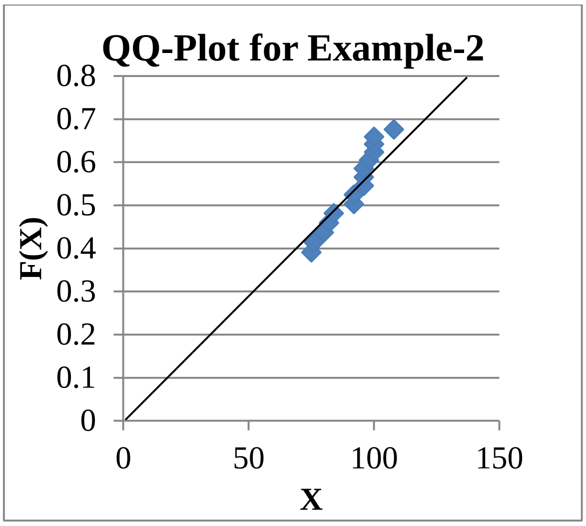
Example-2		
Weeks of life		
Brand - 1	Brand-2	Brand-3
100	76	108
96	80	100
92	75	96
96	84	98
92	82	100

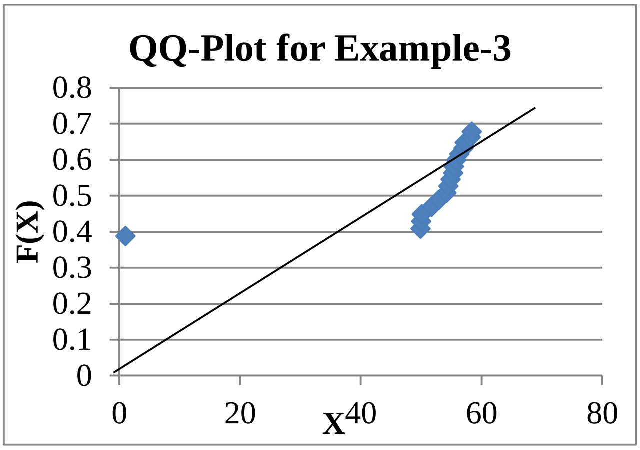
Example 3: An investigation is done by selecting four catalysts which effect the concentration of one component in a three component liquid mixture. The results of concentrations are shown below. Check whether the same affect on the concentration is observed in four catalysts at 5 % level of significance.

Example – 3			
Catalyst			
1	2	3	4
58.2	56.3	50.1	52.9
57.2	54.5	54.2	49.9
58.4	57	55.4	50
55.8	55.3	54.9	51.7

Graphical presentation of Qantile-Quantile Plot are drawn for the data in the illustrated examples:







From the above three graphs shows the goodness of fit of data by qunatile–quantile plot and corresponding correlation coefficient are presented in the following table, which indicates Gumbel Distribution is a better model, revealing significance linear relationship between sample and population quantiles

.

	GD	Normal
Example 1	**0.9595**	**0.2067**
Example 2	**0.9716**	**0.4149**
Example 3	**0.9815**	**0.4447**

In the given data observations are considered as a single sample, a comparison is made between the Normal population and Gumbel population for the determined the control limits. Results are displayed in the Table 3.4 respectively.

Table 3.4

	(LDL, UDL)	No. of. Subgroups fall			
		With in the decision lines	Coverage probability	Outside decision limits	Coverage probability
Example-1 n=5, k=5, α=0.05	[3.379,3.517] [0.8719, 8.5602]	3 5	0.6 1.0	2 0	0.4 0.0
Example-2 n=5, k=3, α=0.05	[87.82,95.52] [25.0305,213.3121]	2 3	0.7 1.0	1 0	0.3 0.0
Example-3 n=4, k=4, α=0.05	[26.14, 82.84] [12.2455, 143.0999]	2 4	0.5 1.0	2 0	0.5 0.0

In the above represented cell the first row values are Normal distribution and second row values are the Gumbel distribution.

5. SUMMARY AND CONCLUSIONS

From the illustrative data, when we use ANOM tables of Ott (1967) [11] the number of homogeneous means for each example are 3,2,2 respectively and heterogeneity are 2,1,2 respectively. Where as when we use ANOM tables of Gumbel Distribution the number of homogeneous means are 5,3,4 respectively there is no heterogeneity means that, any one mean is not deviating from homogeneity i.e, out side the control limits. Therefore there is a possibility of rejection when we use normal distribution constants resulted in homogeneity for within means and deviation from between means. Through quantile-quantile plot technique we made a conclusion that Gumbel Distribution is a better model than Normal. Therefore, Gumbel Distribution is a better decision if all the means to be homogeneous. We recommend the environmentalist hydrologist and experimenters those who are to adopt this techniques. Also suggest for predictive analysis occurrence of natural extreme events such as flood water level and high winds.

Declarations Statements:

I have declared that the following statements are true.

1. Funding: If none, write: Not applicable.
2. Informed Consent Statement: Not applicable.
3. Data Availability Statement: Not applicable.
4. Conflicts of Interest: The author declares no conflict of interest.

REFERENCES

1. Bakir. S.T,(1994).*Analysis of means using ranks for randomized complete block designs*, Communications in Statistics- Simulation and Computation 23, 547-568.
2. Bernard. A.J. and Wludyka. P.S, (2001).*Robust i-sample analysis of means type randomization tests for variances*, Journal of Statistical Computation and Simulation 69, 57-88.
3. Dedewicz. E.J. and Nelson. P.R, *Heteroscedastic analysis of means (hanom)*, American Journal of Mathematical and Management Sciences 23 (2003),143-181.
4. Enrick. N.L,(1976)*An analysis of means in a three-way factorial*, Journal of Quality Technology 8, 189-196.
5. Farnum. N.R, (2004).*Analysis of means tables using mathematical processors*, Quality Engineering 16, 399-405.
6. Guirguis. G.H and Tobias. R.D, (2004).*On the computation of the distribution for the analysis of means*, Communication in Statistics- Simulation and Computation 16, 61-887.
7. Gunst. R.F. Mason. R.L. and Hess. J.L, (1989). *Statistical design and analysis of experiments*, John Wiley and Sons, New York.
8. Montgomery. D.C,(2000).*Design and analysis of experiments*, fifth ed., John Wiley and Sons,.
9. Nelson, P.R. Wludyka. P.S. and Silva. P.R, (2001). *Power curves for analysis of means for variances*, Journal of Quality Technology 33, 60-65.

10. Nelson. P.R. and Dudewicz. E.J, (2002). *Exact analysis of means with unequal variances*, Technometrics 44, 115-119.
11. Ohta. H, (1981). *A procedure for pooling data by analysis of means*, Journal of Quality Technology 13, 115-119.
12. Ott.E.R, (1967). *Analysis of means- a graphical procedure*, Industrial Quality Control 24.
13. Ramig. P.F, *(1983). Application of analysis of means*, Journal of Quality Technology 15, 19-25.
14. Rao. C.V and Prankumar. M, (2002),*Anom-type graphical methods for testing the equality of several correlation coefficients*, Gujarat Statistical Review 29 (2002),47-56.
15. Rao. C.V, *(2005). Analysis of means- a review*, Journal of Quality Technology 37.
16. Schilling. E.G, (1979).*A simplified graphical grant lot acceptance sampling procedure*, Journal of Quality Technology 11, 116-127.
17. Srinivasa Rao, B., Pratapa Reddy, J. and Rosaiah, K. (2012). *"Extreme Value Charts and ANOM Based on Inverse Rayleigh Distribution",* Pakistan Journal of Statistics and Operations Research, VIII (4), 759-766.
18. Srinivasa Rao, B., Pratapa Reddy, J. and Sarath Babu, G. (2012). "*Extreme Value Charts and Analysis of Means (ANOM) Based on the Log Logistic Distribution*," Journal of Modern Applied Statistical Methods, 11(2), Article 20,493-505.
19. B.Srinivasa Rao, P.Sricharani.(2018). *"Extreme value charts and Analysis of means based on Dagum distribution"*, International Journal of Statistics and Applied Mathematics,3(2),351-364.
20. R.Subba Rao, A. Naga Durgamamba, G.Prasad, R.R.L.Kantam.(2018*)." Analysis of Means (ANOM) based on the Size Biased Lomax Distribution",* International Journal of Engineering & Technology, 7(3.31), 129-132.
21. Kalanka P Jayalatha Hon Keung Tony Ng.(2020). "*Analysis of means approach in advanced designs*" **https://doi.org/10.1002/asmb.2540**"
22. Kalanka P. Jayalath and Jacob Turner. (2021). *"Analysis of Means (ANOM) Concepts and Computations"*, Applications and Applied Mathematics: An International Journal, Vol.16, Issue1, 75-96.
23. B.Srinivasa Rao, K.N.V.R.Lakshmi.(2021). "*Analysis of Means(ANOM) based on New-weibull Pareto distribution*", International Journal of Research & Analytical Reviews,8(2),481-491.
24. Anil.A, B.Srinivasa Rao.(2021)." Extreme Value Charts & Analysis of Means based on New Weighted Exponential distribution", Journal of University of Shanghai for Science and Technology, 23(11),825-836.