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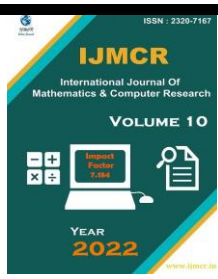

Status-Somber Indices

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ARTICLE INFO	**ABSTRACT**
Published Online: 15 June 2022 Corresponding Author: **V.R.Kulli**	In this paper, we introduce the status Sombor index, modified status Sombor index and their corresponding exponentials of a graph and compute exact formulas for some standard graphs, wheel graphs and friendship graphs. Also we establish some properties of newly defined status Sombor index.

KEYWORDS: status Somber index, modified status Sombor index, status Sombor exponential, modified status Sombor exponential, graph

I. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

In 1972[2], two degree based topological indices were introduced and studied.

The first and second status indices of a graph G were introduced by Ramane et al. in [3], and they are defined as

$$S_1(G)=\sum_{uv\in E(G)}[\sigma(u)+\sigma(v)],$$

$$S_2(G)=\sum_{uv\in E(G)}\sigma_G(u)\sigma_G(v).$$

Recently, some status indices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The Sombor index was introduced by Gutman in [16] and defined it as

$$SO(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)^2+d_G(v)^2}.$$

Recently, some Sombor indices were studied, for example, in [17, 18, 19, 20, 21, 22, 23].

Motivated by the definitions of the status and Somber indices, we introduce the status-Sonber index of a graph and defined it as,

$$SSO(G)=\sum_{uv\in E(G)}\sqrt{\sigma(u)^2+\sigma(v)^2}.$$

We also propose the status Sombor exponential of a graph and it is defined as

$$SSO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\sigma(u)^2+\sigma(v)^2}}.$$

We now define the modified status Sombor index of a graph G as

$$^{m}SSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2}}.$$

We also propose the modified status Sombor exponential of a graph and it is defined as

$$^{m}SSO(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2}}}.$$

The forgotten topological index of a graph is defined as [24]

$$F(G)=\sum_{uv\in E(G)}\left(d_G(u)^2+d_G(v)^2\right).$$

The F-status index of a graph is defined as [25]

$$FS(G)=\sum_{uv\in E(G)}\left(\sigma(u)^2+\sigma(v)^2\right).$$

In this paper, we compute the status Sombor index modified status Sombor index, status Sombor exponential and modified status Sombor exponential of some standard graphs, wheel graphs and friendship graphs. Also we establish some properties of the status Sombor index.

II. RESULTS

2.1. Complete graph K_n on n vertices

Theorem 1. The status Sombor index of a complete graph K_n is

$$SSO(K_n) == \frac{n(n-1)^2}{\sqrt{2}}.$$

Proof: If K_n is a complete graph with n vertices, then $d_{K_n}(u) = n-1$ and □(u) = n – 1 for any vertex u in K_n. Thus

$$SSO(K_n) = \sum_{uv \in E(K_n)} \sqrt{\sigma(u)^2 + \sigma(v)^2} = \frac{n(n-1)}{2}\sqrt{(n-1)^2 + (n-1)^2}$$

$$= \frac{n(n-1)^2}{\sqrt{2}}.$$

2.2. Cycle C_n on n vertices

Theorem 2. Let C_n be a cycle on n vertices. Then

$$SSO(C_n) = \frac{n^3}{2\sqrt{2}}, \qquad \text{if } n \text{ is even,}$$

$$= \frac{n(n-1)^2}{2\sqrt{2}}, \qquad \text{if } n \text{ is odd.}$$

Proof: If C_n is a cycle with n vertices, then $d_{C_n}(u) = 2$ for every vertex u in C_n.

Case 1. Suppose n is even. Then $\sigma(u) = \frac{n^2}{4}$ for any vertex u in C_n. Therefore

$$SSO(C_n) = \sum_{uv \in E(C_n)} \sqrt{\sigma(u)^2 + \sigma(v)^2}$$

$$= \sum_{uv \in E(C_n)} \sqrt{\left(\frac{n^2}{4}\right)^2 + \left(\frac{n^2}{4}\right)^2} = \frac{n^3}{2\sqrt{2}}.$$

Case 2. Suppose n is odd. Then $\sigma(u) = \frac{n^2-1}{4}$ for any vertex u in C_n. Thus

$$SSO(C_n) = \sum_{uv \in E(C_n)} \sqrt{\sigma(u)^2 + \sigma(v)^2}$$

$$= \sum_{uv \in E(C_n)} \sqrt{\left(\frac{(n-1)^2}{4}\right)^2 + \left(\frac{(n-1)^2}{4}\right)^2} = \frac{n(n-1)^2}{2\sqrt{2}}.$$

III. RESULTS FOR WHEEL GRAPHS

A wheel W_n is the join of C_n and K_1. Clearly W_n has n+1vertices and $2n$ edges. A graph W_4 is depicted in Figure 1.

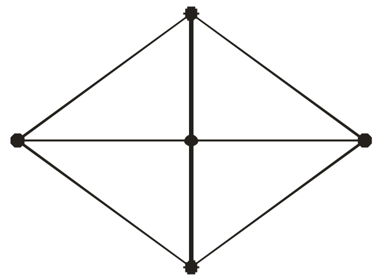

Figure 1. Wheel graph W_4

Let W_n be a wheel with n+1vertices and $2n$ edges. In W_n, there are two types of edges as follows:

E_1 = {uv □ $E(W_n)$ | $d_{W_n}(u) = d_{W_n}(v) = 3$ },

$|E_1| = n$.

E_2 = {uv □ $E(W_n)$ | $d_{W_n}(u) = 3,\ d_{W_n}(v) = n$ },

$|E_2| = n$.

Thus there are two types of status edges as given in Table 1.

Table 1. Status edge partition of W_n

$\sigma(u), \sigma(v) \setminus uv \in E(W_n)$	$(2n-3, 2n-3)$	$(n, 2n-3)$
Number of edges	n	n

Theorem 3. The status Sombor index of a wheel graph W_n is

$$SSO(W_n) = \sqrt{2}n(2n-3) + n\sqrt{5n^2 - 12n + 9}.$$

Proof: By using definition and Table 1, we deduce

$$SSO(W_n) = \sum_{uv \in E(W_n)} \sqrt{\sigma(u)^2 + \sigma(v)^2}$$

$$= n\sqrt{(2n-3)^2 + (2n-3)^2} + n\sqrt{n^2 + (2n-3)^2}$$

$$= \sqrt{2}n(2n-3) + n\sqrt{5n^2 - 12n + 9}.$$

Theorem 4. The status Sombor exponential of a wheel graph W_n is

$$SSO(W_n, x) = nx^{\sqrt{2}(2n-3)} + nx^{\sqrt{5n^2-12n+9}}.$$

Proof: From definition and by using Table 1, we have

$$SSO(W_n, x) = \sum_{uv \in E(W_n)} x^{\sqrt{\sigma(u)^2 + \sigma(v)^2}}$$

$$= nx^{\sqrt{(2n-3)^2 + (2n-3)^2}} + nx^{\sqrt{n^2 + (2n-3)^2}}$$

$$= nx^{\sqrt{2}(2n-3)} + nx^{\sqrt{5n^2-12n+9}}.$$

Theorem 5. The modified status Sombor index of a wheel graph W_n is

$$SSO(W_n) = \frac{n}{\sqrt{2}(2n-3)} + \frac{n}{\sqrt{5n^2 - 12n + 9}}.$$

Proof: By using definition and Table 1, we deduce

$$ {}^{m}SSO(W_n) = \sum_{uv \in E(W_n)} \frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2}} $$

$$ = \frac{n}{\sqrt{(2n-3)^2 + (2n-3)^2}} + \frac{n}{\sqrt{n^2 + (2n-3)^2}} $$

$$ = \frac{n}{\sqrt{2}(2n-3)} + \frac{n}{\sqrt{5n^2 - 12n + 9}}. $$

Theorem 6. The modified status Sombor exponential of a wheel graph W_n is

$$ {}^{m}SSO(W_n) = nx^{\frac{1}{\sqrt{2}(2n-3)}} + nx^{\frac{1}{\sqrt{5n^2-12n+9}}}. $$

Proof: By using definition and Table 1, we deduce

$$ {}^{m}SSO(W_n, x) = \sum_{uv \in E(W_n)} x^{\frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2}}} $$

$$ = nx^{\frac{n}{\sqrt{(2n-3)^2+(2n-3)^2}}} + nx^{\frac{n}{\sqrt{n^2+(2n-3)^2}}} $$

$$ = nx^{\frac{1}{\sqrt{2}(2n-3)}} + nx^{\frac{1}{\sqrt{5n^2-12n+9}}}. $$

IV. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F_n, $n \geq 2$, is a graph that can be constructed by joining n copies of C_3 with a common vertex. A graph F_4 is presented in Figure 2.

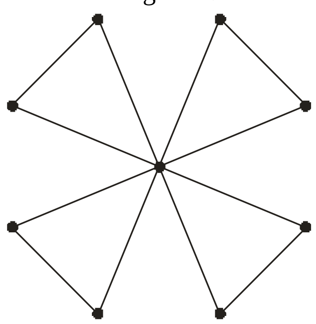

Figure 2. Friendship graph F_4

Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as follows:

$$ E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}, \quad |E_1| = n. $$

$$ E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}, \quad |E_2| = 2n. $$

Therefore, in F_n, there are two types of status edges as given in Table 2.

Table 2. Status edge partition of F_n

$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$	$(4n-2, 4n-2)$	$(2n, 4n-2)$
Number of edges	n	$2n$

In the following theorem, we compute the status Sombor index of a friendship graph F_n.

Theorem 7. The status Sombor index of a friendship graph F_n is

$$ SSO(F_n) = \sqrt{2}n(4n-2) + 4n\sqrt{5n^2 - 4n + 1}. $$

Proof: By using definition and Table 2, we deduce

$$ SSO(F_n) = \sum_{uv \in E(F_n)} \sqrt{\sigma(u)^2 + \sigma(v)^2} $$

$$ = n\sqrt{(4n-2)^2 + (4n-2)^2} + 2n\sqrt{(2n)^2 + (4n-2)^2} $$

$$ = \sqrt{2}n(4n-2) + 4n\sqrt{5n^2 - 4n + 1}. $$

Theorem 8. The status Sombor exponential of a friendship graph F_n is

$$ SSO(F_n, x) = nx^{\sqrt{2}(4n-2)} + 2nx^{2\sqrt{5n^2-4n+1}}. $$

Proof: From definition and by using Table 2, we have

$$ SSO(F_n, x) = \sum_{uv \in E(F_n)} x^{\sqrt{\sigma(u)^2 + \sigma(v)^2}} $$

$$ = nx^{\sqrt{(4n-2)^2+(4n-2)^2}} + 2nx^{\sqrt{(2n)^2+(4n-2)^2}} $$

$$ = nx^{\sqrt{2}(4n-2)} + 2nx^{2\sqrt{5n^2-4n+1}}. $$

Theorem 9. The modified status Sombor index of a friendship graph F_n is

$$ SSO(F_n) = \frac{n}{\sqrt{2}(4n-2)} + \frac{n}{\sqrt{5n^2 - 4n + 1}}. $$

Proof: By using definition and Table 2, we deduce

$$ {}^{m}SSO(F_n) = \sum_{uv \in E(F_n)} \frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2}} $$

$$ = \frac{n}{\sqrt{(4n-2)^2 + (4n-2)^2}} + \frac{2n}{\sqrt{(2n)^2 + (4n-2)^2}} $$

$$ = \frac{n}{\sqrt{2}(4n-2)} + \frac{n}{\sqrt{5n^2 - 4n + 1}}. $$

Theorem 10. The modified status Sombor exponential of a friendship graph F_n is

$$ {}^{m}SSO(F_n) = nx^{\frac{1}{\sqrt{2}(4n-2)}} + 2nx^{\frac{1}{2\sqrt{5n^2-4n+1}}}. $$

Proof: By using definition and Table 2, we deduce

$$ {}^{m}SSO(F_n, x) = \sum_{uv \in E(F_n)} x^{\frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2}}} $$

$$ = nx^{\frac{n}{\sqrt{(4n-2)^2+(4n-2)^2}}} + nx^{\frac{n}{\sqrt{(2n)^2+(4n-2)^2}}} $$

$$ = nx^{\frac{1}{\sqrt{2}(4n-2)}} + 2nx^{\frac{1}{2\sqrt{5n^2-4n+1}}}. $$

V. MATHEMATICAL PROPERTIES OF THE STATUS SOMBOR INDEX

Theorem 11. Let G be a connected graph with m edges. Then

$$SSO(G) \le \sqrt{mFS(G)}.$$

Proof: Using the Cauchy-Schwarz inequality, we obtain

$$\left(\sum_{uv\in E(G)} \sqrt{\sigma(u)^2+\sigma(v)^2}\right)^2 \le \sum_{uv\in E(G)} 1 \sum_{uv\in E(G)} \left(\sigma(u)^2+\sigma(v)^2\right).$$

$$= mFS(G).$$

Thus $$SSO(G) \le \sqrt{mFS(G)}.$$

By means of proof techniques analogous to what earlier was used to the Sombor index [26, 27], we establish Theorem 12 and Theorem 13.

Theorem 12. Let G be a connected graph. Then

$$SSO(G) \ge \frac{1}{\sqrt{2}} S_1(G).$$

Equality holds if and only if G is regular.

Theorem 13. Let G be a connected graph. Then

$$SSO(G) \le \sqrt{2}\left(S_1(G) - S_2(G)\right).$$

VI. CONCLUSION

In this paper, we have introduced the status Sombor index, modified status Sombor index and their exponentials of a graoh. These indices and their exponentials of complete graphs, cycles, wheel graphs, friendship graphs have been computed. Also we have established some properties of the status Sombor index.

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REFERENCES

1. V.R.Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
2. I.Gutman and N.Trinajstic, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chem. Phys. Let.* 17 (1972) 535-538.
3. H.S. Ramane and A.S. Yalnaik, Status connectivity indices graphs and its applications to the boiling point of benzenoid hydrocarbons, *Journal of Applied Mathematics and Computing*, 55 (2017) 607-627.
4. H.S. Ramane, B. Basavagoud and A.S. Yalnaik, Harmonic status index of graphs, *Bulletin of Mathematical Sciences and Applications*, 17 (2016) 24-32.
5. H.S.Ramane, A.S. Yalnaik and R. Sharafdini, Status connectivity indices and coindices of graphs and its computation to some distance balanced graphs, *AKCE International Journal of Graphs and Combinatorics*, (2018) https://doi.org/10.1016j.akcej.2018.09.002.
6. V.R. Kulli, Some new multiplicative status indices of graphs, *International Journal of Recent Scientific Research*, 10(10) (2019) 35568-35573.
7. V.R. Kulli, Computation of status indices of graphs, *International Journal of Mathematics Trends and Technology*, 65(12) (2019) 54-61.
8. V.R.Kulli, Some new status indices of graphs, *International Journal of Mathematics Trends and Technology*, 65(10) (2019) 70-76.
9. V.R.Kulli, Status Gourava indices of graphs, *International Journal of Recent Scientific Research*, 11, 01(B) (2020) 36770-36773.
10. V.R.Kulli, Computation of multiplicative status indices of graphs, *International Journal of Mathematical Archive*, 11(4) (2020) 1-6.
11. V.R.Kulli, Computation of multiplicative (*a*, *b*)-status index of certain graphs, *Journal of Mathematics and Informatics*, 18 (2020) 45-50.
12. V.R.Kulli, Computation of *ABC*, *AG* and augmented status indices of graphs, *International Journal of Mathematics Trends and Technology*, 66(1) (2020) 1-7.
13. V.R.Kulli, Multiplicative *ABC, GA, AG,* augmented and harmonic status indices of graphs, *International Journal of Mathematical Archive*, 11(1) (2020) 32-40.
14. V.R.Kulli, Status Nirmala index and its exponential of a graph, submitted.
15. I.Gutman, Geometric approach to degree based topological indices: Sombor indices, *MATCH Common. Math, Comput. Chem.* 86 (2021) 11-16.
16. V.R.Kulli, On second Banhatti-Sombor indices, *International Journal of Mathematical Archive*, 12(5) (2021) 11-16.
17. V.R.Kulli, Neighborhood Sombor index of some nanostructures, *International Journal of Mathematics Trends and Technology*, 67(5) (2021) 101-108.
18. V.R.Kulli, Sombor indices of two families of dendrimer nanostars, *Annals of Pure and Applied Mathematics*, 24(1) (2021) 21-26.
19. V.R.Kulli, Different versions of Sombor index of some chemical structures, *International Journal of Engineering Sciences and Research Technology*, 10(7) (2021) 23-32.
20. V.R.Kulli, New irregularity Sombor indices and new Adriatic (*a*, *b*)-*KA* indices of certain chemical drugs, *International Journal of Mathematics Trends and Technology*, 67(9) (2021) 105-113.
21. I.Gutman, V.R.Kulli and I.Redzepovic, Sombor index of Kragujevac trees, *Ser. A: Appl. Math. Inform. And Mech.* 13(2) (2021) 61-70.

22. V.R.Kulli and I.Gutman, Revan Sombor index, *Journal of Mathematics and Informatics,* 22 (2022) 23-27.
23. V.R.Kulli, Sombor indices of certain graph operators, *International Journal of Engineering Sciences and Research Technology,* 10(1) (2021) 127-134.
24. B. Furtula and I. Gutman, A forgotten topological index, *J. Math. Chem.* 53 (2015) 1184-1190.
25. V.R. Kulli, The (a, b)-status index of graphs, *Annals of Pure and Applied Mathematics*, 21(2) (2020) 113-118.
26. I. Milovanović, E. Milovanović, M. Matejić, On some mathematical properties ofSombor indices, Bulletin of International Mathematical Virtual Institute11 (2021) 341-353.
27. I. Gutman, Some basic properties of Sombor indices, Open Journal of Discrete Applied Mathematics 4(1) (2021) 1-3.