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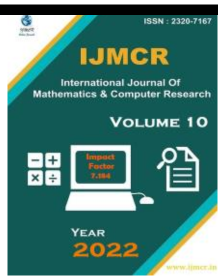

Some Fundamental Properties of Hunaiber Transform and its Applications to Partial Differential Equations

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ABSTRACT
In this paper, we study some basic properties of a new integral transform " Hunaiber transform". Moreover, we apply Hunaiber transform to solve linear partial differential equations with initial and boundary conditions. We solve first order partial differential equations and Second order partial differential equations which are essential equations in mathematical physics and applied mathematics.

KEYWORDS: Hunaiber Transform, Integral Transform, Differential equations, partial differential equations, Linear Partial Differential Equations**.**

I. INTRODUCTION

The source of the integral transforms can be traced back to the work of P. S. Laplace in 1780s and Joseph Fourier in 1822. In recent years, integral and differential equations have been solved using many integral trans-forms**.** The integral transformation method is extremely used to solve different kinds of differential equations in a simple way. As integral transforms converts differential equations into algebraic equations which are more simple than differential equations. Many problems of physical interest are described by integral and differential equa-tions with appropriate or boundary conditions. These problems are usually formulated as initial value problem, boundary value problems, or initial-boundary value pro-blem that appear to be mathematically more vigorous and physically realistic in engineering sciences and applied. There are numerous integral transforms and all of them are suitable to resolve various types' differential equa-tions. Many researchers have turned their attention to solve partial differential equation and to develop new methods for solving such equations. The behavior of the solutions very much depend fundamentally on the classification of partial differential equations, then the problem of classification for partial differential equations well known since the classification controls the adequate number and the type of the conditions in order to determine whether the problem is well posed and has a unique solution. Many researchers developed lot of integral transforms like Laplace, Kamal, Fourier, Elzaki, Sawi, Aboodh, Mahgoub, Shehu, etc. transforms.

Mona Hunaiber introduce new integral transform called as Hunaiber transform.

The Hunaiber transform is a new integral transform similar to the Laplace transform and other integral tran-sforms.

1. 1 Definition.

Let $f(x)$ be piecewise continuous on the interval $0 \leq x \leq \lambda$ for any $\lambda > 0$, and $|f(x)| \leq Me^{ax}$ where $x \geq N$, for any real constant a and some positive constants M and$N$. The Hunaiber transform is defined by

$$\mathrm{H}[f(x)] = \mathrm{F}(\mu^{\alpha}, \beta) = \mu^{\beta} \int_{0}^{\infty} e^{-x\mu^{\alpha}} f(x)\, dx\,, \qquad (1)$$

Where μ complex is variable, β is real number and α is any nonzero real number. Here H is called the Hunaiber transform operator.

It turns out that the Hunaiber transform has very special and useful properties to simplify the process of solving differential equations.

II. SOME PROPERTIES OF HUNAIBER TRANSFORM

(1). Linearity Property If Hunaiber transform of functions $f_1(x)$ and $f_2(x)$ are $F_1(\mu^{\alpha}, \beta)$ and $F_2(\mu^{\alpha}, \beta)$ respectively, then Hunaiber transform of $a\, f_1(x) + b\, f_2(x)$ is given by

$$a\, F_1(\mu^\alpha,\beta) \quad + \quad b\, F_2(\mu^\alpha,\beta), \qquad (2)$$

where a and b are arbitrary constants.

Proof. By definition of Hunaiber transform, we have

$\mathrm{H}[a\, f_1(x) + b\, f_2(x)]$

$$= \mu^\beta \int_0^\infty e^{-x\mu^\alpha} \,(\, a\, f_1(x) + b\, f_2(x)\,)\, dx$$

$$= a\, \mu^\beta \int_0^\infty e^{-x\mu^\alpha}\, f_1(x)\, dx + b\, \mu^\beta \int_0^\infty e^{-x\mu^\alpha}\, f_2(x)\, dx$$

$$= a\, F_1(\mu^\alpha,\beta) \; + \; b\, F_2(\mu^\alpha,\beta).$$

(2). Shifting property If Hunaiber transform of function $f(x)$ is $\mathrm{F}(\mu^\alpha,\beta)$, then ***for*** any real constant $a > 0$, we have

$$\mathrm{H}[e^{ax} f(x)] \; = \mathrm{F}(\mu^\alpha - a, \beta). \qquad (3)$$

Proof. Using the definition of Hunaiber transform, we have

$$\mathrm{H}[e^{ax} f(x)] = \mu^\beta \int_0^\infty e^{-x\mu^\alpha}\, e^{ax} f(x)\, dx$$

$$= \mu^\beta \int_0^\infty e^{-(\mu^\alpha - a)x}\, f(x) dx$$

$$= \mathrm{F}(\mu^\alpha - a, \beta).$$

(3). Change of scale Property

If Hunaiber transform of function $f(x)$ is $\mathrm{F}(\mu^\alpha,\beta)$, then Hunaiber transform of function $f(ax)$ is given by $\frac{1}{a}\, F\left(\frac{\mu^\alpha}{a}, \beta\right)$, where a is positive constant.

Proof. Using the definition of Hunaiber transform, we have

$$\mathrm{H}[f(ax)] = \mu^\beta \int_0^\infty e^{-x\mu^\alpha}\, f(ax) dx.$$

Set $r = ax$, then

$$\mathrm{H}[f(ax)] = \frac{\mu^\beta}{a} \int_0^\infty e^{-\left(\frac{\mu^\alpha}{a}\right) r}\, f(r) dr$$

$$= \frac{1}{a}\, F\left(\frac{\mu^\alpha}{a}\;, \beta\right). \qquad (4)$$

(4). Hunaiber Transform of the function $\mathbf{xf(x)}$

Let $\mathrm{F}(\mu^\alpha,\beta)$ be the Hunaiber transform of function $f(x)$, then

$$\mathrm{H}[xf(x)] \; = \frac{-1}{\alpha\, \mu^{\alpha-1}} \left[\frac{d}{d\mu} \mathrm{F}(\mu^\alpha,\beta) - \frac{\beta}{\mu}\, \mathrm{F}(\mu^\alpha,\beta)\right]. \qquad (5)$$

Proof. Since $\mathrm{H}[f(x)] = \mathrm{F}(\mu^\alpha,\beta)$, then

$$\frac{d}{d\mu} F(\mu^\alpha,\beta) = \frac{d}{d\mu} \left[\mu^\beta \int_0^\infty e^{-x\mu^\alpha} f(x)\, dx \right]$$

$$\frac{d}{d\mu} \mathrm{F}(\mu^\alpha,\beta) = -\alpha\mu^{\alpha-1}\mu^\beta \int_0^\infty e^{-x\mu^\alpha} x\, f(x) d$$

$$+ \; \beta\mu^{\beta-1} \int_0^\infty e^{-x\mu^\alpha} f(x) dx$$

$$\frac{d}{d\mu} \mathrm{F}(\mu^\alpha,\beta) = -\alpha\mu^{\alpha-1}\, \mathrm{H}[xf(x)] + \frac{\beta}{\mu} \mathrm{F}(\mu^\alpha,\beta)$$

Hence, we obtain

$$\mathrm{H}[xf(x)] \; = \frac{-1}{\alpha\, \mu^{\alpha-1}} \left[\frac{d}{d\mu} \mathrm{F}(\mu^\alpha,\beta) - \frac{\beta}{\mu}\; \mathrm{F}(\mu^\alpha,\beta)\right].$$

2.1 Theorem: If $F(x,\mu^\alpha,\beta)$ is a Hunaiber transform of $f(x,y)$ and $f_x(x,y)$ is a first partial derivative of $f(x,y)$ with respect to variable x, then

$$\mathrm{H}[f_x(x,y)] = \; F_x(x,\mu^\alpha,\beta) \qquad (6)$$

Also, we have

$$\mathrm{H}[f_{xx}(x,y)] = F_{xx}(x,\mu^\alpha,\beta) \qquad (7)$$

$$\mathrm{H}[f_{x^n}(x,y)] = F_{x^n}(x,\mu^\alpha,\beta) \qquad (8)$$

2.2 Theorem: If $F(x,\mu^\alpha,\beta)$ is a Hunaiber transform of $f(x,y)$ and $f_y(x,y)$ is a first partial derivative of $f(x,y)$ with respect to variable y, then

$$H\left[f_y(x,y)\right] = \mu^\alpha F(x,\mu^\alpha,\beta) - \mu^\beta\, f(x,0). \qquad (9)$$

Proof. Using the definition(1), we have

$$H[f_y(x,y)] = \mu^\beta \int_0^\infty e^{-x\mu^\alpha}\, f_y(x,y)\, dy$$

By using integration by part, we obtain

$$H\left[f_y(x,y)\right] = \; \mu^\alpha F(x,\mu^\alpha,\beta) - \mu^\beta\, f(x,0).$$

2.3 Theorem: If $F(x,\mu^\alpha,\beta)$ is a Hunaiber transform of $f(x,y)$ and $f_{yy}(x,y)$ is a second partial derivative of $f(x,y)$ with respect to variable y , then

$$H\left[f_{yy}(x,y)\right] = \mu^{2\alpha} F(x,\mu^\alpha,\beta) - \mu^{\alpha+\beta}\, f(x,0) - \mu^\beta\, f_y(x,0) \qquad (10)$$

Proof. Using the definition(1), we have

$$H[f_{yy}(x,y)] = \mu^\beta \int_0^\infty e^{-x\mu^\alpha}\, f_{yy}(x,y)\, dy$$

By using integration by part, we obtain

$$H\left[f_{yy}(x,y)\right] = -\mu^\beta\, f_y(x,0) + \mu^\alpha\mu^\beta \left\{ -f(x,0) + \mu^\alpha \int_0^\infty e^{-x\mu^\alpha}\, f(x,y) dy \right\}$$

$$H\left[f_{yy}(x,y)\right] = \mu^{2\alpha} F(x,\mu^\alpha,\beta) - \mu^{\alpha+\beta}\, f(x,0) - \mu^\beta\, f_y(x,0)\; . \qquad (11)$$

III. APPLICATIONS

In this section, we assume that the inverse Hunaiber transform exists. We apply the inverse Hunaiber tran-sform to find the solution of linear partial differential equations with initial and boundary conditions. We solve first order partial differential equations and the Second order partial differential equations, Laplace, wave, heat equations which are known as three Fundamental equations in mathematical physics and arise in many branches of physics in applied mathematics.

Example3.1. *Consider the first order partial differential equation*

$$f_x(x,y) - 2f_y(x,y) = f(x,y)\,, \qquad (12)$$

With initial condition

$$f(x,0) = e^{-3x} \qquad (13)$$

Take Hunaiber transform to this equation, we gives

$$F_x(x,\mu^\alpha,\beta) - 2\left[\,\mu^\alpha F(x,\mu^\alpha,\beta) - \mu^\beta f(x,0)\right] = F(x,\mu^\alpha,\beta)\,,$$

Where $F(x,\mu^\alpha,\beta)$ is Hunaiber transform of $f(x,y)$. by applied initial condition, we get

$$F_x(x,\mu^\alpha,\beta) - (2\mu^\alpha + 1)\mathrm{F}(x,\mu^\alpha,\beta) = -2\,\mu^\beta\, e^{-3x}$$

This is linear ordinary differential equation, it has the integration factor $e^{-(2\mu^\alpha+1)x}$. Hence

$$F(x,\mu^\alpha,\beta) = e^{-3x}\,\frac{\mu^\beta}{\mu^\alpha + 2}$$

Now, applying the inverse Hunaiber transform of last equation, then the solution of Eq. (12) is

$$f(x,y) = e^{-(3x+2y)}\,.$$

Example 3.2 *Consider the Laplace equation*

$$f_{xx}(x,y) + f_{yy}(x,y) = 0\,, \qquad (14)$$

With conditions

$$f(x,0) = 0\,, \qquad f_y(x,0) = cosx\,. \qquad (15)$$

Take Hunaiber transform to this equation gives

$$F_{xx}(x,\mu^\alpha,\beta) + \mu^{2\alpha}F(x,\mu^\alpha,\beta) - \mu^{\alpha+\beta}\, f(x,0) - \mu^\beta\, f_y(x,0) = 0$$

By applied the conditions (15), we get

$$F_{xx}(x,\mu^\alpha,\beta) + \mu^{2\alpha}F(x,\mu^\alpha,\beta) = \mu^\beta\, cosx$$

It is second order ordinary differential equation, thus its particular solution is

$$F(x,\mu^\alpha,\beta) = \frac{\mu^\beta}{\mu^{2\alpha} - 1}\, cosx$$

Taking inverse Hunaiber transform, we get

$$f(x,y) = cosx\; sinhy\,.$$

Example 3.3 *Consider the wave equation*

$$f_{tt}(x,t) - 4\,f_{xx}(x,t) = 0\,, \qquad x,t > 0. \quad (16)$$

With conditions

$$f(x,0) = \sin\pi x\;, \qquad f_t(x,0) = 0\,. \qquad (17)$$

Take Hunaiber transform to Eq. (16), and use the conditions (17), we get

$$F_{xx}(x,\mu^\alpha,\beta) - \frac{\mu^{2\alpha}}{4}\,F(x,\mu^\alpha,\beta) = -\frac{\mu^{\alpha+\beta}}{4}\, sin\pi x$$

This is the second order differential equation have the particular solution in the form

$$F(x,\mu^\alpha,\beta) = \frac{\mu^{\alpha+\beta}\, sin\pi x}{\mu^{2\alpha} + 4\pi^2}$$

Taking inverse Hunaiber transform, we get

$$f(x,t) = sin\pi x\; cos2\pi t\,.$$

Example 3.4 *Consider the heat equation*

$$f_t(x,t) = f_{xx}(x,t), \qquad (18)$$

With conditions

$$f(0,t) = 0 \quad , \qquad f(x,0) = 3\; sin2\pi t\,. \qquad (19)$$

Applying the Hunaiber transform on both sides of Eq. (18), And substituting the given conditions, we get

$$\mathrm{F}_{xx}(x,\mu^\alpha,\beta) - \mu^\alpha\,\mathrm{F}(x,\mu^\alpha,\beta) = -3\,\mu^\beta\, sin2\pi x$$

It is second order ordinary differential equation. Using the method of undetermined coefficients to solve it. Thus its particular solution is

$$\mathrm{F}(x,\mu^\alpha,\beta) = \frac{3\mu^\beta\, sin2\pi x}{\mu^\alpha + 4\pi^2}\,,$$

Taking inverse Hunaiber transform, we get

$$f(x,t) = 3\; sin2\pi x\; e^{-4\pi^2 t}.$$

IV. CONCLUSION

There are a lot of the integral transforms of exponential type kernels, the Hunaiber transform is new and very powerful among them and there are many problems in applied sciences and engineering can be solved by Hunaiber transform. Fundamental Properties of Hunaiber transform is proved. An application Hunaiber transform to the solution of partial differential equations has been demonstrated.

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