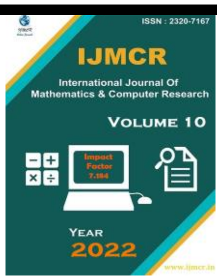
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 10 Issue 07 July 2022, Page no. – 2817-2821

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v10i7.07



HDR Somber Indices and Their Exponentials of a Graph

V. R. Kulli

Department of Mathematics, Gulbarga University, Gulbarga 585106, India

ARTICLE INFO	ABSTRACT
Published Online: 27 July 2022 Corresponding Author: **V.R.Kulli**	In this paper, we introduce the modified HDR-Sombor index, the HDR Sombor exponential and the modified HDR Sombor exponential of a graph. We compute the modified HDR Sombor index, the HDR Sombor exponential, the modified HDR Sombor exponential of chloroquine, hydroxychloroquine and remdesivir. Also we establish some properties of the HDR Sombor index.

KEYWORDS: modified HDR Somber index, modified HDR Sombor exponential, chemical structure, graph.

I. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

The HDR vertex degree of a vertex u in G is $d_{hr}(u) = |\{u,v \in V(G)/\ d(u,v) = \lceil R/2 \rceil$, where $d(u, v)$ is the distance between the vertices u and v in $V(G)$ and R is the radius of G [2].

The HDR Zagreb index was introduced by Alsinai et al. in [2], and it is defined as

$$HDRM_1(G) = \sum_{uv \in E(G)} \left[d_{hr}(u) + d_{hr}(v) \right].$$

Now we call this index as the first HDR Zagreb index.

In [3], Kulli introduced the second HDR Zagreb index of a graph G and it is defined as

$$HDRM_2(G) = \sum_{uv \in E(G)} d_{hr}(u) d_{hr}(v).$$

Recently, some HDR indices were studied, for example, in [4, 5, 6].

The Sombor index was introduced by Gutman in [7] and defined it as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

Recently, some Sombor indices were studied, for example, in [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

The HDR Somber index was introduced in [22] and defined it as,

$$HDRSO(G) = \sum_{uv \in E(G)} \sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}.$$

We propose the HDR Sombor exponential of a graph and it is defined as

$$HDRSO(G,x) = \sum_{uv \in E(G)} x^{\sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}}.$$

We now define the modified HDR Sombor index of a graph G as

$$^{m}HDRSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}}.$$

We also propose the modified HDR Sombor exponential of a graph and it is defined as

$$^{m}HDRSO(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}}}.$$

We put forward the HDR inverse sum indeg index of a graph G and defined it as

$$ISI_H(G) = \sum_{uv \in E(G)} \frac{d_{hr}(u) d_{hr}(v)}{d_{hr}(u) + d_{hr}(v)}.$$

In this paper, we determine the modified HDR Sombor index, the HDR Sombor exponential, the modified HDR Sombor exponential of chloroquine, hydroxychloroquine and remdesivir. We establish some properties of the HDR Sombor index.

II. RESULTS FOR CHLOROQUINE

Let G be the molecular structure of chloroquine. Clearly G has 21 vertices and 23 edges, see Figure 1.

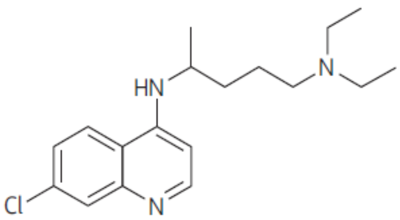


Figure 1. Graph of chloroquine

In G, the edge set $E(G)$ can be divided into five partitions using HDR vertex degree of end vertices of each edge as given in Table 1.

Table 1. Edge partition of G

$d_{hr}(u), d_{hr}(v) \setminus uv \in E(G)$	Number of edges
(2, 3)	2
(3, 4)	7
(3, 5)	5
(3, 3)	2
(4, 4)	2
(1, 2)	3
(1, 1)	2

In the following theorem, we determine the modified HDR Sombor index of chloroquine.

Theorem 1. Let G be the molecular graph of chloroquine. Then $^{m}HDRSO(G) = \frac{2}{\sqrt{13}} + \frac{7}{5} + \frac{5}{\sqrt{34}} + \frac{19}{6\sqrt{2}} + \frac{3}{\sqrt{5}}$.

Proof: From definition and by using Table 1, we obtain

$$^{m}HDRSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}}$$

$$= \frac{2}{\sqrt{2^2+3^2}} + \frac{7}{\sqrt{3^2+4^2}} + \frac{5}{\sqrt{3^2+5^2}} + \frac{2}{\sqrt{3^2+3^2}} + \frac{2}{\sqrt{4^2+4^2}} + \frac{3}{\sqrt{1^2+2^2}} + \frac{2}{\sqrt{1^2+1^2}}$$

$$= \frac{2}{\sqrt{13}} + \frac{7}{5} + \frac{5}{\sqrt{34}} + \frac{19}{6\sqrt{2}} + \frac{3}{\sqrt{5}}.$$

We compute the HDR Sombor and modified HDR Sombor exponentials of chloroquine.

Theorem 2. Let G be the molecular graph of chloroquine. Then

(i) $HDRSO(G,x) = 2x^{\sqrt{13}} + 7x^{5} + 5x^{\sqrt{34}} + 2x^{3\sqrt{2}} + 2x^{4\sqrt{2}} + 3x^{\sqrt{5}} + 2x^{\sqrt{2}}$.

(ii) $^{m}HDRSO(G,x) = 2x^{\frac{1}{\sqrt{13}}} + 7x^{\frac{1}{5}} + 5x^{\frac{1}{\sqrt{34}}} + 2x^{\frac{1}{3\sqrt{2}}} + 2x^{\frac{1}{4\sqrt{2}}} + 3x^{\frac{1}{\sqrt{5}}} + 2x^{\frac{1}{\sqrt{2}}}$.

Proof: (i) Using definition and using Table 1, we have

$$HDRSO(G,x) = \sum_{uv \in E(G)} x^{\sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}}$$

$$= 2x^{\sqrt{2^2+3^2}} + 7x^{\sqrt{3^2+4^2}} + 5x^{\sqrt{3^2+5^2}} + 2x^{\sqrt{3^2+3^2}} + 2x^{\sqrt{4^2+4^2}} + 3x^{\sqrt{1^2+2^2}} + 2x^{\sqrt{1^2+1^2}}$$

$$= 2x^{\sqrt{13}} + 7x^{5} + 5x^{\sqrt{34}} + 2x^{3\sqrt{2}} + 2x^{4\sqrt{2}} + 3x^{\sqrt{5}} + 2x^{\sqrt{2}}.$$

(ii) Using definition and using Table 1, we obtain

$$^{m}HDRSO(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}}}$$

$$= 2x^{\frac{1}{\sqrt{2^2+3^2}}} + 7x^{\frac{1}{\sqrt{3^2+4^2}}} + 5x^{\frac{1}{\sqrt{3^2+5^2}}} + 2x^{\frac{1}{\sqrt{3^2+3^2}}} + 2x^{\frac{1}{\sqrt{4^2+4^2}}} + 3x^{\frac{1}{\sqrt{1^2+2^2}}} + 2x^{\frac{1}{\sqrt{1^2+1^2}}}$$

$$= 2x^{\frac{1}{\sqrt{13}}} + 7x^{\frac{1}{5}} + 5x^{\frac{1}{\sqrt{34}}} + 2x^{\frac{1}{3\sqrt{2}}} + 2x^{\frac{1}{4\sqrt{2}}} + 3x^{\frac{1}{\sqrt{5}}} + 2x^{\frac{1}{\sqrt{2}}}.$$

III. RESULTS FOR HYDROXYCHLOROQUINE

Let H be the graph of hydroxychloroquine and it has 22 vertices and 24 edges, see Figure 2.

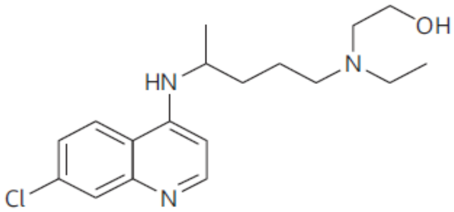


Figure 2. Graph of hydroxychloroquine

The edge set of H can be divided into nine partitions using HDR vertex degree of end vertices of each edge as given in Table 2.

Table 2. Edge partition of H

$d_{hr}(u), d_{hr}(v) \setminus uv \in E(H)$	Number of edges
(2, 3)	1
(3, 3)	3
(3, 4)	7
(3, 5)	5
(4, 4)	2
(1, 3)	1
(1, 2)	2
(2, 2)	2
(1, 1)	1

In Theorem 3, we determine the modified HDR Sombor index of hydroxychloroquine.

Theorem 3. Let H be the molecular graph of hydroxychloroquine. Then

$$^{m}HDRSO(H) = \frac{1}{\sqrt{13}} + \frac{7}{5} + \frac{5}{\sqrt{34}} + \frac{1}{\sqrt{10}} + \frac{2}{\sqrt{5}} + \frac{7}{2\sqrt{2}}.$$

Proof: From definition and by using Table 2, we obtain

$$^{m}HDRSO(H) = \sum_{uv \in E(H)} \frac{1}{\sqrt{d_{hr}(u)^2 + d_{hr}(v)^2}}$$

$$= \frac{1}{\sqrt{2^2+3^2}} + \frac{3}{\sqrt{3^2+3^2}} + \frac{7}{\sqrt{3^2+4^2}} + \frac{5}{\sqrt{3^2+5^2}}$$
$$+ \frac{2}{\sqrt{4^2+4^2}} + \frac{1}{\sqrt{1^2+3^2}} + \frac{2}{\sqrt{1^2+2^2}}$$
$$+ \frac{2}{\sqrt{2^2+2^2}} + \frac{1}{\sqrt{1^2+1^2}}$$
$$= \frac{1}{\sqrt{13}} + \frac{7}{5} + \frac{5}{\sqrt{34}} + \frac{1}{\sqrt{10}} + \frac{2}{\sqrt{5}} + \frac{7}{2\sqrt{2}}.$$

We compute the HDR Sombor and modified HDR Sombor exponentials of hydroxychloroquine.

Theorem 4. Let H be the molecular graph of hydroxychloroquine. Then

(i) $$HDRSO(H,x) = 1x^{\sqrt{13}} + 3x^{3\sqrt{2}} + 7x^{5} + 5x^{\sqrt{34}} + 2x^{4\sqrt{2}} + 1x^{\sqrt{10}} + 2x^{\sqrt{5}} + 2x^{2\sqrt{2}} + 1x^{\sqrt{2}}.$$

(ii) $${}^{m}HDRSO(H,x) = 1x^{\frac{1}{\sqrt{13}}} + 3x^{\frac{1}{3\sqrt{2}}} + 7x^{\frac{1}{5}} + 5x^{\frac{1}{\sqrt{34}}} + 2x^{\frac{1}{4\sqrt{2}}} + 1x^{\frac{1}{\sqrt{10}}} + 2x^{\frac{1}{\sqrt{5}}} + 2x^{\frac{1}{2\sqrt{2}}} + 1x^{\frac{1}{\sqrt{2}}}.$$

Proof: (i) From definition and using Table 2, we get

$$HDRSO(H,x) = \sum_{uv\in E(H)} x^{\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}}$$
$$= 1x^{\sqrt{2^2+3^2}} + 3x^{\sqrt{3^2+3^2}} + 7x^{\sqrt{3^2+4^2}} + 5x^{\sqrt{3^2+5^2}} + 2x^{\sqrt{4^2+4^2}}$$
$$+ 1x^{\sqrt{1^2+3^2}} + 2x^{\sqrt{1^2+2^2}} + 2x^{\sqrt{2^2+2^2}} + 1x^{\sqrt{1^2+1^2}}$$
$$= 1x^{\sqrt{13}} + 3x^{3\sqrt{2}} + 7x^{5} + 5x^{\sqrt{34}} + 2x^{4\sqrt{2}}$$
$$+ 1x^{\sqrt{10}} + 2x^{\sqrt{5}} + 2x^{2\sqrt{2}} + 1x^{\sqrt{2}}.$$

(ii) Using definition and Table 2, we obtain

$${}^{m}HDRSO(H,x) = \sum_{uv\in E(H)} x^{\frac{1}{\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}}}$$
$$= 1x^{\frac{1}{\sqrt{2^2+3^2}}} + 3x^{\frac{1}{\sqrt{3^2+3^2}}} + 7x^{\frac{1}{\sqrt{3^2+4^2}}}$$
$$+ 5x^{\frac{1}{\sqrt{3^2+5^2}}} + 2x^{\frac{1}{\sqrt{4^2+4^2}}} + 1x^{\frac{1}{\sqrt{1^2+3^2}}}$$
$$+ 2x^{\frac{1}{\sqrt{1^2+2^2}}} + 2x^{\frac{1}{\sqrt{2^2+2^2}}} + 1x^{\frac{1}{\sqrt{1^2+1^2}}}$$
$$= 1x^{\frac{1}{\sqrt{13}}} + 3x^{\frac{1}{3\sqrt{2}}} + 7x^{\frac{1}{5}} + 5x^{\frac{1}{\sqrt{34}}} + 2x^{\frac{1}{4\sqrt{2}}}$$
$$+ 1x^{\frac{1}{\sqrt{10}}} + 2x^{\frac{1}{\sqrt{5}}} + 2x^{\frac{1}{2\sqrt{2}}} + 1x^{\frac{1}{\sqrt{2}}}.$$

IV. RESULTS FOR REMDESIVIR

Let R be the molecular structure of remdesivir. Clearly R has 41 vertices and 44 edges, see Figure 3.

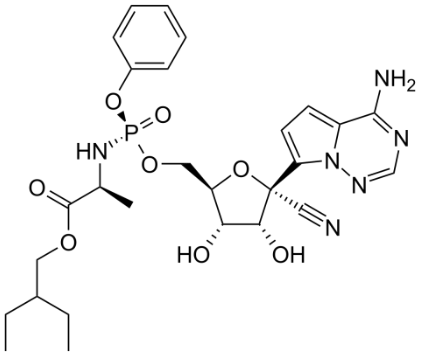


Figure 3. Graph of remdesivir

In R, the edge set of R can be divided into 12 partitions using HDR vertex degree of end vertices of each edge as given in Table 3.

Table 3. Edge partition of R

$d_{hr}(u), d_{hr}(v) \setminus uv \in E(R)$	Number of edges
(1, 2)	7
(1, 3)	5
(2, 2)	2
(2, 3)	8
(3, 3)	4
(3, 5)	3
(3, 4)	4
(4, 5)	4
(5, 5)	3
(5, 6)	2
(5, 7)	1
(6, 7)	1

In Theorem 5, we determine the modified HDR Sombor index of remdesivir.

Theorem 5. Let R be the molecular graph of remdesivir. Then

$${}^{m}HDRSO(R) = \frac{7}{\sqrt{5}} + \frac{5}{\sqrt{10}} + \frac{44}{15\sqrt{2}} + \frac{8}{\sqrt{13}} + \frac{3}{\sqrt{34}} + \frac{4}{5} + \frac{4}{\sqrt{41}} + \frac{2}{\sqrt{61}} + \frac{1}{\sqrt{74}} + \frac{1}{\sqrt{85}}.$$

Proof: From definition and by using Table 3, we obtain

$${}^{m}HDRSO(R) = \sum_{uv\in E(R)} \frac{1}{\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}}$$
$$= \frac{7}{\sqrt{1^2+2^2}} + \frac{5}{\sqrt{1^2+3^2}} + \frac{2}{\sqrt{2^2+2^2}} + \frac{8}{\sqrt{2^2+3^2}}$$
$$+ \frac{4}{\sqrt{3^2+3^2}} + \frac{3}{\sqrt{3^2+5^2}} + \frac{4}{\sqrt{3^2+4^2}} + \frac{4}{\sqrt{4^2+5^2}}$$
$$+ \frac{3}{\sqrt{5^2+5^2}} + \frac{2}{\sqrt{5^2+6^2}} + \frac{1}{\sqrt{5^2+7^2}} + \frac{1}{\sqrt{6^2+7^2}}$$
$$= \frac{7}{\sqrt{5}} + \frac{5}{\sqrt{10}} + \frac{44}{15\sqrt{2}} + \frac{8}{\sqrt{13}} + \frac{3}{\sqrt{34}}$$

$$+\frac{4}{5}+\frac{4}{\sqrt{41}}+\frac{2}{\sqrt{61}}+\frac{1}{\sqrt{74}}+\frac{1}{\sqrt{85}}.$$

We compute the HDR Sombor and modified HDR Sombor exponentials of remdesivir.

Theorem 6. Let R be the molecular graph of remdesivir. Then

(i) $HDRSO(R,x)$

$$=7x^{\sqrt{5}}+5x^{\sqrt{10}}+2x^{2\sqrt{2}}+8x^{\sqrt{13}}+4x^{3\sqrt{2}}+3x^{\sqrt{34}}$$
$$+4x^{5}+4x^{\sqrt{41}}+3x^{5\sqrt{2}}+2x^{\sqrt{61}}+1x^{\sqrt{74}}+1x^{\sqrt{85}}.$$

(ii) $${}^{m}HDRSO(R,x)=7x^{\frac{1}{\sqrt{5}}}+5x^{\frac{1}{\sqrt{10}}}+2x^{\frac{1}{2\sqrt{2}}}+8x^{\frac{1}{\sqrt{13}}}$$
$$+4x^{\frac{1}{3\sqrt{2}}}+3x^{\frac{1}{\sqrt{34}}}+4x^{\frac{1}{5}}+4x^{\frac{1}{\sqrt{41}}}$$
$$+3x^{\frac{1}{5\sqrt{2}}}+2x^{\frac{1}{\sqrt{61}}}+1x^{\frac{1}{\sqrt{74}}}+1x^{\frac{1}{\sqrt{85}}}.$$

Proof: (i) From definition and using Table 3, we get

$$HDRSO(R,x)=\sum_{uv\in E(R)}x^{\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}}$$
$$=7x^{\sqrt{1^2+2^2}}+5x^{\sqrt{1^2+3^2}}+2x^{\sqrt{2^2+2^2}}+8x^{\sqrt{2^2+3^2}}$$
$$+4x^{\sqrt{3^2+3^2}}+3x^{\sqrt{3^2+5^2}}+4x^{\sqrt{3^2+4^2}}+4x^{\sqrt{4^2+5^2}}$$
$$+3x^{\sqrt{5^2+5^2}}+2x^{\sqrt{5^2+6^2}}+1x^{\sqrt{5^2+6^2}}+1x^{\sqrt{6^2+7^2}}$$
$$=7x^{\sqrt{5}}+5x^{\sqrt{10}}+2x^{2\sqrt{2}}+8x^{\sqrt{13}}+4x^{3\sqrt{2}}+3x^{\sqrt{34}}$$
$$+4x^{5}+4x^{\sqrt{41}}+3x^{5\sqrt{2}}+2x^{\sqrt{61}}+1x^{\sqrt{74}}+1x^{\sqrt{85}}.$$

(ii) Using definition and Table 3, we obtain

$${}^{m}HDRSO(R,x)=\sum_{uv\in E(R)}x^{\frac{1}{\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}}}$$
$$=7x^{\frac{1}{\sqrt{1^2+2^2}}}+5x^{\frac{1}{\sqrt{1^2+3^2}}}+2x^{\frac{1}{\sqrt{2^2+2^2}}}+8x^{\frac{1}{\sqrt{2^2+3^2}}}$$
$$+4x^{\frac{1}{\sqrt{3^2+3^2}}}+3x^{\frac{1}{\sqrt{3^2+5^2}}}+4x^{\frac{1}{\sqrt{3^2+4^2}}}+4x^{\frac{1}{\sqrt{4^2+5^2}}}$$
$$+3x^{\frac{1}{\sqrt{5^2+5^2}}}+2x^{\frac{1}{\sqrt{5^2+6^2}}}+1x^{\frac{1}{\sqrt{5^2+7^2}}}+1x^{\frac{1}{\sqrt{6^2+7^2}}}$$
$$=7x^{\frac{1}{\sqrt{5}}}+5x^{\frac{1}{\sqrt{10}}}+2x^{\frac{1}{2\sqrt{2}}}+8x^{\frac{1}{\sqrt{13}}}+4x^{\frac{1}{3\sqrt{2}}}+3x^{\frac{1}{\sqrt{34}}}$$
$$+4x^{\frac{1}{5}}+4x^{\frac{1}{\sqrt{41}}}+3x^{\frac{1}{5\sqrt{2}}}+2x^{\frac{1}{\sqrt{61}}}+1x^{\frac{1}{\sqrt{74}}}+1x^{\frac{1}{\sqrt{85}}}.$$

V. MATHEMATICAL PROPERTIES OF THE HDR SOMBOR INDEX

Theorem 7. Let G be a nontrivial connected graph. Then

$$HDRSO(G)<HDRM_1(G).$$

Proof: For any positive numbers a and b,

$$\sqrt{a+b}<a+b.$$

Applying the above inequality, we obtain

$$\sum_{uv\in E(G)}\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}<\sum_{uv\in E(G)}\left(d_{hr}(u)+d_{hr}(v)\right).$$

Thus

$$HDRSO(G)<HDRM_1(G).$$

Theorem 8. Let G be a connected graph. Then

$$HDRM_1(G)-2ISI_H(G)<HDRSO(G)$$
$$\leq\sqrt{2}HDRM_1(G)-2\sqrt{2}ISI_H(G).$$

The upper bound holds if and only if G is regular.

Proof: For any two numbers a, b>0, we have

$$a^2+b^2<(a+b)^2\ a+b\leq 2(a^2+b^2).$$

Thus $\sqrt{a^2+b^2}<a+b\leq\sqrt{2}\sqrt{a^2+b^2}$

and

$$\left(d_{hr}(u)+d_{hr}(v)\right)^2$$
$$=\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}$$
$$+2d_{hr}(u)d_{hr}(v).$$

Thus $\left(d_{hr}(u)+d_{hr}(v)\right)^2$

$$<\left(d_{hr}(u)+d_{hr}(v)\right)\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}$$
$$+2d_{hr}(u)d_{hr}(v).$$

Hence $\left(d_{hr}(u)+d_{hr}(v)\right)$

$$<\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}+\frac{2d_{hr}(u)d_{hr}(v)}{d_{hr}(u)+d_{hr}(v)}.$$

By the definitions, we have

$$HDRM_1(G)<HDRSO(G)+2ISI_H(G).$$

Also we have

$$\frac{1}{\sqrt{2}}\left(d_{hr}(u)+d_{hr}(v)\right)\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}$$
$$+2d_{hr}(u)d_{hr}(v)\leq\left(d_{hr}(u)+d_{hr}(v)\right)^2.$$

Thus $$\sqrt{d_{hr}(u)^2+d_{hr}(v)^2}+\frac{2\sqrt{2}d_{hr}(u)d_{hr}(v)}{d_{hr}(u)+d_{hr}(v)}$$
$$\leq\sqrt{2}\left(d_{hr}(u)+d_{hr}(v)\right).$$

By the definitions, we have

$$HDRSO(G)+2\sqrt{2}ISI_H(G)\leq\sqrt{2}HDRM_1(G).$$

The equality in the above inequality holds if and only if G is regular.

VI. CONCLUSION

In this paper, we have introduced the modified HDR-Sombor index and the modified HDR Sombor exponential of a graph. We have computed the modified HDR Sombor index, the HDR Sombor exponential, the modified HDR Sombor exponential of chloroquine, hydroxychloroquine and remdesivir. Also we have obtained some properties of the HDR Sombor index.

Acknowledgement: This research is supported by IGTRC No. BNT/IGTRC/ 2022:2212:112 International Graph Theory Research Center, Banhatti 587311, India.

REFERENCES

1. V.R.Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
2. A.Alsinai, H.Ahmed, A.Alwardi and N.D.Soner, HDR degree based indices and Mhr-polynomial for the treatment of COVID-19, *Biointerface Research in Applied Chemistry,* 12(6) (2022) 7214-7225.
3. V.R.Kulli, HDR Zagreb indices of remdesivir, chloroquine, hydroxychloroquine: Research for the treatment of COVID-19, *SSRG International Journal of Applied Chemistry*, 9(1) (2022) 1-9.
4. V.R.Kulli, ABC, GA, AG HDR indices of certain chemical drugs, *International Journal Of Mathematics Trends and Technology,* 68(2) (2022) 80-88.
5. V.R.Kulli, HDR Nirmala index, *International Journal of Mathematics and Computer Research.* 10(70 (2022) 2796-2800.
6. V.R.Kulli, HDR Zagreb indices of remdesivir, chloroquine, hydroxychloroquine: Research for the treatment of COVOD-19, *SSRG International Journal of Applied Chemistry,* 9(1) (2022) 1-9.
7. I.Gutman, Geometric approach to degree based topological indices: Sombor indices, *MATCH Common. Math, Comput. Chem.* 86 (2021) 11-16.
8. V.R.Kulli, On second Banhatti-Sombor indices, *International Journal of Mathematical Archive,* 12(5) (2021) 11-16.
9. V.R.Kulli, Neighborhood Sombor index of some nanostructures, *International Journal of Mathematics Trends and Technology,* 67(5) (2021) 101-108.
10. V.R.Kulli, Sombor indices of two families of dendrimer nanostars, *Annals of Pure and Applied Mathematics*, 24(1) (2021) 21-26.
11. V.R.Kulli, Different versions of Sombor index of some chemical structures, *International Journal of Engineering Sciences and Research Technology,* 10(7) (2021) 23-32.
12. V.R.Kulli, New irregularity Sombor indices and new Adriatic (a, b)-*KA* indices of certain chemical drugs, *International Journal of Mathematics Trends and Technology,* 67(9) (2021) 105-113.
13. I.Gutman, V.R.Kulli and I.Redzepovic, Sombor index of Kragujevac trees, *Ser. A: Appl. Math. Inform. And Mech.* 13(2) (2021) 61-70.
14. V.R.Kulli and I.Gutman, Revan Sombor index, *Journal of Mathematics and Informatics,* 22 (2022) 23-27.
15. V.R.Kulli, Sombor indices of certain graph operators, *International Journal of Engineering Sciences and Research Technology,* 10(1) (2021) 127-134..
16. I. Milovanović, E. Milovanović, M. Matejić, On some mathematical properties of Sombor indices, Bulletin of International Mathematical Virtual Institute11 (2021) 341-353.
17. I. Gutman, Some basic properties of Sombor indices, Open Journal of Discrete Applied Mathematics 4(1) (2021) 1-3.
18. V.R.Kulli, Revan Sombor indices and their exponentials of certain nanotubes, *International Journal of Engineering Sciences and Research Technology,* 11(5) (2021) 22-31.
19. V.R.Kulli, KG Sombor indices of certain chemical drugs, *International Journal of Engineering Sciences and Research Technology,* 11(6) (2022) 27-35.
20. V.R.Kulli, Status Sombor indices, , *International Journal of Mathematicss and Compuer Research,* 10(6) (2022) 2726-2730.
21. V.R.Kulli, Neighborhood Sombor indices, *International Journal of Mathematics Trends and Technology,* 68(6) (2022) 195-202.
22. H.R.Manjunatha, V.R.Kulli and N.D.Soner, The HDR Sombor index, *International Journal of Mathematics Trends and Technology,* 68(4) (2021) 1-6.