



Q - Cubic Bi-Ideals in Near-Rings

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| ARTICLE INFO | ABSTRACT |
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| Published Online: 02 August 2022 | Young Bae Jun made effort in defining a remarkable structure namely cubic structure. Concept of cubic sets which is inter linked with interval-valued fuzzy set and fuzzy set. Interval-valued fuzzy set is another generalization of fuzzy sets that was introduced by Lotfi Asker Zadeh. Motivated by the theory of cubic structure our aim in this paper is to introduce the notion of Q - cubic bi-ideals of near-rings. We also provide some results and study their related properties with examples. |
| Corresponding Author: K. Bharathivelan | |
| KEYWORDS: Near-rings, Q-fuzzy ideals, interval-valued Q-fuzzy ideals, Q-cubic ideals, Homomorphism of Q-cubic ideals. | |

I. INTRODUCTION

The fundamental concept of fuzzy set was introduced by Zadeh [14]. The study of fuzzy algebraic structures has started with the introduction of the concept of fuzzy subgroups in the pioneering paper of Rosenfeld [11]. Abou-Zaid [1] introduced the concept of fuzzy subnear-rings and studied fuzzy ideals in near-rings. The idea of fuzzy ideals of near-rings was first proposed by Kim et al. [4]. After the introduction of the concept of fuzzy sets by Zadeh, several researchers were conducted on the generalization of the notion of fuzzy set. Mason [5] introduced the notion of strong regularity of a near-ring. In 1975 Zadeh [15] introduced the concept of interval-valued fuzzy subsets (in short written by i-v fuzzy sets), where the values of membership functions are intervals of numbers instead of the numbers. Thillaigovindan et al. [13] introduced interval valued fuzzy ideals of near rings. Jun et al. [9] introduced the concept of cubic sets. This structure encompasses interval-valued fuzzy set and fuzzy set. Also Jun et al. [8] introduced the notion of cubic subgroups. Chinnadurai et al. [3] introduced Q-cubic ideals of near-rings. Many researchers who are involved in studying, applying, refining and teaching fuzzy sets have successfully applied this theory in many different fields.

The purpose of this paper to introduce the notion of Q-cubic bi-ideals in near-rings. We investigate some results, examples and properties.

II. PRELIMINARIES

Now we recall some known concepts related to Q-cubic bi-ideals in near-rings from the literature which will be needed

in the sequel. Throughout this paper N stands for a right near-ring.

Definition 2.1. [7] A near-ring is an algebraic system $(R, +, \cdot)$ consisting of a non-empty set R together with two binary operations + and \cdot such that $(R, +)$ is a group not necessarily abelian and (R, \cdot) is a semigroup connected by the following distributive law

$$x \cdot (y + z) = x \cdot y + x \cdot z \text{ valid for all}$$

$x, y, z \in R$. We use the word near-ring to mean left near-ring we denote xy instead of $x \cdot y$.

Definition 2.2. [1] A non-empty subset S of a near-ring R is called a subnear-ring of R if

$$i) x - y \in S$$

$$ii) xy \in S \text{ for all } x, y \in S.$$

Definition 2.3. [12] A subgroup B of N is a bi-ideal of N if $BNB \subseteq B$.

Definition 2.4. [12] A near-ring N is B-simple if it has no proper bi-ideals. That is the only bi-ideals of N are $\{0\}$ and N itself.

Definition 2.5. [6] Let I be an ideal of R. For each $a + I, b + I$ in the factor group R/I , we define $(a + I) + (b + I) = (a + b) + I$

and $(a + I)(b + I) = (ab) + I$. Then R/I is a near-ring

which we call the residue class near-ring of R with respect to I.

Definition 2.6. [2] A mapping $\mu: X \rightarrow [0,1]$ is called a fuzzy subset of X.

Definition 2.7. [1] Let R be a near-ring and μ be a fuzzy subset of R. Then μ is a fuzzy ideal of R if:

- i) $\mu(x - y) \geq \min \{ \mu(x), \mu(y) \}$
- ii) $\mu(y + x - y) \geq \mu(x)$
- iii) $\mu(xy) \geq \mu(y)$
- iiii) $\mu((x + z)y - xy) \geq \mu(z)$ for all $x, y, z \in R$.

A fuzzy subset with (i)-(iii) is called a fuzzy left ideal of R, whereas a fuzzy subset with (i), (ii) and (iv) is called a fuzzy right ideal of R.

Definition 2.8. [10] A fuzzy set μ in N is a fuzzy sub near-ring of N if for all $x, y \in N$,

- i) $\mu(x - y) \geq \min \{ \mu(x), \mu(y) \}$ and
- ii) $\mu(xy) \geq \min \{ \mu(x), \mu(y) \}$

Definition 2.9. [10] A fuzzy set μ in N is a fuzzy bi-ideal of N if for all $x, y \in N$,

- i) $\mu(x - y) \geq \min \{ \mu(x), \mu(y) \}$ and
- ii) $\mu(xyz) \geq \min \{ \mu(x), \mu(z) \}$

Definition 2.10. [2] Let X be a non-empty set. A mapping $\bar{\mu}: X \rightarrow D[0,1]$ is called interval-valued fuzzy set, where $D[0,1]$ denote the family of all closed sub intervals of $[0,1]$ and $\bar{\mu}(x) = [\mu^-(x), \mu^+(x)]$ for all $x \in X$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$ for all $x \in X$.

Definition 2.11. [8] Let X be a non-empty set. A cubic set \mathcal{A} in X is a structure $\mathcal{A} = \{ \langle x, \bar{\mu}_A(x), f_A(x) \rangle : x \in X \}$ which is briefly denoted by $\mathcal{A} = \langle \bar{\mu}_A, f_A \rangle$, where $\bar{\mu}_A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set (briefly, IVF) in X and f is a fuzzy set in X. In this case, we will use $= \langle [\mu^-(x), \mu^+(x)], f_A(x) \rangle \quad \forall x \in X$.
 $\mathcal{A}(x) = \langle \bar{\mu}_A(x), f_A(x) \rangle$

Definition 2.12. [3] Let $\mathcal{A} = \langle \bar{\mu}, \gamma \rangle$ be a cubic set of S. Define $U(\mathcal{A}; \tilde{t}, n) = \{ x \in S | \bar{\mu}(x) \geq \tilde{t} \text{ and } \gamma(x) \leq n \}$ where $\tilde{t} \in D[0,1]$ and $n \in [0,1]$ is called the cubic level set of \mathcal{A} .

Definition 2.13. [3] For any non-empty subset G of a set X, the characteristic cubic set of G is defined to be a structure

$\chi_G(x) = \langle x, \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x) \rangle : x \in X$ which is briefly denoted by $\chi_G(x) = \langle \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x) \rangle$ where

$$\bar{\mu}_{\chi_G}(x) = \begin{cases} [1,1] & \text{if } x \in G \\ [0,0] & \text{otherwise} \end{cases} \text{ and}$$

$$\gamma_{\chi_G}(x) = \begin{cases} 0 & \text{if } x \in G \\ 1 & \text{otherwise} \end{cases}$$

III. MAIN RESULTS

In this section we introduced the new concept of Q-cubic bi-ideals in near-rings and discuss some of its properties. Throughout this paper N denotes near-ring unless otherwise specified.

Definition 3.1. A cubic set $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ in N is a Q-cubic subnear-ring of N if for all $x, y \in N$ and $q \in Q$

- i) $\bar{\mu}(x - y, q) \geq \min \{ \bar{\mu}(x, q), \bar{\mu}(y, q) \}$ and $\omega(x - y, q) \leq \max \{ \omega(x, q), \omega(y, q) \}$
- ii) $\bar{\mu}(xy, q) \geq \min \{ \bar{\mu}(x, q), \bar{\mu}(y, q) \}$ and $\omega(xy, q) \leq \max \{ \omega(x, q), \omega(y, q) \}$

Definition 3.2. A cubic set $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ in N is a Q-cubic bi-ideal of N if for all $x, y, z \in N$ and $q \in Q$

- i) $\bar{\mu}(x - y, q) \geq \min \{ \bar{\mu}(x, q), \bar{\mu}(y, q) \}$ and $\omega(x - y, q) \leq \max \{ \omega(x, q), \omega(y, q) \}$
- ii) $\bar{\mu}(xyz, q) \geq \min \{ \bar{\mu}(x, q), \bar{\mu}(z, q) \}$ and $\omega(xyz, q) \leq \max \{ \omega(x, q), \omega(z, q) \}$

Example 3.3. Let $N = \{a, b, c, d\}$ be the Klein’s four group. Define addition and multiplication in N as follows,

| | | | | |
|---|---|---|---|---|
| + | a | b | c | d |
| a | a | b | c | d |
| b | b | a | d | c |
| c | c | d | b | a |
| d | d | c | a | b |

| | | | | |
|---|---|---|---|---|
| · | a | b | c | d |
| a | a | a | a | a |
| b | a | a | a | a |
| c | a | a | a | a |
| d | a | b | c | d |

Then $(N, +, \cdot)$ is a near-ring. Define a Q-cubic set $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ by $\bar{\mu}(a, q) = [0.8, 0.9]$, $\bar{\mu}(b, q) = [0.6, 0.7]$, $\bar{\mu}(c, q) = [0.5, 0.5] = \bar{\mu}(d, q)$ is an interval-valued Q-

fuzzy bi-ideal of N and $\omega(a, q) = 0.2, \omega(b, q) = 0.6, \omega(c, q) = 0.8 = \omega(d, q)$ is a Q-fuzzy bi-ideal of N. Thus $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a Q-cubic bi-ideal of N.

Theorem 3.4. Every Q-cubic bi-ideal in a regular near-ring N is a Q-cubic sub near-ring of N. Proof: Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a Q-cubic bi-ideal of N and $a, b \in N$. Since N is regular, there exist $x \in N$ such that $a = axa$. Then

$$\begin{aligned} \bar{\mu}(ab, q) &= \bar{\mu}((axa)b, q) \\ &= \bar{\mu}(a(xa)b, q) \\ &\geq \min\{\bar{\mu}(a, q), \bar{\mu}(b, q)\} \text{ and} \\ \omega(ab, q) &= \omega((axa)b, q) \\ &= \omega(a(xa)b, q) \\ &\leq \max\{\omega(a, q), \omega(b, q)\} \end{aligned}$$

Thus $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a Q-cubic subnear-ring of N.

Proposition 3.5. Let N be a strongly regular near-ring. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a Q-cubic bi-ideal in N. Then $\bar{\mu}(x, q) = \bar{\mu}(x^2, q)$ and $\omega(x, q) = \omega(x^2, q)$ for all $x \in N$ and $q \in Q$

Proof: Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a Q-cubic bi-ideal of N and $x \in N, q \in Q$. Since, N is strongly regular, there exist $y \in N$ such that $x = x^2yx^2$. Then

$$\begin{aligned} \bar{\mu}(x, q) &= \bar{\mu}(x^2yx^2, q) \\ &\geq \min\{\bar{\mu}(x^2, q), \bar{\mu}(x^2, q)\} \\ &= \bar{\mu}(x^2, q) \\ &\geq \min\{\bar{\mu}(x, q), \bar{\mu}(x, q)\} \\ &= \bar{\mu}(x, q) \text{ and} \\ \omega(x, q) &= \omega(x^2yx^2, q) \\ &\leq \max\{\omega(x^2, q), \omega(x^2, q)\} \\ &= \omega(x^2, q) \\ &\leq \max\{\omega(x, q), \omega(x, q)\} \\ &= \omega(x, q) \end{aligned}$$

Hence, $\bar{\mu}(x, q) = \bar{\mu}(x^2, q)$ and $\omega(x, q) = \omega(x^2, q)$.

Theorem 3.6. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a Q-cubic bi-ideal of N, then the set

$N_{\mathcal{A}} = \{x \in N, q \in Q \mid \mathcal{A}(x, q) = \mathcal{A}(0, q)\}$ is a Q-cubic bi-ideal of N.

Proof: Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a Q-cubic bi-ideal of N and $x, y \in N$, then $\mathcal{A}(x, q) = \mathcal{A}(0, q)$ and $\mathcal{A}(y, q) = \mathcal{A}(0, q)$. Suppose

$x, y, z \in N_{\mathcal{A}}$.

Then $\bar{\mu}(x, q) = \bar{\mu}(y, q) = \bar{\mu}(z, q) = \bar{\mu}(0, q)$ and $\omega(x, q) = \omega(y, q) = \omega(z, q) = \omega(0, q)$

Since, $\bar{\mu}$ is an interval-valued q-fuzzy bi-ideal of N, $\bar{\mu}(x - y, q) \geq \min\{\bar{\mu}(x, q), \bar{\mu}(y, q)\}$

$$= \min\{\bar{\mu}(0, q), \bar{\mu}(0, q)\} = \bar{\mu}(0, q) \text{ and}$$

ω is a q-fuzzy bi-ideal of N

$$\begin{aligned} \omega(x - y, q) &\leq \max\{\omega(x, q), \omega(y, q)\} \\ &= \max\{\omega(0, q), \omega(0, q)\} = \omega(0, q) \end{aligned}$$

Thus $x - y, q \in N_{\mathcal{A}}$.

$$\begin{aligned} \bar{\mu}(xyz, q) &\geq \min\{\bar{\mu}(x, q), \bar{\mu}(z, q)\} \\ &= \min\{\bar{\mu}(0, q), \bar{\mu}(0, q)\} = \bar{\mu}(0, q) \text{ and} \end{aligned}$$

$$\begin{aligned} \omega(xyz, q) &\leq \max\{\omega(x, q), \omega(z, q)\} \\ &= \max\{\omega(0, q), \omega(0, q)\} = \omega(0, q) \end{aligned}$$

Thus $xyz, q \in N_{\mathcal{A}}$.

Therefore, $N_{\mathcal{A}}$ is a Q-cubic bi-ideal of N.

Theorem 3.7. Let $\{\mathcal{A}_i\} = \langle \bar{\mu}_i, \omega_i \mid i \in \Lambda \rangle$ be a family of Q-cubic bi-ideals of N and then the Q-cubic set $\prod_{i \in \Lambda} \mathcal{A}_i = \langle \bigcap_{i \in \Lambda} \bar{\mu}_i, \bigcup_{i \in \Lambda} \omega_i \rangle$ is also a Q-cubic bi-ideal of N, where Λ is any index set.

Proof: Let $\mathcal{A}_i = \langle \bar{\mu}_i, \omega_i \mid i \in \Lambda \rangle$ be a family of Q-cubic bi-ideals of N.

Let $x, y, z \in N, q \in Q$ and $\bar{\mu} = \bigcap \bar{\mu}_i; \omega = \bigcup \omega_i$ then

$$\begin{aligned} \bar{\mu}(x, q) &= \bigcap \bar{\mu}_i(x, q) = (\inf \bar{\mu}_i)(x, q) = \\ &= \inf \bar{\mu}_i(x, q), \end{aligned}$$

$$\begin{aligned} \omega(x, q) &= \bigcup \omega_i(x, q) = (\sup \omega_i)(x, q) = \\ &= \sup \omega_i(x, q) \end{aligned}$$

$$\begin{aligned} i) \bar{\mu}(x - y, q) &= \inf \bar{\mu}_i(x - y, q) \\ &\geq \inf \{\min\{\bar{\mu}_i(x, q), \bar{\mu}_i(y, q)\}\} \\ &= \min\{\inf \bar{\mu}_i(x, q), \inf \bar{\mu}_i(y, q)\} \\ &= \min\{\bigcap \bar{\mu}_i(x, q), \bigcap \bar{\mu}_i(y, q)\} \end{aligned} \text{ and}$$

$$\begin{aligned} \omega(x - y, q) &= \sup \omega_i(x - y, q) \\ &\leq \sup\{\max\{\omega_i(x, q), \omega_i(y, q)\}\} \\ &= \max\{\sup \omega_i(x, q), \sup \omega_i(y, q)\} \\ &= \max\{\bigcup \omega_i(x, q), \bigcup \omega_i(y, q)\} \end{aligned}$$

$$\omega(x - y, q) = \max\{\omega(x, q), \omega(y, q)\}$$

$$\begin{aligned} ii) \bar{\mu}(xyz, q) &= \inf \bar{\mu}_i(xyz, q) \\ &\geq \inf\{\min\{\bar{\mu}_i(x, q), \bar{\mu}_i(z, q)\}\} \end{aligned}$$

$$\begin{aligned}
 &= \min\{\inf \bar{\mu}_i(x, q), \inf \bar{\mu}_i(z, q)\} \\
 &= \min\{\cap \bar{\mu}_i(x, q), \cap \bar{\mu}_i(z, q)\} \\
 \bar{\mu}(xyz, q) &\geq \min\{\bar{\mu}(x, q), \bar{\mu}(z, q)\} \quad \text{and} \\
 \omega(xyz, q) &= \sup \omega_i(xyz, q) \\
 &\leq \sup \max \{\omega_i(x, q), \omega_i(z, q)\} \\
 &= \max\{\sup \omega_i(x, q), \sup \omega_i(z, q)\} \\
 &= \max\{\cup \omega_i(x, q), \cup \omega_i(z, q)\} \\
 \omega(xyz, q) &\leq \max\{\omega(x, q), \omega(z, q)\}
 \end{aligned}$$

Hence, $\prod_{i \in \lambda} \mathcal{A}_i = \langle \cap_{i \in \lambda} \bar{\mu}_i, \cup_{i \in \lambda} \omega_i \rangle$ is a Q-cubic bi-ideal of N.

Theorem 3.8. Let $f: N \rightarrow N_1$ be a homomorphism of near-rings and $C_f^{-1}: C(N_1) \rightarrow C(N)$ be the inverse Q-cubic transformation induced by f. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a Q-cubic bi-ideal of N_1 by the cubic property then $C_f^{-1}(\mathcal{A}) = \langle C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) \rangle$ is a Q-cubic bi-ideal of N. Proof: Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a Q-cubic bi-ideal of N_1 . For all $x, y, z \in N, q \in Q$ then

$$\begin{aligned}
 i) C_f^{-1}(\bar{\mu}(x - y), q) &= \bar{\mu}(f(x - y), q) \\
 &= \bar{\mu}(f(x, q) - f(y, q)) \\
 &\geq \min\{\bar{\mu}(f(x, q)), \bar{\mu}(f(y, q))\}
 \end{aligned}$$

$$\begin{aligned}
 C_f^{-1}(\bar{\mu}(x - y), q) &\geq \\
 \min\{C_f^{-1}(\bar{\mu}(x), q), C_f^{-1}(\bar{\mu}(y), q)\}
 \end{aligned}$$

$$\begin{aligned}
 C_f^{-1}(\omega(x - y), q) &= \omega(f(x - y), q) \\
 &= \omega(f(x, q) - f(y, q)) \\
 &\leq \max\{\omega(f(x, q)), \omega(f(y, q))\}
 \end{aligned}$$

$$\begin{aligned}
 C_f^{-1}(\omega(x - y), q) &\leq \\
 \max\{C_f^{-1}(\omega(x), q), C_f^{-1}(\omega(y), q)\}
 \end{aligned}$$

$$\begin{aligned}
 ii) C_f^{-1}(\bar{\mu}(xyz), q) &= \bar{\mu}(f(xyz), q) \\
 &= \bar{\mu}(f(x, q)f(y, q)f(z, q)) \\
 &\geq \min\{\bar{\mu}(f(x, q)), \bar{\mu}(f(z, q))\} \\
 &= \min\{C_f^{-1}(\bar{\mu}(x), q), C_f^{-1}(\bar{\mu}(z), q)\}
 \end{aligned}$$

$$\begin{aligned}
 C_f^{-1}(\omega(xyz), q) &= \omega(f(xyz), q) \\
 &= \omega(f(x, q)f(y, q)f(z, q)) \\
 &\leq \max\{\omega(f(x, q)), \omega(f(z, q))\} \\
 &= \max\{C_f^{-1}(\omega(x), q), C_f^{-1}(\omega(z), q)\}
 \end{aligned}$$

Hence, $C_f^{-1}(\mathcal{A}) = \langle C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) \rangle$ is a Q-cubic bi-ideal of N.

Theorem 3.9. Let $f: N \rightarrow N_1$ be an onto near-ring homomorphism, let $C_f: C(N) \rightarrow C(N_1)$ be the Q-cubic transformation respectively induced by f. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a Q-cubic bi-ideal of N which has the cubic property, then $C_f(\mathcal{A})$ is a Q-cubic bi-ideal of N_1 .

Proof: Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a Q-cubic bi-ideal of N.

$$\begin{aligned}
 \text{Since } (C_f(\bar{\mu}))(x', q) &= \sup_{f(x)=x'} \bar{\mu}(x, q) \quad \text{and} \\
 (C_f(\omega))(x', q) &= \inf_{f(x)=x'} \omega(x, q) \quad \text{for } x' \in N_1
 \end{aligned}$$

So, $C_f(\mathcal{A}) = \langle C_f(\bar{\mu}), C_f(\omega) \rangle$ is non-empty.

Let $x', y', z' \in N_1, q \in Q$. Then we have

$$\begin{aligned}
 C_f(\bar{\mu})(x' - y', q) &= \sup_{f(a)=x'-y'} \bar{\mu}(a) \\
 &\geq \sup_{f(x)=x', q, f(y)=y', q} \bar{\mu}(x - y, q) \\
 &\geq \sup_{f(x)=x', q, f(y)=y', q} \min\{\bar{\mu}(x, q), \bar{\mu}(y, q)\}
 \end{aligned}$$

$$\begin{aligned}
 &= \min\{\sup_{f(x)=x', q} \bar{\mu}(x, q), \sup_{f(y)=y', q} \bar{\mu}(y, q)\} \\
 &= \min\{C_f(\bar{\mu})(x', q), C_f(\bar{\mu})(y', q)\}
 \end{aligned}$$

$$\begin{aligned}
 C_f(\omega)(x' - y', q) &= \inf_{f(a)=x'-y'} \omega(a) \\
 &\leq \inf_{f(x)=x', q, f(y)=y', q} \omega(x - y, q)
 \end{aligned}$$

$$\leq \inf_{f(x)=x', q, f(y)=y', q} \max\{\omega(x, q), \omega(y, q)\}$$

$$\begin{aligned}
 &= \max\{\inf_{f(x)=x', q} \omega(x, q), \inf_{f(y)=y', q} \omega(y, q)\} \\
 &= \max\{C_f(\omega)(x', q), C_f(\omega)(y', q)\}
 \end{aligned}$$

$$C_f(\bar{\mu})(x' y' z', q) = \sup_{f(a)=x' y' z'} \bar{\mu}(a)$$

$$\geq \sup_{f(x)=x', q, f(y)=y', q, f(z)=z', q} \bar{\mu}(xyz, q)$$

$$\geq \sup_{f(x)=x', q, f(z)=z', q} \min\{\bar{\mu}(x, q), \bar{\mu}(z, q)\}$$

$$\begin{aligned}
 &= \min\{\sup_{f(x)=x', q} \bar{\mu}(x, q), \sup_{f(z)=z', q} \bar{\mu}(z, q)\} \\
 &= \min\{C_f(\bar{\mu})(x', q), C_f(\bar{\mu})(z', q)\}
 \end{aligned}$$

$$C_f(\omega)(x' y' z', q) = \inf_{f(a)=x' y' z'} \omega(a)$$

$$\leq \inf_{f(x)=x', q, f(y)=y', q, f(z)=z', q} \omega(xyz, q)$$

$$\begin{aligned} &\leq_{f(x)=x',q,f(z)=z',q} \inf \max \{ \omega(x, q), \omega(z, q) \} \\ &= \max \{ \inf_{f(x)=x',q} \omega(x, q), \inf_{f(z)=z',q} \omega(z, q) \} \\ &= \max \{ C_f(\omega)(x', q), C_f(\omega)(z', q) \} \end{aligned}$$

Hence, $C_f(\mathcal{A}) = \langle C_f(\bar{\mu}), C_f(\omega) \rangle$ is a Q-cubic bi-ideal of N_1 .

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