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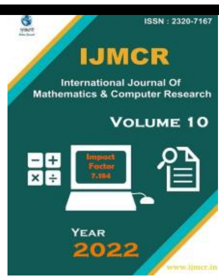

A Note on Neo Balancing Sequence, Generalized Recursive Sequences and Pythagorean Triples

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ABSTRACT

We proved theorems about creating Pythagorean Triples from generalized recursive sequences in this study. Pythagorean triples are a very old concept that has made significant progress in recent mathematical studies. Pythagorean triples can be generated using Neo balancing sequences and generalized recursive sequences, and vice-versa. For greater understanding, a general procedure is presented with appropriate illustrations.

KEYWORDS: Pythagorean Triples, Recurrence Relation, Generalized Neo Balancing Sequence, Generalized Recursive Sequence.

1. INTRODUCTION

Pythagorean triple is a collection of three positive integers that form the sides of a right triangle. Even though Pythagorean triples may be generated using simple algebraic formulas, the concept of generating such triples continues to intrigue mathematicians to his day. This study looks at one such technique that uses recursively generated generalized sequences. To explain the generation technique, theorems and demonstrations were supplied in-depth.

$p^2 + q^2 = r^2$ (1) if and only if three positive integers p, q, r form a Pythagorean triple. A right triangle with two smaller sides of length p, q and a hypotenuse of length r can be formed by any three positive integers that fulfill.

The n^{th} term of a sequence of ratios denoted by M_n and is defined as the positive root of the equation $x^2 - tx + 1 = 0$ (2). Thus, $M_n = \frac{t \pm \sqrt{t^2-4}}{2}$ (3).

The terms of the generalized recursive sequence of order t are defined recursively by the relation $K_{n+2} = tK_{n+1} - K_n + 2 - t, K_0 = 1, K_1 = t, n \geq 1$ (4) where t is some positive integer.

2. NEO BALANCING SEQUENCE AND PYTHAGOREAN TRIPLE

In this part, we will demonstrate how to create the Pythagorean Triple from four successive instances of the Neo Balancing sequence and vice-versa.

The relation $NB_{n+1} = 6NB_n - NB_{n-1} - 4, n \geq 1$ (5) creates a series of real numbers called the generalized balancing number. The typical balancing sequence, whose first few terms are$1, 2, 7, 36, \ldots$ is obtained from (5) if $NB_0 = 1, NB_1 = 2$.

Theorem:2.1 If $NB_n, NB_{n+1}, NB_{n+2}, and\ NB_{n+3}$ denotes four consecutive terms in a Generalized Balancing sequence, then (p, q, r) denotes a Pythagorean Triple,

where $p = 36(NB_{n+1} - 1)^2 - (NB_{n+2} - 1)^2$,
$q = 12(NB_{n+1} - 1)(NB_{n+2} - 1)$, $r = 36(NB_{n+1} - 1)^2 + (NB_{n+2} - 1)^2$ (6)

Proof. Using (5), we observe that
$p^2 + q^2 = (36(NB_{n+1} - 1)^2 - (NB_{n+2} - 1)^2)^2 + \left(12(NB_{n+2} - 1)(NB_{n+1} - 1)\right)^2 = ((NB_{n+2} - 1)^2 + 36(NB_{n+1} - 1)^2)^2 = r^2$.
Hence (p, q, r) is a Pythagorean triple.

Theorem: 2.2 Let (p, q, r) be a Pythagorean triple with the same parity for both p and$r$. The four consecutive terms of the Generalized balancing sequence

$NB_n, NB_{n+1}, NB_{n+2}, and\ NB_{n+3}$ can be constructed using the relations$NB_n = 1 + \sqrt{\frac{r+p}{2}} - \sqrt{\frac{r-p}{2}}$, $NB_{n+1} = 1 + \frac{1}{6}\sqrt{\frac{r+p}{2}}$, $NB_{n+2} = 1 + \sqrt{\frac{r-p}{2}}$ and $NB_{n+3} = 1 + 6\sqrt{\frac{r-p}{2}} - \frac{1}{6}\sqrt{\frac{r+p}{2}}$ (7)

Proof. Using (5) and the values of p, r from (6) and from theorem 2.1, we obtain (7). Also from (7), it is obvious that

$$NB_{n+2} = 6NB_{n+1} - NB_n - 4 \text{ and}$$

$$NB_{n+3} = 6NB_{n+2} - NB_{n+1} - 4.$$

Hence $NB_n, NB_{n+1}, NB_{n+2}, and\ NB_{n+3}$ forms four consecutive terms of generalized balancing sequence. This completes the proof.

Illustration:2.1 Consider the four consecutive Neo Balancing sequences as 2,7,36,205 then the Pythagorean triple would be $p = 71, q = 2520, r = 2521$. Also, if $(71,2520,2521)$ is the given Pythagorean triple then by (7) $NB_n = 2, NB_{n+1} = 7, NB_{n+2} = 36, and\ NB_{n+3} = 205$.
Thus the four consecutive terms of the Neo Balancing sequence 2,7,36,205 corresponds to the Neo Balancing sequence.

3. GENERALIZED RECURSIVE SEQUENCE AND PYTHAGOREAN TRIPLE

In this section, we will provide methods to generate the Pythagorean Triple from four consecutive terms of Generalized Recursive sequence and vice-versa.
The generalized recursive sequence is defined by

$$K_{n+2} = tK_{n+1} - K_n + 2 - K \ (8)$$

for some positive integer$t$. We also notice that the Neo balancing sequence is a special case of

$$K_{n+2} = tK_{n+1} - K_n \text{ For } t = 6$$

with fixed initial values$K_0 = 1, K_1 = 2$.

Theorem: 3.1 Let $K_n, K_{n+1}, K_{n+2}, and\ K_{n+3}$ represent four consecutive terms of the generalized recursive sequence of order t. Then (p, q, r) will be a Pythagorean Triple, where $p = t^2(K_{n+1} - 1)^2 - (K_{n+2} - 1)^2$,

$$q = 2t(K_{n+1} - 1)(K_{n+2} - 1), \text{ and}$$

$$r = t^2(K_{n+1} - 1)^2 + (K_{n+2} - 1)^2 \ (9).$$

Proof.
From the given expression,

$$p^2 + q^2 = (t^2(K_{n+1} - 1)^2 - (K_{n+2} - 1)^2)^2 + \left(2t(K_{n+1} - 1)(K_{n+2} - 1)\right)^2$$
$$= (t^2(K_{n+1} - 1)^2 + (K_{n+2} - 1)^2)^2 = r^2$$

Hence (p, q, r) forms a Pythagorean triple and completes the poof.

Theorem: 4.2 Let (p, q, r) be a Pythagorean triple with the same parity for both p and$r$. The four consecutive terms of the generalized recursive sequence $K_n, K_{n+1}, K_{n+2}, and\ K_{n+3}$
Can be constructed using the relations

$$K_n = 1 + \sqrt{\frac{r+p}{2}} - \sqrt{\frac{r-p}{2}},\ K_{n+1} = 1 + \frac{1}{t}\sqrt{\frac{r+p}{2}},$$

$$K_{n+2} = 1 + \sqrt{\frac{r-p}{2}} \text{ and } K_{n+3} = 1 + t\sqrt{\frac{r-p}{2}} - \frac{1}{t}\sqrt{\frac{r+p}{2}} \ (10).$$

Proof.
From the values of p, r given in (8) we see that $p + r = 2t^2(K_{n+1} - 1)^2$ and so$K_{n+1} = 1 + \frac{1}{t}\sqrt{\frac{r+p}{2}}$.
Similarly, we obtain $K_{n+2} = 1 + \sqrt{\frac{r-p}{2}}$.
Now using (8), we get

$$K_n = tK_{n+1} - K_{n+2} + 2 - K = 1 + \sqrt{\frac{r+p}{2}} - \sqrt{\frac{r-p}{2}}.$$

Finally, in (8), replacing n by $n + 1$, we get $K_{n+3} = 1 + t\sqrt{\frac{r-p}{2}} - \frac{1}{t}\sqrt{\frac{r+p}{2}}$.
Moreover, we see that the recursive equations

$$K_{n+2} = tK_{n+1} - K_n + 2 - K \text{ and}$$

$$K_{n+3} = tK_{n+2} - K_{n+1} + 2 - K$$

Are satisfied with these values.

Hence $K_n, K_{n+1}, K_{n+2}, and\ K_{n+3}$ forms four consecutive terms of the generalized recursive sequence. This completes the proof.

Illustration: 3.1 As an example, we discover four terms of a generalized recursive sequence for the values of $t =$ 1,2,3,4,5,6 if (7,24,25) is the provided Pythagorean triple.
Here $p = 7, r = 25$.
From (10), we obtain the four consecutive terms as follows for $t = 1$, is (2,5,4,0).
From (10), we get the four consecutive terms as follows for $t = 2$, which is (2,3,4,5).
From (10), we get the four consecutive terms as follows for $t = 3$, is $(2, \frac{7}{3}, 4, \frac{26}{3})$.
From (10), we get the four consecutive terms as follows for $t = 4$, which is (2,2,4,12).
From (10), we get the four consecutive terms as follows for $t = 5$, which is $(2, \frac{9}{5}, 4, \frac{76}{5})$.
From (10), we get the four consecutive terms as follows for $t = 6$, which is $(2, \frac{5}{3}, 2, \frac{55}{3})$.

Illustration:3.2 As an example, we discover four terms of a generalized recursive sequence for the values of $t =$ 1,2,3,4,5,6 if (13,84,85) is the provided Pythagorean triple.
Here $p = 13, r = 85$.
From (10), we obtain the four consecutive terms as follows for $t = 1$, is (2,8,7,0).
From (10), we get the four consecutive terms as follows for $t = 2$, which is $(2, \frac{9}{2}, 7, \frac{19}{2})$.

From (10), we get the four consecutive terms as follows for $t = 3$, is $(2, \frac{10}{3}, 7, \frac{50}{3})$.

From (10), we get the four consecutive terms as follows for $t = 4$, which is $(2, \frac{11}{4}, 7, \frac{93}{4})$.

From (10), we get the four consecutive terms as follows for $t = 5$, which is $(2, \frac{12}{5}, 7, \frac{148}{5})$.

From (10), we get the four consecutive terms as follows for $t = 6$, which is $(2, \frac{13}{6}, 7, \frac{215}{6})$.

For all matching values of t, we can obtain four consecutive terms of the generalized recursive sequence.

5. CONCLUSION

Among the few current Pythagorean triple generating methods, the paper dealt with approaches of generating such triples using four consecutive terms generalized recursive sequence. The notion of Neo Balancing sequence was nicely-recognized and mentioned in [6]. But the four consecutive phrases of the Neo Balancing sequence can't produce every recognized Pythagorean triple. Hence we think about the Neo Balancing sequence and via theorems 2.1, and 2.2, we had proved the one-one correspondence between Pythagorean triple and four consecutive terms of Neo Balancing sequence, and subsequently, via theorems 3.1 and 3.2, we had proved the one-one correspondence between Pythagorean triple and four consecutive terms of the generalized recursive sequence. The thinking of the Pythagorean triple from four consecutive terms of generalized recursive sequence and retrieving them returned from a given Pythagorean triple was once a new notion employed in this paper.

REFERENCES

1. A. F. Horadam, Oresme numbers. The Fibonacci Quarterly, 12 (1974), no. 3, 267-271.
2. T. Goy, R. Zatorsky, On Oresme numbers and their connection with Fibonacci and Pell numbers, Fibonacci Quarterly, 57 (2019), no. 3, 268-245.
3. Robert Fronczak, Identities for generalized balancing numbers, Notes on Number Theory and Discrete Mathematics, Vol. 25, 2019, No. 2, 169-180.
4. A. Berczes, K. Liptai, and I. Pink, On generalized balancing sequences, The Fibonacci Quarterly, 48 (2010), no. 2, 121-128.
5. R.Sivaraman, Pythagorean triples and generalized recursive sequences, Mathematical Sciences International Research Journal, Vol. 10 (2021), Issue 2, 1-5.
6. Natdanai Chailangka and Apisit Pakapongpun, Neo Balancing Numbers, International Journal of Mathematics and Computer Science, 16(2021), no. 4, 1653-1664.
7. Titu Andreescu, Dorin Andrica and Ion Cucurezeanu., An introduction to Diophantine Equations, Birhauser, New York, 2010.
8. A.Ozkoc, A.Tekcan, On k-balancing numbers, Notes on Number Theory and Discrete Mathematics, 2017, 23, 38.
9. S.Sriram and P.Veeramallan, Some Special Analysation on a Pythagorean Triangle which Satisfies $\alpha\big((Hypotonuse \times Perimeter) - 4(Area)\big) = (A^2 - B^2)(Perimeter)$ for some Particular different values of α, Advances in Mathematics: Scientific Journal **9** (2020), no.5, 3165– 3174.
10. S.Sriram and P.Veeramallan, Some Special Analysation on a Pythagorean Triangle which satisfies$\lambda\big((Hypotonuse \times Perimeter) - 4(Area)\big) = \mu^2(Perimeter)$, Advances and Applications in Mathematical Sciences Volume 20, Issue 8, June 2021, Pages 1475-1483.
11. S.Sriram and P.Veeramallan, A note on Pythagorean Triples And Generalized Recursive Sequences, Stochastic Modeling & Applications, Vol. 26 No. 3 (January – June, Special Issue 2022 Part - 8)