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Approach to Solve Multi Objective Linear Fractional Programming Problem (MOLFPP)

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1. Abstract:

In this paper we have developed an algorithm to solve multi objective fractional programming problem using fuzzy compromise approach. Here each of the linear fractional programming problems is converted into linear programming problem (LPP) using Charnes and Cooper's transformation and then each of the LPP is solved by Simplex Method using TORA.

Then each of the fractional objective function is expanded about optimal solution vector by Taylor's Series method and converted it into approximate linear programming problem, using partial differentiation. Finally we have solved this problem as Multi Objective Linear Programming Problem using Fuzzy compromise programming approach.

Key Words: *Fractional, Multi objective, Linear Programming.*

2. Introduction:

In decision making process often the objective function is ratio of two linear function and objective function is to be optimised. When there are several fractional objective functions then the problem is called multi objective fractional linear programming problem (MOFLPP). Fractional programming problem can be converted into linear programming problem (LPP) by using variable transformation give by Charnes and Cooper [1]. MOFLPP can be converted into MOLPP using Taylors series method [2].With variable transformation method Chakraborty and Gupta [3] converted MOLFPP to MOLPP using Fuzzy set theoretic approach. Surapati Pramanik and Durga Banerjee [4] gave solution to chance constrained multi objective linear plus fractional programming problem. Singh et.al.[5]developed an algorithm for solving MOLPFPP with the help of Taylor series. Pitram singh ,et.al. [6] gave approach for multi objective linear plus fractional programming problem

MOLPP is solved using fuzzy compromise approach by Lushu Li and K Lai [7]. Multi objective transportation problem is solved by Doke and Jadhav [8]using fuzzy compromise programming approach.

In this paper we will solve MOLFPP using Charnes and Copper transformation then Taylor Series method and finally by fuzzy compromise programming approach.

3. Formulation of MOLFPP

i) Definition: A linear fractional programming problem (**LFPP**) may be defined as under.

Maximize (or Minimize) $Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{j=1}^{n} pjxj \ + \ p0}{\sum_{j=1}^{n} djxj \ + \ d0}$ .... (1)

Subject to constraint $\sum_{j=1}^{n} aijxj \quad \geq \ or \ \ \leq \ or \ = \ bi$ ....(2)

For all i = 1 ,2,3,....,m

$x_{ij} \geq 0$ for all i = 1,2,.....m and j=1,2,...n ....(3)

Where p_j is profit associated with variable X_j and d_j is cost associated with variable X_j . p_0 is basic profit and d_0 is fixed cost. P(x) is total profit and D(x) is total cost and $D(x) \neq 0$ for every $x = (x_1, x_2, x_3, \ldots, x_n) \in S$. Where S denotes a feasible set. Above problem is said in canonical form if $\sum_{j=1}^{n} aijxj \ = \ bi$ For all i = 1 ,2,3,....,m.

Q(x) is called objective function which is fractional. Note that P(x) and D(x) are linear and (2) and (3) are also linear. Hence the name Linear fractional Programming problem (LFPP)

ii) Definition: Suppose $Q_1(x) = \frac{P1(x)}{Q1(x)}$, $Q_2(x) = \frac{P2(x)}{Q2(x)}$,........, $Q_k(x) = \frac{Pk(x)}{Qk(x)}$ are k fractional objective functions to be optimised simultaneously subject to constraint (2) and (3) and $D_k(x) \geq 0$ for every k then the problem is called Multi Objective Linear Fractional Programming Problem (**MOLFPP**).

Note that if the problem is not in standard form then convert it into standard form by adding appropriate slack, surplus and artificial variables.

iii) Definition

i) The linear fractional programming problem is solvable if a) feasible set S is non empty and there exists at least one vector X such that it satisfies constraint (2) and (3) and b) the objective function Q(x) has a finite upper bound over set S.

ii) The System B = { As1,As2,...Asm} of vectors Aj is basis of vectors if As1,As2,...Asm are linearly independent.

iii) The given vector $X = (x_1, x_2, \ldots, x_n)'$ is a basic solution of the system AX = b , if vector X satisfies system $\sum_{j=1}^{n} aijxj \ = \ bi$ for all j=1,2,...,n.

iv) The basic solution is degenerate if at least one of its basic variable equal to zero.

v) A basic solution X of system AX = b is said to be basic feasible solution (BFS) of LFP (1-3) if all elements of X satisfy non negativity constraint.

iv) Theorems

i) A point X of set corresponding to system Ax = b, is its extreme point if and only if it is its basic solution.

ii) Objective function Q (x) is monotonic on any segment of a straight line in feasible set S.

iii) If feasible set S in LFP (1-3) is bounded then the objective function Q(x) attains its maximal value over S in an extreme point of S.

4. Criteria for Optimality of LFPP

A basic feasible solution X is a basic optimal solution of LFPP (1-3) if and only if

$\Delta j(x) \geq 0$

where $\Delta j(x) = \Delta' j$ - Q(x) $\Delta'' j$

$$\Delta' j = \sum_{j=1}^{n} psj\ xij - pj$$

$$\Delta'' j = \sum_{j=1}^{n} dsj\ xij - dj$$

P_{sj} and d_{sj} are profits and cost of j^{th} variable respectively in basis.

5. Solution Procedure for MOLFPP

i) Consider MOLFPP ,Maximize { $Q_1(x), Q_2(x), \ldots, Q_k(x)$ } subject to constraint (2-3)

ii) Find optimal solution of each of the fractional objective function subject to constraint. To solve fractional programming problem convert it to Linear Programming Problem using Charnes- Cooper .

Consider LFPP ,

$$Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{j=1}^{n} pjxj + p0}{\sum_{j=1}^{n} djxj + d0}$$

Subject to constraint $\sum_{j=1}^{n} aijxj \leq bi$, For all i = 1 ,2,3,....,m

$x_{ij} \geq 0$ for all i = 1,2,.....m and j=1,2,...n

Put $x_j = \frac{tj}{to}$ for all j= 1,2,...n.

Then the objective function becomes

Maximize, $F(t) = \sum_{j=0}^{n} pjtj$

Subject to $\sum_{j=0}^{n} djtj = 1$

$$-b_0t_0 + \sum_{j=1}^{n} aijtj \leq 0 \quad , i = 1,2,\ldots,m$$

This problem is LPP and can be solved by any one of the standard algorithm and standard software. These ways solve all k fractional problems individually.

iii) Suppose X_l^* is optimal solution of of $Q_l(x)$ for l =1,2,...,k

iv) Expanding objective function $Q_l(x)$ about X^*_l using Taylor's theorem and ignoring second and higher order terms convert $Q_l(X)$ into linear function.

Consider $Q_l(x) = \frac{Pl(x)}{Ql(x)}$ and $X^*_l = (x_{l1}, x_{l2}, x_{l3}, \ldots, x_{ln})$ be the optimal solution of $Q_l(x)$,then using Taylor Series approach we have

$$Q_l(x) \sim \cong Q_l(x^*_l) + (x_1-x_{1l}) \frac{\partial Ql(Xl)}{\partial x1} + (x_2-x_{2l}) \frac{\partial Ql(Xl)}{\partial x2} +....+ (x_k-x_{kl}) \frac{\partial Ql(Xl)}{\partial xk} + O(h^2).$$

Using this expansion each of the objective function becomes linear function. To avoid complexity of notations write

$Q_l(x) \sim = \check{Z}_l(x)$, l =1,2,3,....,k.

The problem is Multi Objective Linear Programming problem. (MOLPP)

It can be solved by using fuzzy compromise approach as stated in next section.

6. FUZZY COMPROMISE APPROACH FOR MOLPP.

Using Fuzzy Compromise approach for MOLPP.

Consider MOLPP

Maximize Z (x) = [Z_1 (x), Z_2 , …, Z_k (x)] ----------------(5)

Subject to $x \in S$ where S set of feasible solutions.

Solution to (5) is often conflicting as several objectives cannot be optimized simultaneously. To find compromise solutions first solve each of the objective function as marginal or single objective function. In this paper we have converted Linear Fractional Programming problem to Linear Programming problem using Taylor series approach. Note that basic feasible optimal solution of LFPP is same as basic feasible optimal solution of LPP converted using Taylor Series approach.

Suppose x_k * is optimal solution of k^{th} objective function. Find values of each objective at optimal solution of k^{th} objective for all k= 1,2,…,K. Thus we have matrix of evaluation of objectives.

7. MARGINAL EVALUATION FOR SINGLE OBJECTIVE.

For each particular objective we define marginal evaluation function $\varphi_k : X \to [0,1]$ as given below

$$\varphi_k(x) = \begin{cases} 1 & if\ Zk \le Uk \\ \frac{Zk-Uk}{Lk-Uk} & if\ U\boldsymbol{k} < Zk(x) < Lk \\ 0 & if\ Zk(x) \ge Lk \end{cases}$$

Where U_k = Min Z_k (x) k=1,2,…,K

L_k = Max Z_k (x) k=1,2,…,K

According to fuzzy sets, φ_k is fuzzy subset describing fuzzy concept of optimum for objective Z_k on feasible solution space S.

To find compromise solution maximize aggregation operator

$$\mu(x) = \varphi_w [\varphi 1(x), \varphi 2(x), \dots, \varphi k(x)]$$

$$\text{Max } \mu(x) = M^{(\alpha)}_w [\varphi 1(x), \varphi 2(x), \dots, \varphi k(x)]$$

$$\mu(x) = [\sum w_i \varphi^{\alpha}{}_i(x))]^{(1/\alpha)}$$

i) If $\alpha = 1$ then Maximize $\mu(x) = \sum [w_i \varphi_i(x))]$ i.e. weighted A.M. of Fuzzy sets

ii) If $\alpha = 2$ then Maximize $\mu(x) = [\sum w_i \varphi^2{}_i(x))]^{(1/2)}$
i.e. weighted quadratic mean of Fuzzy sets.

This procedure gives weighted arithmetic mean and weighted quadratic mean to find global evaluation of multiple objectives.

Example:

Consider following two objective functions,

Maximize $Q_1(x) = \dfrac{8x1+9x2+4x3+4}{2x1+3x2+2x3+7}$ .... (i)

Maximize $Q_2(x) = \dfrac{4x1+9x2+12x3+7}{3x1+2x2+2x3+5}$ ....(ii)

Subject to $x_1+x_2+2x_3 \le 3$ ....(iii)

$2x_1+x_2+4x_3 \le 4$ ....(iv)

$5x_1+3x_2+x_3 \le 15$ ....(v)

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$ ....(vi)

Step1: Maximize $Q_1(x) = \dfrac{8x1+9x2+4x3+4}{2x1+3x2+2x3+7}$

Use transformation $x_1 = t_1/t_o$, $x_2 = t_2/t_o$, $x_3 = t_3/t_o$ then LFPP becomes

Maximize $Z_1(t) = 4t_0 + 8t_1 + 9t_2 + 4t_3$

Subject to $7t_o + 2t_1 + 3t_2 + 2t_3 = 1$

$-3t_0 + t_1 + t_2 + 2t_3 \le 0$

$-4t_0 + 2t_1 + t_2 4t_3 \le 0$

$-15t_0 + 5t_1 + 3t_2 + t_3 \le 0$

$t_0 \ge 0, t_1 \ge 0, t_2 \ge 0, t_3 \ge 0$

Solving above problem by Simplex Method using TORA we have

$t_0 = 0.07, t_1 = 0.07, t_2 = 0.14, t_3 = 0$

Thus $x_1 = 1$, $x_2 = 2$, $x_3 = 0$ and Maximum value of $Q_1(x) = 2$ Similarly optimal solution of $Q_2(x) = \dfrac{4x1+9x2+12x3+7}{3x1+2x2+2x3+5}$

Subject to given constraint is $x_1 = 0, x_2 = 2, x_3 = ½$ and $X_2^* = (0,2,½)$

and Maximum value of $Q_2(x) = 31/10$

Step 2: Expanding $Q_1(x)$ about $x_1^* = (1, 2, 0)$ its optimal solution by Taylor series approach,

$$Q_1(x) \cong Q_1(x_1^*) + (x_1-1)\frac{\partial Q1(x)}{\partial x1} + (x_2-2)\frac{\partial Q1(x)}{\partial x2} + (x_3-0)\frac{\partial Q1(x)}{\partial x3}$$

$$Q_1(x) \cong Q_1(x_1^*) + (x_1-1)\left\{\frac{\partial Q1(x)}{\partial x1}\right\}_{at\ x1^*} + (x_2-2)\left\{\frac{\partial Q1(x)}{\partial x2}\right\}_{at\ x1^*} +$$

$$(x_3-0)\left\{\frac{\partial Q1(x)}{\partial x3}\right\}_{at\ x1^*}$$

$$Q_1(x) \cong 2 + (x_1-1)\left\{\frac{(2*x1+3*x2+2*x3+7)8-(8*x1+9*x2+4*x3+4)2}{(2*x1+3*x2+2*x3+7)\hat{}2}\right\}_{at\ x1^*}$$

$$+ (x_2-2)\left\{\frac{(2*x1+3*x2+2*x3+7)9-(8*x1+9*x2+4*x3+4)3}{(2*x1+3*x2+2*x3+7)\hat{}2}\right\}_{at\ x1^*}$$

$$+ (x_3-0)\left\{\frac{(2*x1+3*x2+2*x3+7)2-(8*x1+9*x2+4*x3+4)4}{(2*x1+3*x2+2*x3+7)\hat{}2}\right\}_{at\ x1^*}$$

Substituting value of variables and simplifying we have,

$Q_1(x) \cong (4x_1 + 3x_2 - 6x_3 + 20)/15$ ....(vii)

Note that if we solve (vii) subject to (iii,iv,v,vi) by Simplex method using TORA the optimal solution is same as X_1^* i.e. (1,2,0).

Similarly expanding $Q_2(x)$ about its optimal solution $X_2^* = (0,2,½)$ using Taylor Series we

$Q_2(x) \cong (-53x_1 + 28x_2 + 58x_3 + 225)/100$ ....(viii)

Again optimal solution of (viii) subject to (iii,iv,v,vi) is same as X_2^* i.e.

(0,2,1/2).

Step 3: For simplicity of notations objective functions in (vii) and (viii)

We will denote by $\check{Z}_1(x)$ and $\check{Z}_2(x)$

Thus the problem becomes MOLPP.

Maximize { $(\check{Z}_1(x) = (4x_1 + 3x_2 - 6x_3 + 20)/15$ and

$\check{Z}_2(x) = (-53x_1 + 28x_2 + 58x_3 + 225)/100$ }

Subject to (iii, iv, v, vi)

Using fuzzy compromise approach evaluate each of the objective function at X_1^* and X_2^* ,Thus ,

$\check{Z}_1(X_1^*) = 2$, $\check{Z}_1(X_2^*) = \frac{12}{7}$, $\check{Z}_2(X_1^*) = \frac{29}{12}$, $\check{Z}_2(X_2^*) = \frac{31}{10}$

Define L_1 = Max $\check{Z}_1$ = 2 and U_1 = Min $\check{Z}_1 = \frac{12}{7}$

L_2 = Max $\check{Z}_2 = \frac{31}{10}$ and U_2 = Min $\check{Z}_2 = \frac{29}{12}$

Define $\varphi 1(x) = \frac{\check{Z}1 - U1}{L1 - U1}$

$= ((4x_1 + 3x_2 - 6x_3 + 20)/15 - 12/7)/(2 - 12/7)$

$= 0.93333x_1 + 0.7\,x_2 - 1.4\,x_3 - 13.333$

$\varphi 2(x) = \frac{\check{Z}2 - U2}{L2 - U2}$

$= ((-53x_1 + 28x_2 + 58x_3 + 225)/100 - 29/12)/(31/12 - 29/12)$

$= -0.7756\,x_1 + .40977\,x_2 + .84878\,x_3 - .24381$

We have $1(x)$, $\varphi 2(x)$ fuzzy set functions and have solution in S.

To find compromise solution, maximize aggregation operator,

$\mu\,(x) = \varphi_{w}\,[\varphi 1(x), \varphi 2(x)]$

Max $\mu\,(x) = M^{(\alpha)}{}_{w}\,[\varphi 1(x), \varphi 2(x)]$

$\mu\,(x) = [\sum\ wi\,\varphi^{\alpha}{}_{i}\,(x)\,)\,]^{\,(1/\alpha)}$

i) If $\alpha = 1$ then Maximize $\mu\,(x) = \sum\ [\,wi\,\varphi_{i}\,(x)\,)]$ i.e. weighted A.M. of Fuzzy sets

$\mu\,(x) = w_1\,\varphi_1\,(x) + w_2\,\varphi_2\,(x)$

$= w_1\,(0.93333x_1 + 0.7\,x_2 - 1.4\,x_3 - 13.333) +$

$w_2\,(-0.7756\,x_1 + .40977\,x_2 + .84878\,x_3 - .24381)$

such that $w_1 + w_2 = 1$ and w_1 and w_2 are both non negative.

Following table shows possible values of w1 and w2 and respective values of $Q_1(x)$ and $Q_2(x)$

w_1	w_2	x_1	x_2	x_3	$Q_1(x)$	$Q_2(x)$
0	1	0	2	0.5	1.7142	**3.1***
0.1	.9	0	3	0	**1.9375 ** **	**3.0909 ** **
0.8	0.2	0	3	0	1.9375	3.0909
1.0	0	1	2	0	**2 ***	2.42667

Note that optimal value of Q_1 is 2 where as compromise value is 1.9375

And optimal value of Q_2 is 3.1 where as compromise value is 3.0909.

Thus deviation in value of Q_1 is 0.0625 and that of Q_2 is 0.0909.

Thus compromise solution to the problem stated in (i to vi) is

$x_1 = 0$, $x_2 = 3$, $x_3 = 0$

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