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Total Chromatic Number of Fuzzy Bistar Graph & Fuzzy Helm Graph

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ABSTRACT

Fuzzy total chromatic number is the least value of k such that k-fuzzy total coloring exist. In this paper, we discussed the concept of total coloring of graphs to Fuzzy bistar graph and Fuzzy helm graph. Here we define fuzzy chromatic number of the fuzzy Bistar graph and Fuzzy Helm graph.

KEYWORDS: Fuzzy Chromatic number, Fuzzy Bistar graph, Fuzzy Helm graph.

1. INTRODUCTION

Fuzzy sets was introduced by Lofti.A.Zadeh in 1965 at 20th century[3]. The applications of fuzzy sets in the field of cluster analysis, neutral networks etc. were discussed by Zimermann[4]. Zadeh[3] had developed the fuzzy relations on fuzzy sets which had better feature in making fuzzy graph model.

In 1975, Rosenfeld [5] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [6] in 1973. Fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs.

Graph coloring dates back to at least 1852, when Francis Guthric came up with the four color conjecture. Since the graph coloring has been one of the most studied areas of graph theory. Vertex and edge coloring are two of the most popular areas of study in graph theory. They have been shown to be computationally difficult and at the same time practically important.

In 1965, Behad and Vizing have posted independently a new concept of a graph coloring, called a total coloring. This is a mixture type of the ordinary vertex and edge coloring in a sense that a total coloring of a graph G assigns a color to each of vertices and edges.

The fuzzy vertex coloring of a fuzzy graph was defined by Eslahchi and Onagh [7]. In this paper we define total chromatic number of fuzzy bistar graph and fuzzy helm graph in terms of family of fuzzy sets satisfying certain conditions.

2. PRELIMINARIES

Definition: 2.1 (Fuzzy Set)

Let X be a non-empty set. Then a fuzzy set A in X. (i.e., a fuzzy subset A of X) is characterized by a function of the form $\mu_A : X \to [0,1]$, such a function μ_A is called the membership of x in the fuzzy set A. In other words $A = \{(x, \mu_A(x)) / x \in X\}$.

Definition: 2.2 (Fuzzy Graph)

A fuzzy graph $\hat{G} = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all $u, v \in V$ we have $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$.

Definition: 2.3 (Adjacent)

Two vertices u and v in $\hat{G}$ are called adjacent if $(1/2)\min\{\sigma(u), \sigma(v)\} \le \mu(uv)$.

Definition: 2.4 (Degree of a vertex)

The degree of vertex v in $\hat{G}$ denoted by $\deg_G v$, is the number of adjacent vertices to v and the maximum degree of $\hat{G}$ is defined by $\Delta(\hat{G}) = \max\{\deg_G v \mid v \in V\}$

Definition: 2.5 (k-Fuzzy Coloring)

A family $\Gamma = \{\gamma_1, \ldots\ldots, \gamma_k\}$ of fuzzy sets on V is called a k- fuzzy coloring of $\hat{G} = (V, \sigma, \mu)$ if

(a) $\vee \Gamma = 0$

(b) $\gamma_i \wedge \gamma_j = 0$

(c) For every strong edge xy of G, $\wedge \{\gamma_i(x), \gamma_j(y)\} = 0$, $(1 \le i \le k)$.

Definition: 2.6 (k- Fuzzy Total Coloring)

A family $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_k\}$ of fuzzy sets on $V \cup E$ is called a k- fuzzy total coloring of $\hat{G} = (V, \sigma, \mu)$ if

(a) $\max_i \gamma_i(v) = \sigma(v)$ for every $v \in V$ and $\max_i \gamma_i(uv) = \mu(uv)$ for every $uv \in E$.

(b) $\gamma_i \wedge \gamma_j = 0$

(c) For every adjacent vertices u, v of $\min\{\gamma_i(u), \gamma_i(v)\} = 0$ and for every incident edges $\min\{\gamma_i(v_j v_k) / v_j v_k$ are set of incident edges from the vertex $v_j, j = 1,2, ...|V|$ }. The least value of k for which G has k- fuzzy total coloring denoted by $\chi_T(\hat{G})$ is called the fuzzy total chromatic number of $\hat{G}$.

Definition: 2.7 (Fuzzy Bistar Graph)

A bistar graph is a Fuzzy Bistar graph ($(\tilde{B}_{m,n})$ consists of the vertex sets $\{u, v, u_i, v_j / 1 \le i \le m, 1 \le j \le n\}$ with $|u_i| > 1, |v_j| > 1$ where u_i and v_j are the pendent vertices such that $\mu(v,u) > 0$, $\mu(u, u_m) > 0$, $\mu(v, v_n) > 0$ and

$\mu(u_i, u_{i+1}) = 0 \quad for\ 1 \le i \le m$

$\mu(v_j, v_{j+1}) = 0 \quad for\ 1 \le j \le n.$

Definition: 2.8 (Fuzzy Helm Graph)

A helm graph is a Fuzzy Helm graph $(\tilde{H}_n)$ consists of two vertex sets U and V with $|U| > 1$ and $|V| = 1$ such that $\mu(v, u_i) > 0$, $\mu(u_i, u_{i+1}) > 0$ and $\mu(u_{n+1}, u_{n+2}) = 0$ for $1 \le i \le n.$

Definition: 2.9

A family $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_k\}$ of fuzzy sets on $V \cup E$ is called a k- fuzzy total coloring of Fuzzy Bistar graph $\hat{\tilde{B}} = (V, \sigma, \mu)$ if

(a) $\vee \gamma(v) = \sigma(v)$ for every $v \in V$ and $\max_i \gamma_i(uv) = \mu(uv)$ for every $uv \in E$.

(b) $\gamma_i \wedge \gamma_j = 0$

(c) For every adjacent vertices u, v of $\min\{\gamma_i(u), \gamma_i(v)\} = 0$ and for every incident edges $\min\{\gamma_i(v_j v_k) / v_j v_k$ are set of incident edges from the vertex $v_j, j = 1,2, ...|V|$ }.

(d) $\mu(uu_i) \le \min\{\sigma(u), \sigma(u_i)\}$ and $\mu(vv_j) \le \min\{\sigma(v), \sigma(v_j)\}$ for $1 \le i \le m, 1 \le j \le n$

Definition: 2.10

A family $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_k\}$ of fuzzy sets on $V \cup E$ is called a k- fuzzy total coloring of Fuzzy Helm graph $\hat{\tilde{H}} = (V, \sigma, \mu)$ if

(a) $\vee \gamma(v) = \sigma(v)$ for every $v \in V$ and $\max_i \gamma_i(uv) = \mu(uv)$ for every $uv \in E$.

(b) $\gamma_i \wedge \gamma_j = 0$

(c) For every adjacent vertices u, v of $\min\{\gamma_i(u), \gamma_i(v)\} = 0$ and for every incident edges $\min\{\gamma_i(v_j v_k) / v_j v_k$ are set of incident edges from the vertex $v_j, j = 1,2, ...|V|$ }.

(d) $\mu(u_i u_j) \le \min\{\sigma(u_i), \sigma(u_j)\}$ and $\mu(vu_i) \le \min\{\sigma(v), \sigma(u_i)\}$ for $1 \le i \le m, 1 \le j \le n$

Total Coloring of Fuzzy Bistar Graph:

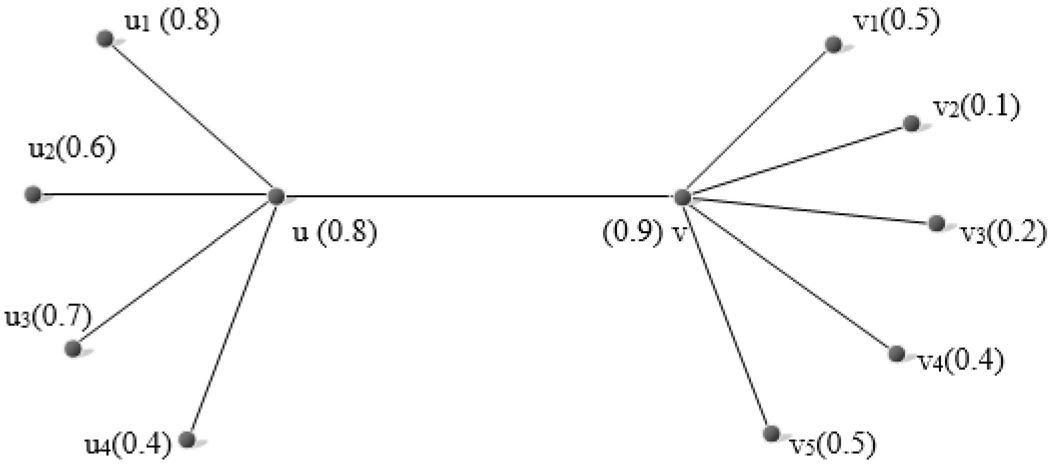
Consider the following example $\hat{\tilde{B}} = (V, \sigma, \mu)$ with vertex set

$V_1 = \{u, u_1, u_2, u_3, u_4\}$

$V_2 = \{v, v_1, v_2, v_3, v_4, v_5\}$ and edge set

$E_1 = \{uu_i / i = 1,2,3,4\}$

$E_2 = \{vv_j / j = 1,2,3,4,5\}$



$\hat{\tilde{B}}_{4,5}$. Here m=4 and n=5

The membership functions are defined as follows:

$\gamma_1(u;v_j) =$ u =0.8
0.4, j= 1,5
0.2, j=2
0.1, j=3
0.4, j=4

$\gamma_1(u_i u_j) =$ 0.2, ij= 5
0.2, ij= 24
0, otherwise

6. CONCLUSION

In this paper, we define an intuitionistic fuzzy Petersen graph, Strong intuitionistic fuzzy Petersen graph and also find a vertex and edge coloring for strong intuitionistic fuzzy Petersen graph with an illustrative example. Finally we get the chromatic number as a crisp number.

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