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Multiplicative Revan-Sombor Indices of Some Benzenoid Systems

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ARTICLE INFO	ABSTRACT
Published Online: 31 October 2022 Corresponding Author: **V.R.Kulli**	In this paper, we introduce the multiplicative modified Revan Sombor index of a graph. Also we compute the multiplicative Revan Sombor index, multiplicative modified Revan Sombor index of triangular benzenoids, benzenoid rhombus, benzenoid hourglass and benzenoid systems.

KEYWORDS: multiplicative Revan Somber index, multiplicative modified Sombor index, benzenoid system.

I. INTRODUCTION

Let G be a finite, simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Let $\Delta(G)(\delta(G))$ denote the maximum (minimum) degree among the vertices of G. The Revan vertex degree of a vertex u in G is defined as $r_G(u) = \Delta(G) + \delta(G) - d_G(u)$. The Revan edge connecting the Revan vertices u and v will be denoted by uv. For additional definitions and notations, the reader may refer to [1].

A topological index is a numerical parameter mathematically derived from the graph structure. In organic chemistry, topological indices have been found to be useful in chemical documentation, isomer discrimination, structure property relationships, structure activity relationships and pharmaceutical drug design. There has been considerable interest in the general problem of determining topological indices.

The first and second Revan indices of a graph G were introduced by Kulli in [2], and they are defined as

$$R_1(G) = \sum_{uv\in E(G)} \left[r_G(u) + r_G(v)\right],$$

$$R_2(G) = \sum_{uv\in E(G)} r_G(u) r_G(v).$$

Recently, some Revan indices were studied in [3, 4, 5].

The Revan Sombor index was proposed by Kulli et al. in [6] and defined it as

$$RSO(G) = \sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}.$$

Recently, some Sombor indices were studied in [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30].

The modified Revan Sombor index was introduced by Kulli in [31] and defined it as

$${}^{m}RSO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}.$$

In [32], the multiplicative Revan Sombor index of a graph G is defined as

$$RSOII(G) = \prod_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}.$$

We can express the multiplicative Revan Sombor index as

$$RSOII(G) = \prod_{uv\in E(G)} \sqrt{\left[\Delta+\delta-d_G(u)\right]^2 + \left[\Delta+\delta-d_G(v)\right]^2}.$$

Inspired by work on Revan and Sombor indices, we propose the multiplicative modified Revan Sombor index of a graph as follows:

The multiplicative modified Revan Sombor index of a graph G is defined as

$${}^{m}RSOII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}.$$

We can express the multiplicative modified Revan Sombor index as

$$ {}^{m}KGII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}}. $$

Recently, some multiplicative indices were studied in [33, 34, 35, 36].

In this paper, we compute the multiplicative Revan Sombor index, multiplicative modified Revan Sombor index of triangular benzenoids. benzenoid rhombus, benzenoid hourglass and benzenoid systems. For more information about these benzenoids see [37].

II. RESULTS FOR TRIANGULAR BENZENOID T_p

In this section, we consider the graph of triangular benzenoid T_p in which p is the number of hexagons in the base graph. Clearly T_p has $\frac{1}{2}p(p+1)$ hexagons. The graph of T_4 is shown in Figure 1.

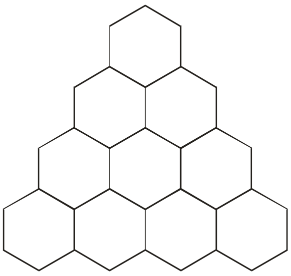

Figure 1. The graph of T_4

Let G be the graph of a triangular benzenoid T_p. By calculation, we find that G has $p^2 + 4p + 1$ vertices and $\frac{3}{2}p(p+3)$ edges. From Figure 1, we see that $\Delta(G) = 3$ and $\delta(G) = 2$. Also by calculation, we find that the edge set $E(G)$ can be divided into three partitions:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.

$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6p - 6$.

$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(u) = 3\}$, $|E_3| = \frac{3}{2}p(p-1)$.

In the following theorem, we compute the multiplicative Revan Sombor index of T_p.

Theorem 1. The multiplicative Revan Sombor index of T_p is given by

$$ RSOII(T_p)=\left(3\sqrt{2}\right)^6\times\left(\sqrt{13}\right)^{6p-6}\times\left(2\sqrt{2}\right)^{\frac{3}{2}p(p-1)}. $$

Proof. By definition, we have

$$ RSOII(T_p)=\prod_{uv\in E(T_p)}\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2} $$

$$ =\left[\sqrt{(3+2-2)^2+(3+2-2)^2}\right]^6 $$

$$ \times\left[\sqrt{(3+2-2)^2+(3+2-3)^2}\right]^{6p-6} $$

$$ \times\left[\sqrt{(3+2-3)^2+(3+2-3)^2}\right]^{\frac{3}{2}p(p-1)}. $$

After simplification, we obtain the desired result.

In the following theorem, we compute the multiplicative modified Revan Sombor index of T_p.

Theorem 2. The multiplicative modified Revan Sombor index of T_p is given by

$$ {}^{m}RSOII(T_p)=\left(\frac{1}{3\sqrt{2}}\right)^6\times\left(\frac{1}{\sqrt{13}}\right)^{6p-6}\times\left(\frac{1}{2\sqrt{2}}\right)^{\frac{3}{2}p(p-1)}. $$

Proof. By definition, we have

$$ {}^{m}KGII(T_p)=\prod_{uv\in E(T_p)}\frac{1}{\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}} $$

$$ =\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-2)^2}}\right)^6 $$

$$ \times\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-3)^2}}\right)^{6p-6} $$

$$ \times\left(\frac{1}{\sqrt{(3+2-3)^2+(3+2-3)^2}}\right)^{\frac{3}{2}p(p-1)} $$

gives the desired result after simplification.

III. RESULTS FOR BENZENOID RHOMBUS R_p

In this section, we consider the graph of benzenoid rhombus R_p which is obtained from two copies of a triangular benzenoid T_p by identifying hexagons in one of their base rows. The graph of benzenoid rhombus R_4 is presented in Figure 2.

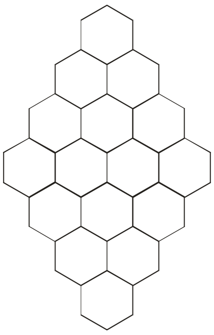

Figure 2. The graph of R_4

Let G be the graph of a benzenoid rhombus R_p. By calculation, we obtain R_p has $2p^2 + 4p$ vertices and $3p^2+4p-1$ edges. From Figure 2, we see that $\Delta(G)=3$ and $\delta(G)=2$. By calculation, we obtain that the edge set $E(R_p)$ can be divided into three partitions:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 8(p - 1)$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(u) = 3\}$, $|E_3| = 3p^2 - 4p + 1$.

In the following theorem, we compute the multiplicative Revan Sombor index of R_p.

Theorem 3. The multiplicative Revan Sombor index of R_p is given by

$$RSOII\left(R_p\right)=\left(3\sqrt{2}\right)^{6}\times\left(\sqrt{13}\right)^{8(p-1)}\times\left(2\sqrt{2}\right)^{3p^2-4p+1}.$$

Proof. By definition, we have

$$RSOII\left(R_p\right)=\prod_{uv\in E(R_p)}\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}$$

$$=\left[\sqrt{(3+2-2)^2+(3+2-2)^2}\right]^{6}$$

$$\times\left[\sqrt{(3+2-2)^2+(3+2-3)^2}\right]^{8(p-1)}$$

$$\times\left[\sqrt{(3+2-3)^2+(3+2-3)^2}\right]^{3p^2-4p+1}.$$

After simplification, we obtain the desired result.

In the following theorem, we compute the multiplicative modified Revan Sombor index of R_p.

Theorem 4. The multiplicative modified Revan Sombor index of R_p is given by

$$^{m}RSOII\left(R_p\right)=\left(\frac{1}{3\sqrt{2}}\right)^{6}\times\left(\frac{1}{\sqrt{13}}\right)^{8(p-1)}\times\left(\frac{1}{2\sqrt{2}}\right)^{3p^2-4p+1}.$$

Proof. By definition, we have

$$^{m}KGII\left(R_p\right)=\prod_{uv\in E(R_p)}\frac{1}{\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}}$$

$$=\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-2)^2}}\right)^{3}$$

$$\times\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-3)^2}}\right)^{3(p-1)}$$

$$\times\left(\frac{1}{\sqrt{(3+2-3)^2+(3+2-3)^2}}\right)^{\frac{3}{2}p(p-1)}$$

gives the desired result after simplification.

IV. RESULTS FOR BENZENOID HOURGLASS X_p

In this section, we consider the graph of benzenoid hourglass X_p which is obtained from two copies of a triangular benzenoid T_p by overlapping hexagons. The graph of benzenoid hourglass is shown in Figure 3.

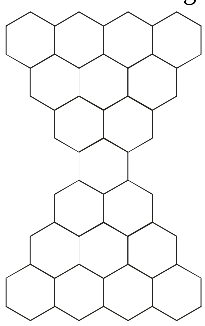

Figure 3. The graph of X_p

Let G be the graph of X_p. By calculation, we find that X_p has $2(p^2+4p-2)$ vertices and $3p^2+9p-4$ edges. From Figure 3, we see that $\Delta(G)=3$ and $\delta(G)=2$. By calculation, we obtain that the edge set $E(X_p)$ can be divided into three partitions:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 8$
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 12p - 16$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(u) = 3\}$, $|E_3| = 3p^2 - 3p + 4$.

In the following theorem, we compute the multiplicative Revan Sombor index of X_p.

Theorem 5. The multiplicative Revan Sombor index of X_p is given by

$$RSOII\left(X_p\right)=\left(3\sqrt{2}\right)^{8}\times\left(\sqrt{13}\right)^{12p-16}\times\left(2\sqrt{2}\right)^{3p^2-3p+4}.$$

Proof. By definition, we have

$$RSOII\left(X_p\right)=\prod_{uv\in E\left(X_p\right)}\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}$$

$$=\left[\sqrt{(3+2-2)^2+(3+2-2)^2}\right]^{8}$$

$$\times\left[\sqrt{(3+2-2)^2+(3+2-3)^2}\right]^{12p-16}$$

$$\times\left[\sqrt{(3+2-3)^2+(3+2-3)^2}\right]^{3p^2-3p+4}.$$

After simplification, we obtain the desired result.

In the following theorem, we compute the multiplicative modified Revan Sombor index of X_p.

Theorem 6. The multiplicative modified Revan Sombor index of X_p is given by

$$^{m}RSOII\left(X_p\right)=\left(\frac{1}{3\sqrt{2}}\right)^{8}\times\left(\frac{1}{\sqrt{13}}\right)^{12p-16}\times\left(\frac{1}{2\sqrt{2}}\right)^{3p^2-3p+4}.$$

Proof. By definition, we have

$$^{m}KGII\left(X_p\right)=\prod_{uv\in E\left(X_p\right)}\frac{1}{\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}}$$

$$=\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-2)^2}}\right)^{8}$$

$$\times\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-3)^2}}\right)^{12p-16}$$

$$\times\left(\frac{1}{\sqrt{(3+2-3)^2+(3+2-3)^2}}\right)^{3p^2-3p+4}$$

gives the desired result after simplification.

V. RESULTS FOR BENZENOID SYSTEMS

We focus on the chemical graph structure of a jagged rectangle benzenoid system, denoted by $B_{m,n}$ for all m, n, in N. Three chemical graphs of a jagged rectangle benzenoid system are depicted in Figure 4.

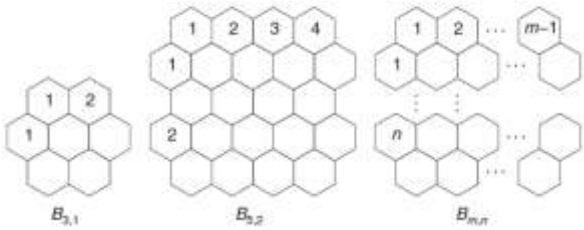

Figure 4

Let $G = B_{m,n}$. Clearly the vertices of G are either of degree 2 or 3, see Figure 4. By calculation, we obtain that G has $4mn + 4m + m – 2$ vertices and $6mn + 5m + n – 4$ edges. From Figure 4, we see that $\Delta(G)=3$ and $\delta(G)=2$. By calculation, we obtain that the edge set $E(B_{m,n})$ can be divided into three partitions:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2n+4$.

$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 4m+4n – 4$.

$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(u) = 3\}$, $|E_3| = 6mn + m – 5n – 4$.

In the following theorem, we compute the multiplicative Revan Sombor index of $B_{m,n}$.

Theorem 7. The multiplicative Revan Sombor index of $B_{m,n}$ is given by

$$RSOII\left(B_{m,n}\right)=\left(3\sqrt{2}\right)^{2n+4}\times\left(\sqrt{13}\right)^{4m+4n-4}\times\left(2\sqrt{2}\right)^{6mn+m-5n-4}.$$

Proof. By definition, we have

$$RSOII\left(B_{m,n}\right)=\prod_{uv\in E\left(B_{m,n}\right)}\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}$$

$$=\left[\sqrt{(3+2-2)^2+(3+2-2)^2}\right]^{2n+4}$$

$$\times\left[\sqrt{(3+2-2)^2+(3+2-3)^2}\right]^{4m+4n-4}$$

$$\times\left[\sqrt{(3+2-3)^2+(3+2-3)^2}\right]^{6mn+m-5n-4}.$$

After simplification, we obtain the desired result.

In the following theorem, we compute the multiplicative modified Revan Sombor index of $B_{m,n}$.

Theorem 8. The multiplicative modified Revan Sombor index of $B_{m,n}$ is given by

$$^{m}RSOII\left(B_{m,n}\right)=\left(\frac{1}{3\sqrt{2}}\right)^{2n+4}\times\left(\frac{1}{\sqrt{13}}\right)^{4m+4n-4}\times\left(\frac{1}{2\sqrt{2}}\right)^{6mn+m-5n-4}.$$

Proof. By definition, we have

$$^{m}KGII\left(B_{m,n}\right)=\prod_{uv\in E\left(B_{m,n}\right)}\frac{1}{\sqrt{\left[\Delta+\delta-d_G(u)\right]^2+\left[\Delta+\delta-d_G(v)\right]^2}}$$

$$=\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-2)^2}}\right)^{2n+4}\times\left(\frac{1}{\sqrt{(3+2-2)^2+(3+2-3)^2}}\right)^{4m+4n-4}$$

$$\times\left(\frac{1}{\sqrt{(3+2-3)^2+(3+2-3)^2}}\right)^{6mn+m-5n-4}$$

gives the desired result after simplification.

VI. CONCLUSION

In this paper, we have introduced the multiplicative modified Revan Sombor index of a graph. We have computed the multiplicative Revan Sombor index and the multiplicative modified Revan Sombor index for triangular benzenoids, benzenoid rhombus, benzenoid hourglass and benzenoid systems.

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