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A Study on Manifold Kaehleriain Space

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ABSTRACT

This work delves into the space-time theory of the 4-dimensional Kaehler manifold. Since the isotropic pressure, energy density, and energy momentum tensor all vanish in a perfect fluid Kaehler space-time manifold, we have established that this space-time manifold is an Einstein manifold and studied the Einstein equation with a cosmological constant in it. Finally, we demonstrated that, on a conformally flat, perfectly fluid Kaehler space-time manifold, the velocity vector field is infinitesimally spatially isotropic. Ideal fluid dilution to the point where only Ricci and minimal symmetry hold. We have proven that Kaehler space-time manifolds have either 0 scalar curvature or a connection between the respective rho and alpha vector fields via g(rho,alpha) = 4. To conclude, we have proved that a Kaehler space-time manifold cannot have both perfect fluidity and non-zero scalar curvature (weak Ricci symmetry).

KEYWORDS: Kaehlerian manifolds, H-projective, recurrent, curvature, Killing vector fields

INTRODUCTION

In this work, we computed metrics on Kaehlerian manifolds, complex hyper surfaces, and other manifolds implanted into special transformations with additive recurrent curvature features and holomorphic projective correspondences. Hereafter abbreviated as (M, g, F) is the Kaehler manifold of dimension $2n \geq 4$ with Riemannian metric $g = (g_{ij})$ and nearly complex structure $F = F_{ij}$, where $F_{ij} = -F_{ji}$. The formulas for the Riemannian and Ricci tensors are provided here.

$$R^{h}_{kji} = \partial_k \Gamma^{h}_{ji} - \partial_j \Gamma^{h}_{ki} + \Gamma^{h}_{ka}\Gamma^{a}_{ji} - \Gamma^{h}_{ja}\Gamma^{a}_{ki}$$

$$P_{ji} = R^{a}_{aji} \qquad (1.1)$$

Weyl projective curvature tensors with regard to Riemannian connection is recurrent with respect to quarter-symmetric metric connection in hyperbolic Kaehlerian manifold with Riemannian connection.

$$W_{rjkhPi} + W_{irkhPj} + W_{ijrhPk} + W_{ijkrPh} = 0$$

2. Hypersurfaces on the Kaehlerian manifold have an induced virtually contact metric structure. As an example, think about the local coordinate system $\{X^n\}$ in the 2n-dimensional differentiable manifold $\overline{M}^{2n}$. if for some $\overline{M}^{2n}$ there is a tensor field F^{k}_{λ} that satisfies

$$(2.1)\ F^{\mu}_{\lambda} F^{k}_{\mu} = -\delta^{k}_{\lambda}$$

The location of the tensor The structure of manifold M is the same as F, hence both M and F are said to be almost complex. In particular, it is well known that there is a Riemannian metric tensor satisfying for any nearly complex manifold M.

$$(2.2) G_{k\lambda}\, F^{k}_{\mu} F^{\lambda}_{v} = G_{\mu v}$$

The Hermitian metric is used here for our measurements. Since the above-described pair (F,G) shares many properties with the manifold M, we also refer to M as a nearly Hermitian manifold. If and only if this condition holds, then there exists a nearly complex structure F on a nearly Hermitian manifold.

$$(2.3)\ \overline{\nabla} F_{\lambda k} = 0$$

Only if the manifold is metric will the covariant differentiation with regard to the Hermitian Kaehlerian be represented by $\overline{\nabla}$.[1-3]

OBJECTIVES

1. The unique differential geometric features of Kaehlerian manifolds are our primary area of interest in this paper.
2. We next apply this expression to calculate the Ricci tensor on an extremely compact Kahlerian manifold.
3. Third, changing the Kaehlerian manifold to the Peterson-Codazzi one.

REVIEW OF LITERATURE

It is a peculiar object, with peculiar proportions and peculiar characteristics. Theoretically, M2n+ι must satisfy the condition that the structure group of its tangent bundle reduces to (7(n)xl, where U(n) is the real representation of the unitary group of n complex variables, in order for it to be considered a contact manifold. Using a collection of tensor fields denoted by f, S. Sasaki has defined the virtually touch manifold condition. Thus, the structure of an almost contact manifold is admitted by every orientable hypersurface of a nearly complex manifold. The study of hypersurfaces of nearly contact manifolds is one of the many fruitful aspects of this subject. Indeed, the present author and others, including M. Kurita, Y. Tashiro, S. Tachibana, and others, have proven various theorems on a particular hypersurface of an essentially complex manifold. We say that a hypersurface is contact if and only if the contact metric produced by the hypersurface is a contact metric on an essentially Hermitian manifold. [4-9]

Izumi and Kazanari formulated and computed the simplest holomorphic projective transformations on compact Kaehlerian manifolds. Projective analogues of holomorphic ammeters have also been the subject of research by Malave Guzman. Negi's curiosity on pseudo-analytic vectors on pseudo-Kaehlerian manifolds was piqued as a result of this. Negi et al. define and show an analytic HP-transformation for nearly Kaehlerian spaces. Measurements and calculations on a Kahlerian manifold coupled to H-projective recurrent curvature killing vector fields prove the existence of holomorphic characteristics of Einsteinian and constant curvature manifolds. Kaehlerian holomorphically projective recurrent curvature manifolds with almost complex topologies can be built by using the geometric features of the harmonic and scalar curvatures induced by the dominant vectorial fields [10, 12].

This article was written by M.M. Praveena, C.S. Bagewadi, and M.R. Krishnamurthy (2021). A Kählerian solison is a disturbance in the spacetime continuum. We also consider the scenario of solitons in a Kählerian space-time manifold that is substantially pseudo-symmetric. The 23 curvature tensors of projective, conharmonic, and conformal varieties are typically taken to be equal to zero within nearly pseudo symmetric Kählerian space-time manifolds. We find that the values of the cosmological constant, the energy density, the gravitational constant, and the pressure at an isotropic pressure of 25 all affect the stability, growth, and shrinking of solitons. [13]

RESEARCH METHODOLOGY

Secondary sources, such as books, educational and development publications, government papers, and print and online reference materials, were our primary means of acquiring knowledge about the Kahlerian Einstein manifold. In this work, we will focus on Kahle's Einstein manifold. Analytic holomorphic projective transformations on a Kahler-Einstein manifold have analytic vectors. If the associated vector of an analytic holomorphically projective transformation meeting the condition is not a unit vector, then the manifold is not an Einstein manifold, and vice versa for Kahlerian manifolds.

RESULT AND DISCUSSION

The following theorem states that any holomorphically projective Kaehlerian manifold can be transformed into a Peterson-Codazzi manifold. [14]

THEOREM 1: Given that Kn is a Kaehlerian manifold, we define x to be a holomorphically projective recurrent curvature killing vector field with a linked to vectorial field V.

$$L_x(\nabla_j \mathrm{P}_{ki} - \nabla_k \mathrm{P}_{ji}) = \left\{(n+2)\mathrm{R}^{\;\;\;a}_{jki} - \mathrm{P}_{ki}\delta^{\;a}_{ki} + \mathrm{P}_{ji}\delta^{\;a}_{k} - F^{\;a}_{i}\,\mathrm{H}_{ki} + F^{\;a}_{k}\,\mathrm{H}_{ji} + 2F^{\;a}_{i}\,\mathrm{H}_{jk}\right\}V_a. \quad (2.1)$$

Proof. The following is the result of applying the generic commutation relation to a tensor of type (0,2):

$$(L_x\nabla_j \mathrm{P}_{ki} - L_x\nabla_k \mathrm{P}_{ji}) - (\nabla_j L_x \mathrm{P}_{ki} - \nabla_k L_x \mathrm{P}_{ji}) = \left(L_x\Gamma^{\;a}_{kj}\right)\mathrm{P}_{ja} - \left(L_x\Gamma^{\;a}_{ji}\right)\mathrm{P}_{ka} \qquad (2.2)$$

Assuming instead that X is a holomorphically projective curvature transformation as in (1.5), we obtain:

$$L_x\Gamma^{\;a}_{ji} = \delta^{\;a}_{j}\,V_i + \delta^{\;a}_{i}\,V_j - F^{\;a}_{j}F^{\;h}_{i}\,V_h - F^{\;a}_{i}F^{\;h}_{j}\,V_h \qquad (2.3)$$

To add, the theorem can be treated (1.2)

$$L_x\mathrm{P}_{ji} = -(n+2)\nabla_j \mathrm{V}_i \qquad (2.4)$$

Using logic, we are able to determine $L_x\Gamma^{\;a}_{ki}$ and $L_x\, P_{ki}$.

By plugging in (2.3) and (2.4) into (2.2), we obtain:

$$\left(L_x\nabla_j \mathrm{P}_{ki} - L_x\nabla_k \mathrm{P}_{ji}\right) - \left(\nabla_j[-(n+2)\nabla_k \mathrm{V}_i] - \nabla_k[-(n+2)\nabla_j \mathrm{V}_i]\right) = \left(\delta^{\;a}_{k}\,V_i + \delta^{\;a}_{i}\,V_k - F^{\;a}_{k}F^{\;h}_{i}\,V_h - F^{\;a}_{i}F^{\;h}_{k}\,V_h\right)P_{ja} - \left(\delta^{\;a}_{j}\,V_i + \delta^{\;a}_{i}\,V_j - F^{\;a}_{j}F^{\;h}_{i}\,V_h - F^{\;a}_{i}F^{\;h}_{j}\,V_h\right)P_{ka}$$

THEOREM 2: We derive a formula for computing the Ricci tensor in a dense Kahlerian manifold Knconfessing holomorphic projective curvature transformations with corresponding vector V if Anthis is recurrent curvature.

Proof: Here we have: $\nabla_k \; R^h \quad = R^h \; V_k,$

Using the formula a = j and counting up from 1, we get:

$\nabla_k P_{ji} = P_{ji} V_k,$ ……………………………………………(1)

Although, we have

$$\nabla_k P_{ji} = \partial_k(P_{ji}) - \Gamma^a_{kj} P_{ai} - \Gamma^a_{ki} P_{ja}$$

Concerningg^{ij} then outcome is:

$\nabla_k P_{ji} = \partial_k(P_{ji})$……………………………………(2)

Applying (1) and (2), we get the following conclusions:

$\partial(P_{ji}) - P_{ji} \; \partial_k f = 0.$

This partial differential equation has a solution, the Ricci tensor, which is used to calculate the scalar curvature tensor. [15]

CONCLUSION

This paper demonstrates that, under certain conditions, the Ricci tensor in a hyperbolic Kaehlerian manifold is pure with regard to a quarter-symmetric metric connection.

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