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Integral Type Generalized Contraction Mappings in Modular Spaces

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ARTICLE INFO	ABSTRACT
Published Online: 16 December 2022 Corresponding Author: **A. S .Saluja**	In this paper we study the existence of common fixed point for ρ-compatible mapping satisfying a generalized quasi contraction condition of integral type in modular spaces Our results extend and generalize the results of Beygmohammadi and Razani [4] and Razani and Moradi [25].

KEYWORDS: Common fixed point, modular space, ρ-compatible, generalized quasi-contraction of integral type.
MSC (2010): 47H10, 54H25.

1. INTRODUCTION

In 1922, S. Banach, proved a contraction principle, which ensures the existence and uniqueness of a fixed point of a self map on complete metric space, under some appropriate conditions. This principle is known as 'Banach Fixed Point Theorem'. This theorem states that 'if T be a self mapping of a complete metric space (X,d) and if there exist a number c, with $0 \leq c < 1$, such that $d(Tx,Ty) \leq cd(x,y)$ for all x, y ∈ X, then T has a unique fixed point in X . During the last 80 years, this result was extended and generalize through a lot of fixed point and common fixed point theorems which have been established by many authors in different spaces by taking more general contractive conditions. In the year 1986, Jungck [9] introduced the notion of compatible mappings and utilized the same to improve commutativity condition in common fixedpoint theorems. This concept has been frequently employed to prove the existence of common fixed points. In 2002, Branciari [5] gave an analogue of Banach's contraction principle for an integral type inequality, which is stated as follows:

Let (X,d) be a complete metric space, k ∈ [0, 1), f : X →X a mapping such that for each x, y ∈ X,

$$\int_0^{d(f(x)-f(y))} \varphi(t)dt \leq k\int_0^{d(x,y)} \varphi(t)dt,$$

Where, $\varphi : R^+ \to R^+$be a Lebesgue integrable mapping which is summable, non-negative and for all ε > 0, $\int_0^{\varepsilon} \varphi(t)dt > 0$. Then f has a unique fixed point u ∈ X, such that for each x ∈X, $\lim_{n\to\infty} f^n x = u$.

In the year 1950, The notion of modular space, as a generalization of a metric space, was introduced by Nakano [21], later in 1959, which is redefined and generalized by Musielak and Orlicz [19]. Fixed point theorems in modular spaces, generalizing the classical Banach fixed point theorem in metric space, have been studied extensively. Razani and Maradi [25] studied fixed point theorems for ρ-compatible mappings of integral type in modular spaces, Beygmohammadi and Razani [4] proved the existence of common fixed point for mappings defined on a complete modular spaces satisfying contractive inequality of integral type.

In this paper we prove some common fixed point theorems for generalized quasi contractive mappings of integral type.

We start with a brief recollection of basic definitions and facts in modular spaces from [5], [8], [9], [10], [13], [14], [19], [25], [26] and [27].

2. PRELIMINARIES

Definition 2.1. Let X be a vector space over the field R (or C). A functional $\rho : X \to [0, \infty]$ is called a modular if for any arbitrary x and y in X, the following conditions are satisfied :

(i) ρ(x) = 0 if and only if x = 0,

(ii) $\rho(\alpha x) = \rho(x)$ for all scalar α with |∝| = 1,

(iii) $\rho(\alpha x + \beta y) \leq \rho(x) + \rho(y)$, whenever $\alpha, \beta \geq 0$ and $\alpha + \beta = 1$, If one replace (iii) by (iv)

(iv) $\rho(\alpha x + \beta y) \leq \alpha^s\rho(x) + \beta^s\rho(y)$, for $\alpha, \beta \geq 0$ and $\alpha^s + \beta^s = 1$, where s ∈ (0, 1] then, the modular ρ is called s–convex modular, and if s = 1, then ρ is called convex modular.

If ρ is modular in X, then the set defined by

$$X_\rho = \{x \in X : \rho(\lambda x) \to 0 \text{ as } \lambda \to 0\}$$

is called a modular space. Clearly, the modular space X_ρ is a subspace of space X.

Note that,ρ be an increasing function. If $0 < \alpha < \beta$, then property (iii) with y = 0 implies that

$$\rho(ax) = \rho(\frac{\alpha}{\beta}(\beta x)) \leq \rho(\beta x)$$

Definition 2.2. A modular ρ is called satisfy the Δ_2- condition if, $\rho(2x_n) \to 0$, as $n \to \infty$, whenever $\rho(x_n) \to 0$ as $n \to \infty$.

Definition 2.3. Let X_ρ be a modular space. Then,

(1) The sequence $\{x_n\}_{n\in N}$ is said to be ρ-convergent to $x \in X_\rho$, if $\rho(x_n - x) \to 0$, as $n \to \infty$.

(2) The sequence $\{x_n\}_{n\in N}$ is said to be ρ-Cauchy, if $\rho(x_n - x_m) \to 0$, as $n, m \to \infty$.

(3) A subset S of X_ρ is said to be ρ- complete, if each ρ- Cauchy sequence in S, is ρ- convergent in S.

Definition 2.4. Let S be subset of X_ρ and $f : S \to S$, then f is called a ρ- contraction if for each $x, y \in X_\rho$, there exists a $q < 1$, such that $\rho(f(x) - f(y)) \leq q\rho(x - y)$.

Definition 2.5. Let X_ρ be a modular space, where ρ satisfies the Δ_2- condition. Two mappings $S, f : X_\rho \to X_\rho$ are said to be ρ-compatible, if $\rho(Sfx_n - fSx_n)) \to 0$, as $n \to \infty$, whenever $\{x_n\}_{n\in N}$ be a sequence in X_ρ, such that $fx_n \to z$ and $Sx_n \to z$ for some $z \in X_\rho$.

Definition 2.6. Two self-maps $S, h : X_\rho \to X_\rho$ of a modular space X_ρ are (i, j, k) – generalized contraction of integral type, if there exists $0 < k < 1$ and $i, j \in R^+$ with $j > i$, such that

(2.6.1) $\int_0^{\rho(j(Sx-Sy))} \varphi(t)dt \leq k \int_0^{M(x,y)} \varphi(t)dt$ for all $x, y \in X_\rho$

Where, $M(x, y) = \max \{\rho(i(hx - hy)), \rho(i(Sx - hx)), \rho(i(Sy - hy)), \frac{\rho(i(Sy- hx))+ \rho(i(hy-Sx))}{2}, \frac{\rho(i(Sx - hx))[1 + \rho(i(Sy - hy))}{1 + \rho(i(hx - hy))}, \frac{\rho(i(hx - Sy))[1 + \rho(i(Sx - hy))}{1 + \rho(i(hx - hy))}\}$

and $\varphi : R^+ \to R^+$ be a Lebesgue integrable mapping which is summable, non-negative and

(2.6.2) $\int_0^c \varphi(t)dt > 0$, for all $c > 0$.

3. MAIN RESULT

Theorem 3.1. Let X_ρ be a ρ -complete modular space, where ρ satisfies the Δ_2-condition. Suppose $S, h: X_\rho \to X_\rho$ are (i, j, k) – generalized contraction of integral type such that $S(X_\rho) \subset h(X_\rho)$. If one of S or h is continuous, then S and h have a unique common fixed point.

Proof : Choose $\frac{j}{2} > i$ and let $\alpha \in R^+$ be the conjugate of $\frac{j}{i}$, i.e. $\frac{i}{j} + \frac{1}{\alpha} = 1$. Then $\frac{j}{2} > i$ implies that $\alpha i < j$.

Now, we choose an arbitrary point x_0 in X_ρ and construct inductively the sequence $\{Sx_n\}_{n\in N}$ as follows:

$$Sx_n = hx_{n+1} \text{ and } S(X_\rho) \subseteq h(X_\rho)$$

Thus, we have from (2.6.1)

$$\int_0^{\rho(j(Sx_{n+1}-Sx_n))} \varphi(t)dt \leq k \int_0^{M(x_{n+1}, x_n)} \varphi(t)dt$$

Where,

$M(x_{n+1}, x_n) = \max \{\rho(i(hx_{n+1} - hx_n)), \rho(i(Sx_{n+1} - hx_{n+1})), \rho(i(Sx_n - hx_n)), \frac{\rho(i(Sx_n- hx_{n+1}))+\rho(i(Sx_{n+1}- hx_n))}{2}, \frac{\rho(i(Sx_{n+1}-hx_{n+1}))[1+ \rho(i(Sx_n-hx_n))]}{1+ \rho(i(hx_{n+1}-hx_n))}, \frac{\rho(i(hx_{n+1}- Sx_n))[1+ \rho(i(Sx_{n+1}- hx_n))]}{1+ \rho(i(hx_{n+1}-hx_n))}\}$

$= \max \{\rho(i(hx_{n+1} - hx_n)), \rho(i(hx_{n+2} - hx_{n+1})), \rho(i(hx_{n+1} - hx_n)), \frac{\rho(i(hx_{n+1}- hx_{n+1})) + \rho(i(hx_n-hx_{n+2}))}{2}, \frac{\rho(i(hx_{n+2}-hx_{n+1}))[1+ \rho(i(hx_{n+1}-hx_n))]}{1+ \rho(i(hx_{n+1}-hx_n))}, \frac{\rho(i(hx_{n+1}-hx_{n+1}))[1+ \rho(i(hx_n-hx_{n+2}))}{1+ \rho(i(hx_{n+1}-hx_n))}\}$

(3.1.1) $M(x_{n+1}, x_n) = \max \{\rho(i(hx_{n+1} - hx_n)), \rho(i(hx_{n+2} - hx_{n+1})), \frac{\rho(i(hx_{n+2}-hx_n))}{2}\}$

Moreover, by $\alpha i < j$,

$\rho(i(hx_n - hx_{n+2})) = \rho(i(hx_n - hx_{n+2})) = \rho(i(Sx_{n-1} - Sx_{n+1}))$

$\backslash = \rho\left(\alpha\frac{i}{\alpha}(Sx_{n+1} - Sx_n) + i\frac{j}{j}(Sx_n - Sx_{n-1})\right)$

$\backslash \leq \rho(\alpha i(Sx_{n+1} - Sx_n)) + \rho(j(Sx_n - Sx_{n-1}))$

$\backslash \leq \rho(j(Sx_{n+1} - Sx_n)) + \rho(j(Sx_n - Sx_{n-1}))$

Then,

$$M(x_{n+1}, x_n) \leq \rho(j(Sx_n - Sx_{n-1}))$$

So that

(3.1.2) $\int_0^{\rho(j(Sx_{n+1}-Sx_n))} \varphi(t)dt \leq k \int_0^{\rho(j(Sx_n-Sx_{n-1}))} \varphi(t)dt \leq k^2 \int_0^{\rho(j(Sx_{n-1}-Sx_{n-2}))} \varphi(t)dt$

By induction, we have

(3.1.3) $\int_0^{\rho(j(Sx_{n+1}-Sx_n))} \varphi(t)dt \leq k^n \int_0^{\rho(j(Sx_1-Sx_0))} \varphi(t)dt$

On taking the limit $n \to \infty$,

we get $\lim_{n \to \infty} \rho(j(Sx_n - Sx_{n+1}) = 0$.

Now, suppose that $I < j' < 2i$.

Since ρ is an increasing function, then we have

(3.1.4) $\rho(j'(Sx_n - Sx_{n-1})) \leq \rho(j(Sx_n - Sx_{n+1})$, whenever, $j' < 2i \leq j$

On taking the limit $n \to \infty$, we get $\lim_{n \to \infty,} \rho(j'(Sx_n - Sx_{n+1}) = 0$, for $i < j' < 2i$.

Thus, we have

(3.1.5) $\lim_{n \to \infty,} \rho(j(Sx_n - Sx_{n+1}) = 0$, for any $j > i$

Now, we show that $\{Sx_n\}_{n\in N}$ is ρ - Cauchy in X_ρ.

If not, then there exists an $\varepsilon > 0$ and two subsequences {p(s)} and {q(s)} of integers, with $s \leq p(s) < q(s)$, such that

(3.1.6) $\rho(j(Sx_{p(s)} - Sx_{q(s)}) \geq \varepsilon$, for s = 1, 2, 3, . . .

Thus, we can assume that

(3.1.7) $\rho(j(Sx_{p(s)} - Sx_{q(s)-1})<\varepsilon$

In order to show that q(s) be the smallest number exceeding p(s) for which (3.1.6) holds and

$$\sum_s = \{n \in N: \exists\, p(s) \in N: \rho(i(Sx_n - Sx_{p(s)}) \geq \varepsilon$$

and n > p(s) ≥ s}.

Obviously, $\sum_s \neq \emptyset$ and since $\sum_s \subset$ N, then by well ordering principle, the smallest element of $\sum_s$ is denoted by p(s), and clearly (3.1.6) holds.

Then,

(3.1.8) $\int_0^{\rho(j(Sx_{p(s)}-Sx_{q(s)}))} \varphi(t)dt \leq k \int_0^{M(x_{p(s)}, x_{q(s)})} \varphi(t)dt,$

where,

(3.1.9) $M(x_{p(s)}, x_{q(s)}) = \max \{\rho\left(i(hx_{p(s)} - hx_{q(s)})\right), \rho\left(i(Sx_{p(s)} - hx_{p(s)})\right), \rho\left(i(Sx_{q(s)} - hx_{q(s)})\right),$

$\frac{\rho\left(i(Sx_{q(s)}-hx_{p(s)})\right) + \rho\left(i(hx_{q(s)}- Sx_{p(s)})\right)}{2},$

$\frac{\rho(i(Sx_{p(s)}-hx_{p(s)}))[1+ \rho\left(i(Sx_{q(s)}-hx_{q(s)})\right)}{1+ \rho\left(i(hx_{p(s)}-hx_{q(s)})\right)},$

$\frac{\rho(i(hx_{p(s)}-Sx_{p(s)}))[1+ \rho\left(i(Sx_{p(s)}- hx_{q(s)})\right)}{1+ \rho(i(hx_{p(s)}-hx_{q(s)}))}\}$

Note that

$$\rho\left(i(Sx_{p(s)-1} - Sx_{q(s)-1})\right) = \rho\left(i(Sx_{p(s)-1} - Sx_{p(s)} + Sx_{p(s)} - Sx_{q(s)-1})\right)$$

$$\leq \rho\left(\left(\alpha\frac{i}{\alpha}(Sx_{p(s)-1} - Sx_{p(s)})\right) + \left(i\frac{j}{j}(Sx_{p(s)} - Sx_{q(s)-1})\right)\right)$$

$$\leq \rho\left(\alpha i(Sx_{p(s)-1} - Sx_{p(s)})\right) + \rho\left(j(Sx_{p(s)} - Sx_{q(s)-1})\right)$$

On taking limit s→∞, using Δ_2- condition and (3.1.5), we get

$$\lim_{s\to\infty,} \rho\left(\alpha i(Sx_{p(s)-1} - Sx_{p(s)})\right) \to 0$$

and $\lim_{s\to\infty,} \rho\left(i(Sx_{q(s)-1} - Sx_{q(s)})\right) \to 0$

Therefore, as s→∞,

(3.1.10) $\int_0^{M(x_{p(s)}, x_{q(s)})} \varphi(t)dt \leq \int_0^{\varepsilon} \varphi(t)dt$

On the other hand, by the inequality (3.1.5), as s →∞,

(3.1.11) $\int_0^{\varepsilon} \varphi(t)dt \leq \int_0^{\rho\left(j(Sx_{p(s)}-Sx_{q(s)})\right)} \varphi(t)dt$

Therefore from (2.6.2), (3.1.5), (3.1.10) and (3.1.11), we have

$$\int_0^{\varepsilon} \varphi(t)dt \leq \int_0^{\rho\left(j(Sx_{p(s)}-Sx_{q(s)})\right)} \varphi(t)dt \leq k\int_0^{M(x_{p(s)}, x_{q(s)})} \varphi(t)dt \leq k\int_0^{\varepsilon} \varphi(t)dt$$

Which is a contradiction.

Therefore, by Δ_2- condition the sequence $\{Sx_n\}_{n\in N}$ is ρ -Cauchy.

Then by the ρ -completeness of X_ρ, there exists a point u∈ X_ρ, such that

$$\rho(j(Sx_n - u))\to 0, \text{ as } n \to\infty,.$$

Now, we show that u be a fixed point of S.

If S is continuous, then S^2x_n→Su and Shx_n→Su.

Since, $\rho((hSx_n - Shx_n))\to 0$, then by ρ- compatibility, hSx_n→Su .

Note that,

$$\int_0^{\rho(j(Sx_n-S^2x_n))} \varphi(t)dt \leq k \int_0^{M(x_n,Sx_n)} \varphi(t)dt$$

Where,

$M(x_n, Sx_n) = \max \{\rho(i(hx_n - hSx_n)), \rho(i(Sx_n - hx_n)), \rho(i(hSx_n - SSx_n)),$

$\frac{\rho(i(SSx_n- hx_n))+ \rho(i(Sx_n-hSx_n))}{2}, \frac{\rho(i(Sx_n-hx_n))[1+ \rho(i(SSx_n-hSx_n))}{1+ \rho(i(hx_n-hSx_n))},$

$\frac{\rho(i(hx_n- SSx_n))[1+ \rho(i(Sx_n-hSx_n))}{1+ \rho(i(hx_n-hSx_n))}\}$

Limit n →∞, yields $\int_0^{\rho(j(u-Su))} \varphi(t)dt \leq k \int_0^{\rho(j(u-Su))} \varphi(t)dt$

And so, Su = u.

Since $S(X_\rho) \subset h(X_\rho)$, then there exists a point $w\in X_\rho$, such that

u = Su = hw.

Now, we have

$$\int_0^{\rho(j(S^2x_n-Sw))} \varphi(t)dt \leq k \int_0^{M(Sx_n,w)} \varphi(t)dt$$

Where,

$M(Sx_n, w) = \max \{\rho(i(hSx_n - hw)), \rho(i(S^2x_n - hSx_n)), \rho(i(Sw - hw)),$

$\frac{\rho(i(Sw- hSx_n))+ \rho\left(i(hw- S^2x_n)\right)}{2},$

$\frac{\rho(i(S^2x_n -hSx_n))[1+ \rho(i(Sw-hw))}{1+ \rho(i(hSx_n -hw))},$

$\frac{\rho(i(hSx_n - Sw))[1+ \rho\left(i(S^2x_n - hw)\right)}{1+ \rho(i(hSx_n -hw))}\}$

On taking the limit n →∞, we get

$$\int_0^{\rho(j(u-Sw))} \varphi(t)dt \leq k \int_0^{\rho(j(u-Sw))} \varphi(t)dt$$

Thus, u = Sw = hw and hence hu = hSw = Shw = Su = u

Moreover, if h is continuous instead of S, by a similar proof as above, we have hu = Su = u.

Now, to prove the uniqueness of common fixed point, let v be another common fixed point of S and h. Then,

$M(u, v) = \max \{\rho(i(u - v)), \rho(i(u - u)), \rho(i(v - v)), \frac{\rho(i(v-u))+ \rho(i(v-u))}{2},$

$\frac{\rho(i(u-u))[1+ \rho(i(v-v))}{1+ \rho(i(u-v))}, \frac{\rho(i(u- v))[1+ \rho(i(u-v))}{1+ \rho(i(u-v))}\}$

$= \rho(i(u - v))$

Therefore,

$$\int_0^{\rho(j(u-v))} \varphi(t)dt \leq k \int_0^{\rho(j(u-v))} \varphi(t)dt$$

Which implies, u = v.

4. GENERALIZATION

Now, we prove another version of above theorem 3.1. We need the following lemma [27].

Lemma 4.1 [27] Let t > 0, $\varphi(t)dt <$ t if and only if $\lim_{n\to\infty} \varphi^n(t) = 0$, where φ^n denotes the n- times repeated compositions of φ with itself.

Theorem 4.2. Let X_ρ be a ρ - complete modular space, where ρ satisfies the Δ_2-condition. Suppose i, j ∈ R^+ with j > i and S, h : $X_\rho \to X_\rho$, such that $S(X_\rho) \subset h(X_\rho)$ and

(4.2.1) $\int_0^{\rho(j(Sx-Sy))} \varphi(t)dt \leq \varphi(\int_0^{(M(x,y)} \varphi(t)dt)$, for all x, y $\in X_\rho$, where

$$M(x,y) = \max\{\rho(i(hx-hy)), \rho(i(Sx-hx)), \rho(i(Sy-hy)), \frac{\rho(i(Sy-hx))+\rho(i(hy-Sx))}{2}, \frac{\rho(i(Sx-hx))[1+\rho(i(Sy-hy))}{1+\rho(i(hx-hy))}, \frac{\rho(i(hx-Sy))[1+\rho(i(Sx-hy))}{1+\rho(i(hx-hy))}\}$$

And φ : $R^+ \to R^+$ be a continuous nondecreasing and right continuous function such that $\varphi(t)dt <$ t for any t > 0. If one of h or S is continuous, then h and S have a unique common fixed point.

Proof. As in the proof of Theorem 3.1, we have from (3.1.2)

$$\int_0^{\rho(j(Sx_{n+1}-Sx_n))} \varphi(t)dt \leq \varphi(\int_0^{\rho(j(Sx_n-Sx_{n-1}))} \varphi(t)dt)$$

$$\leq \varphi^2 \int_0^{\rho(j(Sx_{n-1}-Sx_{n-2}))} \varphi(t)dt$$

By induction,

$$\int_0^{\rho(j(Sx_{n+1}-Sx_n))} \varphi(t)dt \leq \varphi^n \int_0^{\rho(j(Sx_1-Sx_0))} \varphi(t)dt$$

Taking the limit n →∞, then yields by lemma 4.1, we get

$$\lim_{n\to\infty} \int_0^{\rho(j(Sx_{n+1}-Sx_n)} \varphi(t)dt \leq 0.$$

Using the same method of Theorem 3.1, S and h have a unique common fixed point.

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