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Generalised Girth Domination Number of Graphs

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Abstract:

The concept of complete graphs with real life application was introduced in[17] and the Forbidden pairs and the existence of a dominating cycle was introduced in[19] . In this paper, We introduce a new domination parameter called girth domination number , That is, if all the edges of the girth graph are the edges of any other cycles in a graph G and let G is a connected graph then C_{ni} is the girth graph of G if $C_{ni} \leq C_{nj}$, i≠j. A subset S of V of a non trivial graph G is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating set in G. A subset S of V of a nontrivial graph G is said to be girth dominating set, if every vertex in V-S is adjacent to at least one vertex of girth graph is called the girth dominating set. The minimum cardinality taken over all girth dominating set is called the girth domination number and is denoted by $\gamma_g(G)$.We determine this number for some standard graphs and obtain bounds for general graphs. Its relationship with other graph theoretical parameters are also investigated.

Mathematics Subject Classification: 05C69

Keywords: *Dominating set, Domination number ,Girth dominating set and Girth domination number,*

1 Introduction:

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge[3] in 1958 and 1962 and O.Ore[8] in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in [7] listed over 1200 papers related to domination in graphs in over 75 variation.

Throughout this paper , G(V , E) a finite , simple , connected and undirected graph where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G has n vertices and m edges. Degree of a vertex v is denoted by d(v), the *maximum degree* of a graph G is denoted by Δ(G). Let C_n a cycle on n vertices , P_n a path on n vertices by and a complete graph on n vertices by K_n. A graph is *connected* if any two vertices are connected by a path . A maximal connected subgraph of a graph G is called a *component* of G .The *number of components* of G is denoted by ω(G). The *complement* $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G. A tree is a connected acyclic graph. A *bipartite graph* is a graph whose vertex set can be divided into two disjoint sets V_1 and another in V_2 . A *complete bipartite graph* is a bipartite graph with partitions of order $|V_1|$ =m and $|V_2|$=n, is denoted by $K_{m,n}$. A star denoted by $K_{1,n-1}$ is a *tree* with one root vertex and n-1 pendant vertices. A bistar , denoted by B(m,n) is the graph obtained by joining the root vertices of the stars denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A *wheel* graph denoted by W_n is a graph with n vertices formed by connecting a single vertex to all vertices of C_{n-1}. A *Helm* graph denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle of W_n.

The *chromatic number* of a graph G denoted by χ(G) is the smallest number of colors needed to colour all the vertices receive different colours . For any real number x , $\lceil x \rceil$ denotes the largest integer greater than or equal to x and $\lfloor x \rfloor$ the smallest integer less than or equal to x. A Nordhaus- Gaddum – type result is a lower or upper bound on the sum or product of a parameter of a graph and its complement. Throughout this paper, we only consider undirected graphs vertices of a graph G in which adjacent with no loops .The basic definitions and concepts used in this study are adopted from[11].

Given a graph G = (V(G) , E(G)) , the cardinality $|V(G)|$ of the vertex set V(G) is the order of G is n. The distance d_G(u ,v) between two vertices u and v of G is the length of the *shortest path* joining u and v .If d_G(u ,v) = 1, u and v are said to be adjacent.

For a given vertex v of a graph G , The open neighbourhood of v in G is the set N_G(v) of all vertices of G that are adjacent to v.

The degree deg_G(v) of v refers to$|N_G(v)|$, and $\Delta(G) = \max\{deg_G(v) : v \epsilon V(G)\}$. The closed neighbourhood of v is the set N_G[v] = N_G(v) ∪ v for $S \subseteq V(G)$, N_G(S) =$\cup_{v \epsilon s} N_G(v)$ and $N_G[v]$= N_G(S)∪ S. If $N_G[v]$==V(G), then S is a dominating set in G. The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $\gamma(G)$.

Definition [18]:In a connected graph G, a chord of a spanning tree T is a line of G which is not in T.Clearly the subgraph of G consists of T and any chord of T has exactly one cycle.

Definition [18]:If T is a regular of degree 2,every component is a cycle and regular graphs of degree 3 are called cubic.

Definition [18]:If all the edges of the girth are the edges of any other cycles in a graph G.

Theorem[18]:Let x be a line of a connected graph G, The following statements are equivalent(1)x is a bridge of G.(2)x is not on any cycle of G.(3)There exist points u and v of G s.t the line x is on every path joining u and v.(4)There exists a partition of v into subsets U and W s.t for any points u∈Uand w∈W the line x is on every path joining u and w.

Theorem[18]:Let G be a connected graph with atleast three points. The following statements are equivalent. (1)G is a block (2)Every two points of G lie on a common cycle (3)Every point and line of G lie on a common cycle (4) Every two lines of G lie on a common cycle (5)Given two points and one line of G, there is a path joining the points which contains the line (6)For every three distinct points of G, There is a path joining any two of them which contains the third.

A dominating set S in a graph G is an independent dominating set if for every pair of distinct vertices u and v in S, u and v are non adjacent in G the minimum cardinality $\gamma_i(G)$ of an independent dominating set in G is called the *independent domination number* of G.

Definition[14]The Corona G $\circ H$ of a graphs G and H is the graph obtained by taking one copy of G and $|V(G)|$ copies of H and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H . It is customary to denote by H_v that copy of H whose vertices are adjoined with the vertex v of G. In effect G $\circ H$ is composed of the subgraphs H_v+v joined together by the edges of G. Moreover V(G$\circ H) = \cup_{v \epsilon V(G)} V(H_v + v)$

2.MAIN RESULTS

Definition:2.0: A set S⊂V(G) is called a girth dominating set of G if every vertex in V-S is adjacent to at least one vertex in the girth graph of G. The minimum cardinality of a girth dominating set of G is called girth domination number of G denoted by $\gamma_g(G)$.

Example 2.1: For any graph $|G|$=K_4 =$C_3 + v = 4$ and $\cup N(v_i) = C_3$ has girth dominating set of G with $\gamma_g(G)$=n-1=3 for n= 4 if Max{d(u_i, u_j)} ≥n-2,$i \neq j$ where $u_i \in C_3$.

Result 2.2: For any graph |G| =K_n =$C_{n-2} + v = n$, $C_{n-2} = n - v$and $\cup N(v_i) = C_{n-2}$ has girth dominating set of G with $\gamma_g(G)$ ≥n-2 for n ≥ 5 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the girth dominating set and its $\gamma_g(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4,Similarly we can have the girth dominating set with $\gamma_g(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k$, for every $v_i \in V - S$.

Example 2.3: For any graph $|G|$=$K_n \odot K_{1,n-1}$=$[C_{n-1} + v] \odot K_{1,n-1}$ = $2n - 1$, $[C_{n-1}] \odot K_{1,n-1} + v = 2n - 1$ and $\cup N(v_i) = C_{n-1}$ has a girth dominating set of G with $\gamma_g(G)$=n-1 for n = 4 and we have $\cup N(v_i) = C_{n-1}$ has girth dominating set of G with $\gamma_g(G)$ ≥n-2 for n= 5 for every $v_i \in V - S$ and N(V-S) ≠V-S. Since if Max{d(u_i, u_j)} ≥ n-3 and $(u_i, u_j) \in C_{n-2}$ then we can have the girth dominating set and its $\gamma_g(G)$=3 and $|N(u_i) \cap (V - S)| \geq 1$,i≠1. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4,Similarly we can have the girth dominating set with $\gamma_g(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k$, k=3,4....

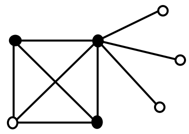

Fig1: $|G|$=$K_n \odot K_{1,n-1}$

Result 2.4: For any graph $|G|$=K_n =$C_{n-2} + 2v = n$, $C_{n-2} = n - 2$ and $\cup N(v_i) = C_{n-2}$ has girth dominating set of G with $\gamma_g(G)$ ≥n-2 for n≥ 5 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the girth dominating set and its $\gamma_g(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4,Similarly we can have the girth dominating set with $\gamma_g(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k$, for every $v_i \in V - S$.

Result 2.5: For any graph $|G|$=$K_n - 5e = |(C_{n-3} + 3v) - 5e|$=$K_6 - 5e = n$,$(C_{n-3} + 3v) = n$ and $C_{n-3} = n - 3$ with $\cup N(v_i) = C_{n-3}$has girth dominating set of G with $\gamma_g(G)$ ≥n-3 for n≥ 6 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the girth dominating set and its $\gamma_g(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4,Similarly we can have the girth dominating set with $\gamma_g(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k$, for every $v_i \in V - S$

Result 2.6: For any graph $|G|$=$K_n - 2e = (C_{n-3} + 3v) - 2e$=$K_6 - 2e = n$,$(C_{n-3} + 3v) = n$ and $C_{n-3} = n - 3$ with $\cup N(v_i) = C_{n-3}$ has girth dominating set of G with $\gamma_g(G)$ ≥n-3 for n≥ 6 for every $v_i \in V - S$. Since if Max{d(u_i, u_j)} = 2 then we can have the girth dominating set and its $\gamma_g(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4,Similarly we can have the girth dominating set with $\gamma_g(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k$, for every $v_i \in V - S$.

Example 2.7: For any wheel graph G=W_n , n≥ 4 is a girth dominating set with $\gamma_g(G)$=3 since Max{d(u_i, u_j)} = 2 then we can have the girth dominating set and its $\gamma_g(G)$=3. and $|\cup N(v_i)| = 3$, for every $v_i \in V - S$

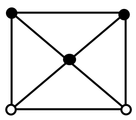

Fig 2: G=W_n , n≥ 4

Example 2.8: For any complete bipartite graph G=$K_{m,n}$ is a girth dominating set and its girth domination number $\gamma_g(G)$=4 for m,n ≥ 3 since Max{d(u_i, u_j)} = 3 and Min{d(u_i, u_j)} = 1 and $|\cup N(v_i)| = 4$, for every $v_i \in V - S$

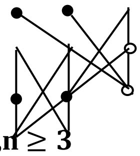

Fig 3: G=$K_{m,n}$; m,n ≥ 3

Example 2.9: For any Helm graph G=H_n is not a girth dominating set for n≥ 4 Since $|N(u_i) \cap (V - S)| \neq 1$, i≠1 where V-S =$(H_n - C_3)$

Example 2.10: For any graph G= $K_4 - e$ is a girth dominating set and its girth domination number $\gamma_g(G)$=3 with $|M|$=1 then we have $|N(u_i) \cap (V - S)| = 1$,i≠1

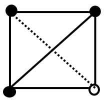

Fig 4: G= $K_4 - e$

Example 2.11: For any graph G= $K_5 - ie$; i=1,2,3 is a girth dominating set and its girth domination number $\gamma_g(G)$=3

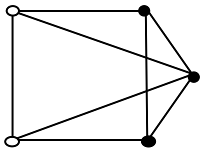

Fig 5: G= $K_5 - 2e$

Example 2.12: For any graph G= $K_5 - 4e$ is a girth dominating set and its girth domination number $\gamma_g(G)$=3 with $|M|$=1 then we have $|N(u_i) \cap (V - S)| = 1$,i≠1

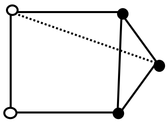

Fig 6: $K_5 - 4e$

Lemma 2.13: Let G is a connected graph then C_{ni} is the girth graph of G if $C_{ni} \leq C_{nj}$, i≠j.

Lemma 2.14: Let G be any graph and C_{ni} is the cycle then $\cap C_{ni}$ ≤e if $C_{ni} \leq C_{nj}$, i≠j.

Lemma2.15: Let G be any connected graph and C_{ni} is the cycle then $\cap C_{ni}$ ≤2v if $C_{ni} \leq C_{nj}$, i≠j.

Lemma 2.16: Let G be a connected graph and $\exists u_i \in C_{ni}$ is the girth dominated if $\cup_{i=1}^{n} N(v_i)$=u_i

Where v_i ∈V-S

Proof: Given G be a connected graph , By lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is girth dominated which gives $|N(u_i) \cap (V - S)| = 1$ and $N(v_i)$=u_i Where $u_i \in C_{ni}$ and v_i ∈V-S, Hence $\cup_{i=1}^{n} N(v_i)$=u_i

Lemma 2.17: For any graph $|G| = C_{n-1} + v = n$ then S=C_{n-1}=n-1 where C_{n-1} is the girth dominating set of a graph G and $v \in V - S$ if then $|N(u_i) \cap (V - S)| = 1$

Proof: For any graph $|G|$=n and if there exists a cycle $C_{n-1} \leq C_n$ that is $C_{n-1} + v$=n we have S=$C_{n-1} = n - v$ means that the graph is G-v, and the vertex v is non adjacent with any vertex of G. If v_i is adjacent with atleast one vertex .Hence $G = C_{n-1} + v$ then $|N(u_i) \cap (V - S)| = 1$

Lemma 2.18: For any graph $|G| = C_{n-1} + v = n$ then S=C_{n-1}=n-1 where C_{n-1} is the girth dominating set of a graph G and $v \in V - S$ if $|N(u_i) \cap (V - S)| = 2$.

Proof: For any graph $|G|$=n and if there exists a cycle $C_{n-1} \leq C_n$ that is $C_{n-2} + 2v$=n where $v \in V - S$ we have S=$C_{n-2} = n - 2$ means that the graph is G-2v, There fore the vertices v_i i≥ 2 is non adjacent with any two vertices of G. If v_i is adjacent with atleast 2 vertices then $|N(u_i) \cap (V - S)| = 2$.

Lemma 2.19: Let G be any complete graph and $\exists u_i \in C_{ni}$ ⊆S is the girth dominating set of G then $|N(u_i) \cap (V - S)| \geq 1$ where v_i ∈V-S.

Proof: Given G be a connected graph by lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is girth dominated ,which gives $\bigcup_{i=1}^{n} N(v_i)$=u_i where $u_i \in C_{ni}$, i= 1,2,..n and then For any graph $|G|$=n and if there exists a cycle $C_{n-1} \leq C_n$ that is $C_{n-1} + v$=n we have S=$C_{n-1} = n - v$ means that the graph is G-v, and the vertex v is non adjacent with any vertex of G. If v_i is adjacent with atleast one vertex then $|N(u_i) \cap (V - S)| \geq 1$ where v_i ∈V-S.

Theorem 2.20: For any graph G .Let S be a girth dominating set of G if S =$\{u_i\}$ for i=1,2....n. is $\gamma_g(G - v) \leq \gamma_g(G)$

Proof: For any graph G .Let S be a girth dominating set of G and V-S =$\{v_i\}$ for i=1,2,...n. we have Given G be a connected graph by lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is girth dominated ,which gives $\bigcup_{i=1}^{n} N(v_i)$=u_i where $u_i \in C_{ni}$, i= 1,2,..n and then $|N(u_i) \cap (V - S)| \geq 1$ where v_i ∈V-S.

If any one of v_i is removed then the edges incident on v_i is removed then it may not be the girth dominating set of G, Since $\bigcup N(v_i) = S$ which gives N(v_i) $\neq \{u_i\}$. There fore $\bigcup N(v_i) \neq S$. Hence $\gamma_g(G - v) \leq \gamma_g(G)$.

Theorem 2.21: For any graph G with girth cycle is girth dominated if it has atleast one matching

Proof : For any graph with girth cycle will be the dominating set S of G. If every vertex of V-S is adjacent to atleast one vertex of S. If not then there exist one v ∈ $V - S$ is not adjacent to one vertex of S then we made a matching to this vertex with S then it gives the girth dominating set of G with $|M|$=1.Hence we can find atleast one matching to find the girth dominating set of G, that is $|M|$ ≥1

Theorem 2.22: Every girth dominating set and it is of chromatic number $\chi(G)$ ≥3

Proof: Given the graph G is a girth dominating set since it is girth dominated the graph G must have at least a girth cycle of C_3.If every cycle C_n ,Since if Max{d(u_i, u_j)} = 2 then we can have the girth dominating set and its $\gamma_g(G)$=3 must have 3 colourable and every vertex of V-S is adjacent to at least one vertex of S have 4^{Th} vertex may have the 3^{rd} colour other than the colours which had in the girth cycle. Hence there must have at least 3 colours needed to colour the girth dominated graph ,that is $\chi(G)$ ≥3.

Lemma 2.23 : If $|N(u_i) \cap (V - S)| \geq 1$, Then $(u_1, u_2 \dots, u_{n-i})$ are the girth dominating set of any graph G.

Proof: For any graph $|G|$=n and if there exists a cycle $C_{n-i} \leq C_n$, $\left|C_{n-i} + \bigcup_{1}^{n-3} v_i\right|$ =n where $v_i \in V - S$ and we have S=C_{n-i} =n-v_i;S=n-(n-3), $i < n - 2$ means that the graph G-v_i. There fore any $v_i$is non adjacent with any vertex of G. If v_i is adjacent with C_{n-i} by at least one vertex then $|N(u_i) \cap (V - S)| \geq 1$ then $\cup N(v_i) = (u_1, u_2 \dots, u_{n-i})$ that is $\cup N(v_i) = S$.

Theorem 2.24: Let S be any girth dominating set in a graph G then their domination numbers are **(i)** $\gamma_g(G)$ ≥n-1if G≅ K_n =[$C_{n-1} + v$] for n≥ 4.**(ii)** $\gamma_g(G)$ ≥n-2 if G≅ K_n =[$C_{n-2} + 2v$] for n≥ 4. **(iii)** $\gamma_g(G)$ ≥n-3if G≅ K_n -

5e=[$(C_{n-3}+3v)-5e$] for n≥ 6.**(iv)** $\gamma_g(G)$ ≥n-3if G≅ K_n-2e =[$(C_{n-3}+3v)-2e$] for n≥ 6.**(v)** $\gamma_g(G)$ ≥n-1if G≅ K_n-e =[$(C_{n-1}+v)-e$] for n≥ 4 with $|M|$=1.

Proof: Given S be a girth dominating set of any graph G. By lemma 2.13 ,For any graph $|G|$=n and if there exists a cycle $C_{n-i} \leq C_n$, $|C_{n-i}+\cup_1^{n-3} v_i|$ =n where $v_i \in V-S$ and we have S=C_{n-i} =n-v_i; S=n-(n-3), $i \leq n-3$ means that the graph G-v_i. There fore any v_i is non adjacent with any vertex of G. If v_i is adjacent with C_{n-i} by atleast one vertex then $|N(u_i) \cap (V-S)| \geq 1$ then $\cup N(v_i) = (u_1, u_2 \dots, u_{n-i})$ that is, $\cup N(v_i) = S$.

Case(i) Since the graph $|G|$ ≅K$_n$=$|C_{n-i}+\cup_1^{n-3} v_i|$ for n≥ 4 and . Hence by lemma 2.23 if n= 4 , we have $\gamma_g(G)$=n-1 and Max{d(u_i, u_j)} ≥ n-2, $i \neq j$.

If there exists a cycle $C_{n-i} \leq C_n$, $|C_{n-i}+\cup_1^{n-3} v_i|$ =n where $v_i \in V-S$ and we have S=C_{n-i} =n-v_i;S=n-(n-3), $i \leq n-3$ means that the graph is G-v_i. Hence we must have $v_i \in V-S$ such that $|V-S| \geq 1$ and $\cup N(v_i) = (u_1, u_2, \dots \dots u_{n-i})$, $|\cup N(v)| = n-i$ where $u_i \in C_{n-i}$, i=1,2,....n-3 and $v_i \in V-S$ and $\gamma_g(G)$=n-i. Hence by lemma 2.23 if n≥ 5 , we have $\gamma_g(G)$ ≥n-2 and Max{d(u_i, u_j)} ≥ n-3, $i \neq j$ and $\gamma_g(G)$ ≥n-i and Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$.

Case(ii) Given graph $|G| \cong K_n$ =$|C_{n-i}+\cup_1^{n-3} 2v_i|$ for n≥ 4

Since the graph $|G|$ ≅K$_n$=$|C_{n-2}+\cup_1^{n-3} 2v_i|$ for n= 4 we have by lemma 2.23 if n= 4 , we cannot have S=n-(n-3), $i \leq n-3$ and also $\gamma_g(G)$=n-1 and Max{d(u_i, u_j)} ≥ n-2, $i \neq j$.

If there exists a cycle $C_{n-i} \leq C_n$, $|C_{n-i}+\cup_1^{n-3} 2v_i|$ =n where $v_i \in V-S$ and we have S=C_{n-i} =n-v_i;S=n-(n-3), $i \leq n-3$ means that the graph is G-v_i. Hence we must have $v_i \in V-S$ such that $|V-S| \geq 1$ and $\cup N(v_i) = (u_1, u_2, \dots \dots u_{n-i})$, $|\cup N(v)| = n-i$ where $u_i \in C_{n-i}$, i=1,2,....n-3 and $v_i \in V-S$ and $\gamma_g(G)$=n-i. Hence by lemma 2.23 if n≥ 5 , we have $\gamma_g(G)$ ≥n-2 and Max{d(u_i, u_j)} ≥ n-3, $i \neq j$ and $\gamma_g(G)$ ≥n-i and Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$.

Case(iii) Given graph $|G| \cong K_n$ -5e=[$|C_{n-i}+\cup_1^{n-3} 3v_i| - 5e$] for n = 6 and $\cup N(v_i) = (u_1, u_2, \dots \dots u_{n-i})$, $|\cup N(v)| = n-(n-3) = 3$ where $u_i \in C_{n-i}$, i=1,2,....n-3 and $v_i \in V-S$, we have $\gamma_g(G)$=n-3 and Max{d(u_i, u_j)} ≥n-4,$i \neq j$.

If there exists a cycle $C_{n-i} \leq C_n$, $|C_{n-i}+\cup_1^{n-3} 2v_i|$ =n where $v_i \in V-S$ and we have S=C_{n-i} =n-v_i;S=n-(n-3), $i \leq n-3$ means that the graph is G-v_i. Hence we must have $v_i \in V-S$ such that $|V-S| \geq 1$ and $\cup N(v_i) = (u_1, u_2, \dots \dots u_{n-i})$, $|\cup N(v)| = n-i$ where $u_i \in C_{n-i}$, i=1,2,....n-3 and $v_i \in V-S$ and $\gamma_g(G)$=n-i. Hence by lemma 2.23 if n≥ 6 , we have $\gamma_g(G)$ ≥n-3 and Max{d(u_i, u_j)} ≥ n-4, $i \neq j$ and $\gamma_g(G)$ ≥n-i and Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$.

Case (iv) Given graph $|G| \cong K_n$ -2e =[$|C_{n-i}+\cup_1^{n-3} 3v_i| - 2e$] for n ≥ 6 and $\cup N(v_i) = (u_1, u_2, \dots \dots u_{n-i})$, $|\cup N(v)| = n-i$ where $u_i \in C_{n-i}$, i=1,2,....n-3 and $v_i \in V-S$. Hence $\gamma_g(G)$=n-i and Max{d(u_i, u_j)} ≥n-(i+1), $i \neq j$. Hence if n≥ 6 , we have $\gamma_g(G)$=n-1 and Max{d(u_i, u_j)} ≥n-2, $i \neq j$. If $|N(u_i) \cap (V-S)| \neq 1$, there must have $|M| \leq n-3$, then we can have $|N(u_i) \cap (V-S)| = 1$. Hence $\gamma_g(G)$ ≥n-3 with $|M| \leq n-3$.

Case (v) Given graph G≅ K_n-e =[$|C_{n-i}+\cup_1^{n-3} v_i| - e$] for n≥ 4 and $\cup N(v_i) = (u_1, u_2, \dots \dots u_{n-i})$, $|\cup N(v)| = n-i$ where $u_i \in C_{n-i}$, i=1,2,....n-3 and $v_i \in V-S$. Hence $\gamma_g(G)$=n-i and Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$. Hence if for n≥ 5 , we have $\gamma_g(G)$=n-1 and Max{d(u_i, u_j)} ≥n-2,$i \neq j$. If $|N(u_i) \cap (V-S)| \neq 1$, there must have $|M| \leq n-3$, then we can have $|N(u_i) \cap (V-S)| = 1$ Hence $\gamma_g(G)$ ≥n-2 with $|M| \leq n-3$.

Theorem 2.25: If $\cup N(v_i) = \cup_{i=1}^{3} u_i$ and $|\cup N(v_i)| \geq 3$ then $\cup u_i$ are the girth dominating set of any graph G.

Proof: Given G be any graph and we have in $|G| = |C_{n-k} + \cup_{i=1}^{k} v_i|$=n then S=$C_{n-k} = n - \cup_{i=1}^{k} v_i$;n≥ $k + 3$ and S=n-k, S ≥ $k + 3 - k$, S ≥ 3 and if $\cup_{i=1}^{k} v_i$ is non adjacent with any vertices of C_{n-k} then we have If $|N(u_i) \cap (V - S)| \neq n - k$ .Hence N(v_i) must have adjacent with C_{n-k} by atleast one vertex which gives $|N(u_i) \cap (V - S)| = 1$ and already we have S≥ 3, There fore we have $\cup N(v_i) = \cup_{i=1}^{3} u_i$ and $|\cup N(v_i)| \geq 3$ and in general we have $\cup N(v_i) = (u_1, u_2, \ldots \ldots u_{n-i})$, $|\cup N(v)| = n - i$ where $u_i \in C_{n-i}$, i=1,2,.... and $v_i \in V - S$. Hence $\gamma_g(G)$ ≥n-i and Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$.

Theorem 2.26: If $|\cup N(v_i)| \geq k$ then $\cup_{i=1}^{k} u_i$ are the girth dominating set of G, Then any two vertices of C_n and Max{d(u_i, u_j)} $= k - 1$, k ≥ 3 that is i,j are non adjacent and d($u_{i,} u_j$) $\neq 1$.

Proof: Given $|\cup N(v_i)| \geq k$ then $\cup_{i=1}^{k} u_i$ are the girth dominating set of G and we have $|G| = |C_n + \cup_{i=1}^{r} v_i|$; n ≥ r+k and k ≥ 3 we have S=n-r which implies that S≥ $r + k - r$, that is S≥ k and if $\cup_{i=1}^{r} v_i$ is non adjacent with any vertices of C_n then we have If $|N(u_i) \cap (V - S)| \neq n - r$. Hence N(v_i) must have adjacent with C_{n-k} by atleast one vertex, that is $|N(u_i) \cap (V - S)| = 1$ and $|\cup N(v_i)| \geq k$ and we have S≥ k, $\cup N(v_i)$= $\cup_{i=1}^{k} u_i$ and we must have d($u_{i,} u_j$) $\neq 1$, i≠j and nonadjacent if d($u_{i,} u_j$) $= 2$ then we can have the girth dominating set and $\gamma_g(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_g(G)$=4, Similarly we can have the girth dominating set with $\gamma_g(G)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k$, for every $v_i \in V - S$.

Theorem 2.27: Every Connected graph is of girth dominating set $C_3$with $|M| \leq 3$.

Proof: Let $C_n$be the girth subgraph of G and S = C_n with V-S=G-S. If $|\cup N(v - S)| = 3$ then C_n is the girth dominating set of G with n≥ 3. If d(u_i, u_j)=1 where $u_i \in C_n$ then C_n is the girth subgraph of G . If d(u_i, u_j)≥2 then add a chord to the subgraph which gives d(u_i, u_j)=1 and $|\cup N(v - S)| = 3$ then it is the girth dominating set of G. If N(V-S)= C_3 and if $|\cup N(v - S)| < 2$ add one edge of matching with any one vertex of C_3 again if $|\cup N(v - S)| < 2$ add another vertex of C_3 ,Since the girth graph is of cycle 3 we can add maximum 3 matching ,then it becomes girth dominating set of G , That is $|M| \leq 3$. Otherwise the graph has no girth dominating set.

Corollary 2.28: Every connected graph is of girth dominating set C_n with $|M| \leq n$; n≥ 3.

Proof is obvious from Theorem 2.26 and 2.27.

Theorem 2.29: Every Corona graph of a girth graph G is not girth dominating set, that is $|\cup N(H_v)| \neq n - k$.

Proof: Let G be a girth graph of any number of vertices and S be the set of n-k vertices of girth cycle then $|N(u_i) \cap N(H_v)| \neq n - k$ and V-S=G-S $\cup H_v$ .Hence we have $\cup$ N($H_v) = V$ which implies that $|\cup N(H_v)| \neq (u_1, u_2, \ldots \ldots u_{n-k}) = n - k$. Hence for any graph $|G|$=n and if there exists a cycle $C_{n-1} \leq C_n$ that is $C_{n-i} + \cup_{i=k}^{n-3} iv$=n we have S=$C_{n-k} = n - iv$,i=1,2.....,k and k≥ 2 means that the graph is G-iv and the vertex v is non adjacent with any vertex of G then it gives $|N(u_i) \cap (V - S)| = 1$.

Hence every vertex of V-S is non adjacent to at least one vertex of S that is , $|\cup N(H_v)| = (u_1, u_2, \ldots \ldots u_{n-k}, v_i)$ $\neq n - k$ and $|N(u_i) \cap (N(H_v)| \neq n - k$ .Hence it is not a girth dominating set of G by the definition girth domination but if S=G then $|N(u_i) \cap (V - S)| = 1$ which gives the girth dominating set G.

Lemma 2.30: Let G be any connected graph with girth dominating set then $\cup N(S) = (S, V - S)$

Proof: For any graph G .Let S be a girth dominating set of G and V-S ={v_i} for i=1,2,...n. we have Given G be a connected graph by lemma 2.13, $\exists C_{ni} \leq C_{nj}$ and i≠j and given it is girth dominated ,which gives $\cup_{i=1}^{n} N(v_i)$=u_i where $u_i \in C_{ni}$, i= 1,2,..n and we have N(u_i)=(u_i, v_i) and ∪N(u_i)=($S, V - S$) and $|N(u_i) \cap (V - S)| \geq 1$ where v_i ∈V-S.

Theorem 2.31: Let S_1 and S_2 be any two girth dominating set of same order in G_1 and G_2 respectively then $[\cup N(S_1)\cdot \cup N(S_2)] = \cup N(S) = [S, V-S]$

Proof: Let G_1 and G_2 be any two graphs of n_1 and n_2 vertices and its girth dominating sets are S_1 and S_2 respectively. If we have $\cup N(S_1) = (S_1, V-S_1)$ and $\cup N(S_2) = (S_2, V-S_2)$ and its $[\cup N(S_1)\cdot \cup N(S_2)] = [S_1\cdot S_2, (V_1-S_1)\cdot(V_2-S_2)]$=[S, $V_1\cdot V_2 - S$] where $S_1.S_2$ =S and $v_1\cdot v_2$ =V which gives $\cup N(S) = [S, V-S]$

4.Conclusion:

In this paper we found an upper bound for the girth domination number and relationship between girth domination numbers of graphs and characterized the corresponding extremal graphs. Similarly girth domination numbers with other graph theoretical parameters can be considered.

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