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# t-Derivations in *BF*-Algebras

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| ARTICLE INFO | ABSTRACT |
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In this paper, we define the concept of $(\ell, r)$-$t$-derivation and $(r, \ell)$-$t$-derivation in *BF*-algebras. Then, we investigate some properties of $(\ell, r)$ and $(r, \ell)$-$t$-derivation in *BF*-algebras. Also, we obtain some properties of regular of $t$-derivation in *BF*-algebra. Finally, we investigate the properties of the concept in $BF_1$-algebras.

**KEYWORDS:** *BF*-algebra, $(\ell, r)$ and $(r, \ell)$- $t$-derivation, $t$-derivation

## I. INTRODUCTION

In 2002, Neggers and Kim [1] introduced a new algebraic structure called *B*-algebra. *B*-algebra was defined as a nonempty set $A$ with a constant 0 and a binary operation $*$ when satisfies the axioms: $(B1)$ $a * a = 0$, $(B2)$ $a * 0 = a$, and $(B3)$ $(a * b) * c = a * (c * (0 * b))$ for all $a, b, c \in A$. Walendziak [2] defined and study one of the generalizations of *B*- algebra: *BF*- algebra. It's satisfying $(B.1), (B.2)$ and $(BF)$ $0 * (a * b) = b * a$ for all $a, b \in A$.

The concept of derivative was first introduced in the study of rings and near rings [3]. However, as abstract algebras developed, the concept of derivation has been studied in other algebraic structures. Al-Shehrie [4] defined and studied the concept derivation of *B*-algebra. The result obtained is to define $(\ell, r)$ and $(r, \ell)$-derivation, regular in *B*-algebra and obtained the properties. Some development in the concept of $t$-derivation is presented in the following studies [5]–[7]. In 2014, Soleimani and Jahangiri [8] introduced a new notion called $t$-derivation of *B*-algebra. They defined a self-map $d_t$ in *B*-algebra $(A; *, 0)$ with $d_t(a) = a * t$ for all $a, t \in A$ then they defined the concept of $(\ell, r)$ and $(r, \ell)$-$t$-derivation on *B*-algebra. Recently, the concept of $t$-derivation on the other type of algebras was also studied by another author, see for examples [9] and [10].

In this paper, we define the concept of $(\ell, r)$ and $(r, \ell)$-$t$-derivation, also $t$-derivation in *BF*-algebra. Then, we investigate the properties.

## II. PRELIMINARIES

Some theories about *B*-algebra, *BF*-algebra also the derivation concept of *B*-algebra and $t$-derivation are given in this section.

**Definition 2.1.** [1] $A$ is a nonempty set with a constant 0 and with a binary operation $*$ is *B*- algebra if satisfies the axioms:
$(B1)$ $a * a = 0$,
$(B2)$ $a * 0 = a$,
$(B3)$ $(a * b) * c = a * (c * (0 * b))$
for all $a, b, c \in A$.

**Lemma 2.2.** [1] If $(A; *, 0)$ is *B*- algebra, then
(i) $0 * (0 * a) = a$,
(ii) $(a * b) * (0 * b) = a$,
(iii) $b * c = b * (0 * (0 * c))$,
(iv) $a * (b * c) = (a * (0 * c)) * b$,
(v) If $a * c = b * c$, then $a = b$,
(vi) If $a * b = 0$, then $a = b$,
For all $a, b, c \in A$.

**Proof.** The proof of this Lemma has been given in [1].

**Example 2.1.** Let $A = \{0, 1, 2, 3\}$ be a set with the following table:

Table 2.1: Cayley's table for $(A; *, 0)$.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 1 | 2 | 0 |

Then $(A; *, 0)$ is a *B*-algebra.

The concept of derivation in *B*-algebra was studied in [4]. Let $(A; *, 0)$ be a *B*-algebra, defined $a \wedge b = b * (b * a)$ for all $a, b \in A$.

**Definition 2.3.**[4] Let $(A;*,0)$ is a $B$-algebra. A self-map $d$ of $A$ is a $(\ell,r)$-derivation of $A$ if it satisfying

$$d(a*b)=(d(a)*b)\wedge\big(a*d(b)\big),$$

for all $a,b\in A$ and is a $(r,\ell)$-derivation of $A$ if it satisfying

$$d(a*b)=\big(a*d(b)\big)\wedge(d(a)*b),$$

for all $a,b\in A$. Then $d$ is called a derivation of $A$ if $d$ is a $(\ell,r)$ and also $(r,\ell)$-derivation of $A$.

**Definition 2.4.** [4] Let $(A;*,0)$ be a $B$-algebra. $d$ is a self-map of $A$ and it's said to be regular if $d(0)=0$.

**Definition 2.5.**[8] Let $(A;*,0)$ is a $B$-algebra. $d_t$ is a self-map of $A$, for any $t\in A$ defined by $d_t(a)=a*t$ for all $a\in A$.

**Definition 2.6.**[8] Let $(A;*,0)$ be $B$-algebra. For any $t\in A$, $d_t$ is a self-map of $A$ and it's called $(\ell,r)$-$t$-derivation of $A$ if it's satisfying

$$d_t(a*b)=(d_t(a)*b)\wedge\big(a*d_t(b)\big),$$

for all $a,b\in A$ and is called $(r,\ell)$-$t$-derivation of $A$ if it's satisfying

$$d_t(a*b)=\big(a*d_t(b)\big)\wedge(d_t(a)*b),$$

for all $a,b\in A$. Then $d_t$ is a $t$-derivation of $A$ when $d_t$ is a $(\ell,r)$ and also a $(r,\ell)$-$t$-derivation of $A$.

One of the other generalizations of $B$-algebra is $BG$-algebra. In the following, the definition of $BG$-algebra will be given.

**Definition 2.7.**[11] A $BG$-algebra is a non-empty set $A$ with a binary operation $*$ and a constant 0 if satisfies the following axioms $(B1),(B2)$ and $(BG)(a*b)*(0*b)=a$ for all $a,b\in A$.

**Definition 2.8.**[2] Let $A$ is a $BF$-algebra. For all $a,b\in A$ satisfying:

$(B1)$ $a*a=0$,

$(B2)$ $a*0=a$,

$(BF)$ $0*(a*b)=b*a$.

**Examples 2.2.** Given $A=\{0,x,y,z\}$ is a set with this following Cayley table:

**Table 2.2: Cayley table for $(A;*,0)$**

| $*$ | 0 | $x$ | $y$ | $z$ |
|---|---|---|---|---|
| 0 | 0 | $x$ | $y$ | $z$ |
| $x$ | $x$ | 0 | z | $y$ |
| $y$ | $y$ | $z$ | 0 | $x$ |
| $z$ | $z$ | $y$ | $x$ | 0 |

Based on Table 2.2, it can be proved that $A$ is $BF$-algebra.

**Proposition 2.9.**[2] Let $(A;*,0)$ is a $BF$-algebra, then the following hold:

(i) $0*(0*a)=a$,

(ii) If $0*a=0*b$, then $a=b$,

(iii) If $a*b=0$, then $b*a=0$.

For all $a,b\in A$.

**Proof.** The proof of this proposition has been proved in [2].

**Definition 2.10.**[2] Let $A$ be a $BF$-algebra. Then $A$ is $BF_1$ -algebra if it's a $BG$ - algebra.

**Theorem 2.11.**[2] Let $(A;*,0)$ be a $BF_1$ -algebra, then

(i) If $a*b=0$, then $a=b$,

(ii) If $a*b=c*b$, then $a=c$,

(iii) If $b*a=b*c$, then $a=c$

For all $a,b,c\in A$.

**Proof.** The proof of this Theorem has been given in [2].

**Theorem 2.12.**[2] If $(A;*,0)$ is a $B$-algebra, then $(A;*,0)$ is a $BF$ algebra.

**Proof.** This Theorem has been proven in [2].

**Theorem 2.13.**[2] For all $a,b,c\in A$, any $BF$-algebra which is that satisfies the identity $(a*c)*(b*c)=a*b$ is also a $B$-algebra.

**Proof.** This Theorem has been proven in [2].

## III. MAIN RESULT

The following is given the concept of $t$-derivation in $BF$-algebra and investigate the properties of $(\ell,r)$-$t$-derivation and $(r,\ell)$-$t$-derivation in $BF$-algebra and $BF_1$ -algebra.

**Definition 3.1.** For any $t\in A$, let $d_t$ be a self-map of $BF$-algebra $A$ and defined by $d_t(a)=a*t$ for all $a\in A$.

Let $(A:*,0)$ be a $BF$-algebra. Defined $a\wedge b=b*(b*a)$ for all $a,b\in A$.

**Definition 3.2.** Let $(A;*,0)$ is a $BF$-algebra, a self-map $d_t$ of $A$ for any $t\in A$ is called $(\ell,r)$-$t$-derivation of $A$, then

$$d_t(a*b)=(d_t(a)*b)\wedge(a*d_t(b))$$

for all $a,b\in A$ and is called $(r,\ell)$-$t$-derivation of $A$, then

$$d_t(a*b)=(a*d_t(b))\wedge(d_t(a)*b)$$

for all $a,b\in A$. $d_t$ is a $t$-derivation of $A$ if $d_t$ is a $(\ell,r)$-$t$-derivation and also a $(r,\ell)$-$t$- derivation of $A$.

**Definition 3.3.** A self-map $d_t$ of $BF$-algebra $A$ is regular if $d_t(0)=0$.

The following is given a property of $(\ell,r)$-$t$-derivation and property of $(r,\ell)$-$t$-derivation in $BF$-algebra and some properties of $BF_1$ -algebra.

**Theorem 3.4.** Let $A$ is a $BF$-algebra. $d_t$ is a $(\ell,r)$-$t$-derivation of $A$. Let $d_t$ regular, then we have $d_t(a)=d_t(a)\wedge a$ for all $a\in A$.

**Proof.** Suppose $(A;*,0)$ is a *BF*-algebra and $d_t$ is a $(\ell,r)$-*t*-derivation of *A*. Since $d_t$ is a regular, by $(B.2)$ obtained

$$\begin{aligned} d_t(a) &= d_t(a*0) \\ &= (d_t(a)*0)\wedge(a*d_t(0)) \\ &= d_t(a)\wedge(a*0) \\ d_t(a) &= d_t(a)\wedge a. \end{aligned}$$

Thus, it is proved that if $d_t$ regular, then $d_t(a)=d_t(a)\wedge a$ for all $a\in A$.

**Theorem 3.5.** Let $d_t$ is a $(r,\ell)$-*t*-derivation in *BF*- algebra *A* and $d_t$ regular, then we define $d_t(a)=a\wedge d_t(a)$ for all $a\in A$.

**Proof.** Suppose $d_t$ is a $(r,\ell)$-*t*-derivation in *BF*-algebra *A* and $d_t$ is a regular, then by $(B.2)$ obtained

$$\begin{aligned} d_t(a) &= d_t(a*0) \\ &= (a*d_t(0))\wedge(d_t(a)*0) \\ &= (a*0)\wedge d_t(a) \\ d_t(a) &= a\wedge d_t(a). \end{aligned}$$

Thus, it is proved that if $d_t$ is regular, then $d_t(a)=a\wedge d_t(a)$ for all $a\in A$.

**Lemma 3.6.** Let $(A;*,0)$ be a *BF₁*-algebra, then $d_t$ is an injective function.

**Proof.** Let $d_t(a)=d_t(b)$ for all $a,b\in A$, then by Theorem 2.11(ii) obtained

$$\begin{aligned} d_t(a) &= d_t(b) \\ a*t &= b*t \\ a &= b \end{aligned}$$

Thus, it is proved that $d_t$ is an injective function.

**Theorem 3.7.** Let $(A;*,0)$ is a *BF₁*-algebra. $d_t$ is a $(\ell,r)$-*t*-derivation of *A*. $d_t$ regular if and only if $d_t$ is an identity.

**Proof.** Suppose $d_t$ is a $(\ell,r)$-*t*-derivation in *BF₁*-algebra *A*. Because $d_t$ regular, then by $(B1),(B2)$ and Theorem 2.11 (iii) obtained

$$\begin{aligned} d_t(0) &= d_t(a*a) \\ 0 &= (d_t(a)*a)\wedge\big(a*d_t(a)\big) \\ \big(a*d_t(a)\big)*\big(a*d_t(a)\big) &= \big(a*d_t(a)\big) \\ &\quad *\big[\big(a*d_t(a)\big)*(d_t(a)*a)\big] \\ \big(a*d_t(a)\big) &= \big(a*d_t(a)\big)*(d_t(a)*a) \\ \big(a*d_t(a)\big)*0 &= \big(a*d_t(a)\big)*(d_t(a)*a) \\ &= d_t(a)*a \\ d_t(a)*d_t(a) &= d_t(a)*a \\ d_t(a) &= a. \end{aligned}$$

Thus, it is proved that $d_t$ is an identity. Then, on the other side, suppose that $d_t$ is an identity with $d_t(a)=a$ for all $a\in A$, then $d_t(0)=0$. So, it is proved that $d_t$ regular. This completes the proof.

**Theorem 3.8.** Let $d_t$ is a $(r,\ell)$-*t*-derivation in *BF₁*-algebra *A*. $d_t$ regular if and only if $d_t$ identity.

**Proof.** Suppose $d_t$ is a $(r,\ell)$-*t*-derivation of *BF₁*-algebra *A*. Because $d_t$ regular, then by Theorem 3.5, axioms $(B.1)$, $(B.2)$ and Theorem 2.11(iii) obtained

$$\begin{aligned} d_t(a) &= a\wedge d_t(a) \\ d_t(a)*0 &= d_t(a)*(d_t(a)*a) \\ 0 &= d_t(a)*a \\ d_t(a)*d_t(a) &= d_t(a)*a \\ d_t(a) &= a. \end{aligned}$$

Thus, it is proved that $d_t$ is an identity. Then, conversely suppose that $d_t$ is identity with $d_t(a)=a$ for all $a\in A$, then $d_t(0)=0$. Therefore, it is proved that $d_t$ is regular.

**Theorem 3.9.** Let $d_t$ is a *t*-derivation in *BF₁*-algebra *A*. $d_t$ regular if only if $d_t$ is identity.

**Proof.** Suppose $(A;*,0)$ be a *BF₁*-algebra, by Theorem 3.7 we have that if $d_t$ is a $(\ell,r)$-*t*-derivation of *A*, then $d_t$ regular if and only if $d_t$ identity and from Theorem 3.8 we have that if $d_t$ is a $(r,\ell)$-*t*-derivation of *A*, so $d_t$ is regular if and only if $d_t$ identity. Therefore, it is proved that if $d_t$ is a *t*-derivation in *A*, then $d_t$ regular if and only if $d_t$ identity.

## IV. CONCLUSION

In this paper, we can conclude in general in *BF*-algebra, the $(\ell,r)$-*t*-derivation's properties are different from $(r,\ell)$-*t*-derivation's properties. However, in *BF₁*-algebra they have the properties that prevail in $(\ell,r)$ and also in $(r,\ell)$-*t*-derivation: if and only if $d_t$ regular then $d_t$ identity. Therefore, if $d_t$ is a *t*-derivation in *A*, then $d_t$ regular if and only if $d_t$ identity in *BF₁*-algebra.

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