International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 11 Issue 01 January 2023, Page no. – 3132-3137

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v11i1.04



New Operations on Fuzzy Environment

J. Vadivel Naidu¹, K. Bharathivelan²

^{1,2}Department of Mathematics, CK College of Engineering and Technology, Cuddalore -608002, Tamilnadu,

| ARTICLE INFO | ABSTRACT | | |
|------------------------|---|--|--|
| Published Online: | In this paper we introduce the operations Φ and \bullet on interval valued intuitionistic fuzzy soft set | | |
| 09 January 2023 | and establish some properties of these operators. We also develop a decision-making method based | | |
| | on information measure of interval valued intuitionistic fuzzy soft set. | | |
| Corresponding Author: | | | |
| J. Vadivel Naidu | | | |
| KEYWORDS: Interval val | ued intuitionistic fuzzy soft set, Φ and $ullet$ operators, information measure, decision making | | |
| technique. | | | |

1. INTRODUCTION

The theory of intuitionistic fuzzy set was introduced by Atanassov [1, 4]. The concept of interval valued intuitionistic fuzzy set was developed by the same author [2, 3]. Soft set theory was first introduced by Molodtsov [13]. Motivated by these theories, the theory of fuzzy soft set [9, 10, 14, 16] and the theory of intuitionistic fuzzy soft set [11, 12] have been developed. Yang et al. [15] presented the concept of interval valued fuzzy set and soft set models [5, 6].

In this paper we define Φ and \bullet operators on interval valued intuitionistic fuzzy soft set (IVIFSS) and investigate some properties of these operators. We introduce the notion of an information measure on IVIFSS and study its basic properties. We also develop a decision-making method based on information measure of IVIFSS.

2. PRELIMINARIES

In this section we recall some definitions and results needed for our study.

Definition 2.1 [8]. Let X be a non-empty set. A fuzzy set A is characterized by its membership function $\mu_A: X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in X. A is completely determined by the set of tuples, $A = \{(x, \mu_A(x)): x \in X\}.$

Definition 2.2 [1]. Let X be a non empty set. An intuitionistic fuzzy set A is an object of the form

 $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \}, \text{ where the functions}$ $\mu_A : X \to [0,1] \text{ and } \nu_A : X \to [0,1] \text{ define the degree of membership and degree of non-membership of the element}$ $x \in X \text{ respectively, and for every}$

 $x \in X, 0 \le \mu_A(x) + \nu_A(x) \le 1.$

Let U be the universe of objects and E the set of parameters in relation to objects in U. Parameters are often attributes, characteristics or properties of objects.

Definition 2.3 [9]. Let F(U) be the set of all fuzzy subsets of U and $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to F(U)$.

For any parameter $\alpha \in A$, $F(\alpha)$ is a fuzzy subset of U and it is called fuzzy value set of the parameter $\alpha \in A, F(\alpha) = \{(x, \mu_{F(\alpha)}(x)) : x \in U\}, \mu_{F(\alpha)}(x)$

denotes the membership degree that an object x holds on the parameter α , where $x \in U$ and $\alpha \in A$.

Definition 2.4 [11]. Let U be an universe and E a set of parameters. Let P(U) denote the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. A pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F: A \rightarrow P(U)$.

Notation: For any set A of real numbers, we denote $\underline{A} = \inf A$ and $\overline{A} = \sup A$.

Definition 2.5 [2]. An interval valued intuitionistic fuzzy set on an universe X is an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},\$ where $\mu_A(x) = \left[\underline{\mu}_A(x), \overline{\mu}_A(x)\right]$ and $\nu_A(x) = \left[\underline{\nu}_A(x), \overline{\nu}_A(x)\right],\$ $\underline{\mu}_A(x) = \inf \mu_A(x), \overline{\mu}_A(x) = \sup \mu_A(x), \mu_A(x), \nu_A(x) : A - D([0,1])$ stands for the set of all closed subintervals of [0, 1] which satisfy the condition,

 $0 \le \sup \mu_A(x) + \sup \nu_A(x) \le 1.$

Definition 2.6 [7]. Let U be the universe and E a set of parameters. Let IVIFS(U) denote the set of all interval valued intuitionistic fuzzy sets over U and $A \subseteq E$. A pair (F, A) is an interval valued intuitionistic fuzzy soft set over U, where F is a mapping given by $F: A \rightarrow IVIFS(U)$. For any parameter $\alpha \in A, F(\alpha)$ is an interval valued intuitionistic fuzzy soft set.

Definition 2.7 [7]. The complement of an IVIFSS (F, A)

denoted by $(F, A)^c$ is defined as

$$(F, A)^c = \left\{ \left\langle x, \left[\underline{\nu}_A(x), \overline{\nu}_A(x) \right], \left[\underline{\mu}_A(x), \overline{\mu}_A(x) \right] \right\rangle : x \in U \text{ and } \alpha \in A \right\}$$

Definition 2.8 [17]. For

 $A \in IVIFS(U)$, $U = \{x_1, x_2, ..., x_n\}$, an information measure to indicate the degree of fuzziness of A is defined as

$$\stackrel{\rightarrow}{IM} \stackrel{([0,1])}{(A)} = \frac{1}{n} \sum_{i=1}^{n} \frac{\min\{\underline{\mu}_{A}(x_{i}), \underline{\nu}_{A}(x_{i})\} + \min\{\overline{\mu}_{A}(x_{i}), \overline{\nu}_{A}(x_{i})\}}{\max\{\underline{\mu}_{A}(x_{i}), \underline{\nu}_{A}(x_{i})\} + \max\{\overline{\mu}_{A}(x_{i}), \overline{\nu}_{A}(x_{i})\}}$$

3. Φ and \bullet operators on interval valued intuitionistic fuzzy soft set

In this section, we define Φ and \bullet operators on IVIFSS and study some properties of these operators.

Definition 3.1. Let (F, A) and (G, B) be two IVIFSS over the same universe U. Then the operation Φ on (F, A) and (G, B) is denoted by $(F, A)\Phi(G, B)$ and is defined as $(F, A)\Phi(G, B) = (H, C)$, where $C = A \cup B$ and $\forall \alpha \in C$

$$(\mathbf{H}, C) = \begin{cases} \left\{ \left\langle m, \left[\frac{\left(\underline{\mu}_{F(\alpha)}(m) + \underline{\mu}_{G(\alpha)}(m)\right)}{2}, \frac{\left(\overline{\mu}_{F(\alpha)}(m) + \overline{\mu}_{G(\alpha)}(m)\right)}{2}\right] \right\rangle | m \in U \right\} & \text{if } \alpha \in A \cap B \\ \left\{ \left[\frac{\left(\underline{\nu}_{F(\alpha)}(m) + \underline{\nu}_{G(\alpha)}(m)\right)}{2}, \frac{\left(\overline{\nu}_{F(\alpha)}(m) + \overline{\nu}_{G(\alpha)}(m)\right)}{2}\right] \right\rangle | m \in U \right\} & \text{if } \alpha \in A \cap B \\ \left\{ \left\langle m, \left[\underline{\mu}_{F(\alpha)}(m), \overline{\mu}_{F(\alpha)}(m)\right], \left[\underline{\nu}_{F(\alpha)}(m), \overline{\nu}_{F(\alpha)}(m)\right] \right\rangle | m \in U \right\} & \text{if } \alpha \in A - B \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{G(\alpha)}(m)\right], \left[\underline{\nu}_{G(\alpha)}(m), \overline{\nu}_{G(\alpha)}(m)\right] \right\rangle | m \in U \right\} & \text{if } \alpha \in B - A. \end{cases} \end{cases}$$

Example 3.2. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be the universal set and $E = \{$ beautiful, moderate, cheap, costly, wooden $\}$ be the set of parameters. Let $A = \{$ moderate, beautiful $\}$,

 $B = \{$ beautiful, cheap $\}$. IVIFSS (F, A) describes the attractiveness of the objects where as the IVIFSS (G, B) describes its cost. The tabular representation of IVIFSS (F, A) is as follows:

| U | moderate | Beautiful |
|-----------------------|----------------------------|---------------------------|
| <i>x</i> ₁ | [0.6, 0.62], [0.2, 0.3] | [0.8, 0.85], [0.1, 0.13] |
| <i>x</i> ₂ | [0.82, 0.84], [0.05, 0.11] | [0.6, 0.64], [0.25, 0.3] |
| <i>x</i> ₃ | [0.14, 0.22], [0.3, 0.35] | [0.57, 0.61], [0.3, 0.36] |
| X_4 | [0.75, 0.79], [0.15, 0.17] | [0.3, 0.35], [0.6, 0.63] |
| <i>x</i> ₅ | [0.6, 0.64], [0.3, 0.34] | [0.5, 0.6], [0.3, 0.37] |

Table 1

"New Operations on Fuzzy Environment"

| U | beautiful | Cheap |
|-----------------------|----------------------------|----------------------------|
| <i>x</i> ₁ | [0.85, .0.9], [0.02, 0.08] | [0.9, 0.94], [0.01, 0.05] |
| <i>x</i> ₂ | [0.6, 0.7], [0.2, 0.25] | [0.75, 0.8], [0.13, 0.15] |
| <i>x</i> ₃ | [0.7, 0.75], [0.1, 0.18] | [0.76, 0.87], [0.08, 0.12] |
| X_4 | [0.68, 0.73], [0.2, 0.24] | [0.29, 0.34], [0.6, 0.63] |
| <i>x</i> ₅ | [0.24, 0.3], [0.6, 0.64] | [0.82, 0.86], [0.1, 0.12] |

The tabular representation of IVIFSS (G, B) is as follows:

Table 2

The tabular representation of IVIFSS $(F,A)\Phi(G,B)$ is

| U | moderate | beautiful | Cheap |
|-----------------------|----------------------------|-------------------------------|----------------------------|
| <i>x</i> ₁ | [0.6, 0.62], [0.2, 0.3] | [0.825, 0.875], [0.06, 0.105] | [0.9, 0.94], [0.01, 0.05] |
| <i>x</i> ₂ | [0.82, 0.84],[0.05, 0.11] | [0.6, 0.67], [0.225, 0.275] | [0.75, 0.8], [0.13, 0.15] |
| <i>x</i> ₃ | [0.14, 0.22], [0.3, 0.35] | [0.635, 0.68], [0.2, 0.27] | [0.76, 0.87], [0.08, 0.12] |
| <i>x</i> ₄ | [0.75, 0.79], [0.15, 0.17] | [0.49, 0.54], [0.4, 0.435] | [0.29, 0.34], [0.6, 0.63] |
| <i>x</i> ₅ | [0.6, 0.64], [0.3, 0.34] | [0.37, 0.45], [0.45, 0.505] | [0.82, 0.86], [0.1, 0.12] |

Table 3

Proposition 3.3. For any two non-empty IVIFSS (F, A) and (G, B) over U, we have the following: (1) $(F, A)\Phi(G, B) = (G, B)\Phi(F, A)$;

$$(2) \left[(F,A)^{c} \Phi (G,B)^{c} \right]^{c} = (F,A) \Phi (G,B).$$

Proof. (1) Proof is obvious.
$$(2) \operatorname{Let}(F,A) = \left\{ \left\langle m, \left[\underline{\mu}_{F(\alpha)}(m), \overline{\mu}_{F(\alpha)}(m) \right] \right| \left[\underline{\nu}_{F(\alpha)}(m), \overline{\nu}_{F(\alpha)}(m) \right] \right\rangle | m \in U \right\} \text{ and } \\
(G,B) = \left\{ m, \left[\underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{G(\alpha)}(m), \overline{\nu}_{G(\alpha)}(m) \right] \right\} | m \in U \right\} \text{ be two IVIFSS.} \\
\left\{ \left\{ \left\langle m, \left[\frac{\left(\underline{\nu}_{F(\alpha)}(m) + \underline{\nu}_{G(\alpha)}(m) \right)}{2}, \frac{\left(\overline{\nu}_{F(\alpha)}(m) + \overline{\mu}_{G(\alpha)}(m) \right)}{2} \right] \right\} | m \in U \right\} \text{ if } \alpha \in A \cap B \\
\left\{ \left\{ \left\langle m, \left[\underline{\nu}_{F(\alpha)}(m), \overline{\nu}_{F(\alpha)}(m) \right], \left[\underline{\mu}_{F(\alpha)}(m), \overline{\mu}_{F(\alpha)}(m) \right] \right\} | m \in U \right\} \text{ if } \alpha \in A - B \\
\left\{ \left\{ \left\langle m, \left[\underline{\nu}_{F(\alpha)}(m), \overline{\nu}_{F(\alpha)}(m) \right], \left[\underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{G(\alpha)}(m) \right] \right\} | m \in U \right\} \text{ if } \alpha \in B - A. \\
\right\} \right\}$$

"New Operations on Fuzzy Environment"

$$\begin{bmatrix} (F,A)^{c} \Phi (G,B)^{c} \end{bmatrix}^{c} = \begin{cases} \left\{ \left\langle m, \left[\frac{\left(\underline{\mu}_{F(\alpha)}(m) + \underline{\mu}_{G(\alpha)}(m) \right)}{2}, \frac{\left(\overline{\mu}_{F(\alpha)}(m) + \overline{\mu}_{G(\alpha)}(m) \right)}{2} \right] \right\rangle | m \in U \\ \left\{ \left[\frac{\left(\underline{\nu}_{F(\alpha)}(m) + \underline{\nu}_{G(\alpha)}(m) \right)}{2}, \frac{\left(\overline{\nu}_{F(\alpha)}(m) + \overline{\nu}_{G(\alpha)}(m) \right)}{2} \right] \right\rangle | m \in U \\ \left\{ \left\langle m, \left[\underline{\mu}_{F(\alpha)}(m), \overline{\mu}_{F(\alpha)}(m) \right] \left[\underline{\nu}_{F(\alpha)}(m), \overline{\nu}_{F(\alpha)}(m) \right] \right\rangle | m \in U \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{G(\alpha)}(m), \overline{\nu}_{G(\alpha)}(m) \right] \right\rangle | m \in U \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{G(\alpha)}(m), \overline{\nu}_{G(\alpha)}(m) \right] \right\rangle | m \in U \\ \right\} & \text{if } \alpha \in A - B \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{G(\alpha)}(m), \overline{\nu}_{G(\alpha)}(m) \right] \right\rangle | m \in U \\ \right\} & \text{if } \alpha \in B - A. \end{cases} \\ \text{proof.}$$

Definition 3.4. Let (F, A) and (G, B) be two non empty IVIFSS over U. Then the operation \bullet is defined as $(F, A) \bullet (G, B) = (K, C)$, where $C = A \cup B$ and $\forall \alpha \in C$

$$(\mathbf{K}, C) = \begin{cases} \left\{ \left\langle m, \left[\frac{2\underline{\mu}_{F(\alpha)}(m) \cdot \underline{\mu}_{G(\alpha)}(m)}{\underline{\mu}_{F(\alpha)}(m) + \underline{\mu}_{G(\alpha)}(m)}, \frac{2\overline{\mu}_{F(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m)}{\overline{\mu}_{F(\alpha)}(m) + \overline{\mu}_{G(\alpha)}(m)} \right], \\ \left[\frac{2\underline{\nu}_{F(\alpha)}(m) \cdot \underline{\nu}_{G(\alpha)}(m)}{\underline{\nu}_{F(\alpha)}(m) + \underline{\nu}_{G(\alpha)}(m)}, \frac{2\overline{\nu}_{F(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m)}{\overline{\nu}_{F(\alpha)}(m) + \overline{\nu}_{G(\alpha)}(m)} \right] \right\rangle | m \in U \end{cases} \quad if \ \alpha \in A \cap B \\ \left\{ \left\langle m, \left[\underline{\mu}_{F(\alpha)}(m), \overline{\mu}_{F(\alpha)}(m) \right] \left[\underline{\nu}_{F(\alpha)}(m), \overline{\nu}_{F(\alpha)}(m) \right] \right\rangle | m \in U \right\} \quad if \ \alpha \in A - B \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{G(\alpha)}(m) \right] \right\} \left[\underline{\nu}_{G(\alpha)}(m), \overline{\nu}_{G(\alpha)}(m) \right] \right\rangle | m \in U \right\} \quad if \ \alpha \in B - A. \end{cases} \end{cases}$$

Example 3.5. The tabular representation of the IVIFSS $(K, C) = (F, A) \bullet (G, B)$ in Example 3.2 is given below:

| U | moderate | beautiful | Cheap |
|-----------------------|----------------------------|--------------------------------|----------------------------|
| x_1 | [0.6, 0.62], [0.2, 0.3] | [0.824, 0.874], [0.033, 0.099] | [0.9, 0.94], [0.01, 0.05] |
| <i>x</i> ₂ | [0.82, 0.84],[0.05, 0.11] | [0.6, 0.669], [0.222, 0.273] | [0.75, 0.8], [0.13, 0.15] |
| <i>x</i> ₃ | [0.14, 0.22], [0.3, 0.35] | [0.628, 0.673], [0.15, 0.24] | [0.76, 0.87], [0.08, 0.12] |
| x_4 | [0.75, 0.79], [0.15, 0.17] | [0.416, 0.473], [0.3, 0.348] | [0.29, 0.34], [0.6, 0.63] |
| <i>x</i> ₅ | [0.6, 0.64], [0.3, 0.34] | [0.324, 0.4], [0.4, 0.469] | [0.82, 0.86], [0.1, 0.12] |

Table 4

Proposition 3.6. For any two non-empty IVIFSS (F, A) and (G, B) over U

(1) $(F, A) \bullet (G, B) = (G, B) \bullet (F, A);$ (2) $[(F, A)^{c} \bullet (G, B)^{c}]^{c} = (F, A) \bullet (G, B).$ *Proof.* The proof of (1) is obvious. "New Operations on Fuzzy Environment"

$$\left[(F,A)^{c} \bullet (G,B)^{c} \right]^{c} = \begin{cases} \left\{ \left\langle m, \left[\frac{2\underline{\nu}_{F(\alpha)}(m) \cdot \underline{\nu}_{G(\alpha)}(m)}{\underline{\nu}_{F(\alpha)}(m) + \underline{\nu}_{G(\alpha)}(m)}, \frac{2\overline{\nu}_{F(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m)}{\overline{\nu}_{F(\alpha)}(m) + \overline{\nu}_{G(\alpha)}(m)} \right] \right\rangle | m \in U \right\} & if \alpha \in A \cap B \\ \left\{ \left\langle m, \left[\underline{\nu}_{F(\alpha)}(m) \cdot \underline{\mu}_{G(\alpha)}(m), \overline{\mu}_{F(\alpha)}(m) \right] \right] \left[\underline{\mu}_{F(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m) \right] \right\rangle | m \in U \right\} & if \alpha \in A \cap B \\ \left\{ \left\langle m, \left[\underline{\nu}_{F(\alpha)}(m) \cdot \overline{\nu}_{F(\alpha)}(m) \right] \right] \left[\underline{\mu}_{F(\alpha)}(m) \cdot \overline{\mu}_{F(\alpha)}(m) \right] \right\rangle | m \in U \right\} & if \alpha \in A - B \\ \left\{ \left\langle m, \left[\underline{\nu}_{G(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m) \right] \right] \left[\underline{\mu}_{G(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m) \right] \right\rangle | m \in U \right\} & if \alpha \in B - A. \end{cases} \\ \left[\left\{ \left\langle m, \left[\frac{2\underline{\mu}_{F(\alpha)}(m) \cdot \underline{\mu}_{G(\alpha)}(m)}{\underline{\mu}_{F(\alpha)}(m) + \underline{\mu}_{G(\alpha)}(m)}, \frac{2\overline{\mu}_{F(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m)}{\overline{\mu}_{F(\alpha)}(m) + \overline{\mu}_{G(\alpha)}(m)} \right] \right\} | m \in U \right\} & if \alpha \in A - B \\ \left\{ \left\langle m, \left[\frac{2\underline{\nu}_{F(\alpha)}(m) \cdot \underline{\nu}_{G(\alpha)}(m)}{\underline{\mu}_{F(\alpha)}(m) + \underline{\mu}_{G(\alpha)}(m)}, \frac{2\overline{\nu}_{F(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m)}{\overline{\mu}_{F(\alpha)}(m) + \overline{\mu}_{G(\alpha)}(m)} \right] \right\} | m \in U \right\} & if \alpha \in A \cap B \\ \left\{ \left\langle m, \left[\frac{2\underline{\nu}_{F(\alpha)}(m) \cdot \underline{\nu}_{G(\alpha)}(m)}{\underline{\nu}_{F(\alpha)}(m) + \underline{\nu}_{G(\alpha)}(m)}, \frac{2\overline{\nu}_{F(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m)}{\overline{\nu}_{F(\alpha)}(m) + \overline{\nu}_{G(\alpha)}(m)} \right] \right\} | m \in U \right\} & if \alpha \in A \cap B \\ \left\{ \left\langle m, \left[\underline{\mu}_{F(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{F(\alpha)}(m) \cdot \overline{\nu}_{F(\alpha)}(m) \right] \right\} | m \in U \right\} & if \alpha \in A - B \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{G(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m) \right] \right\} | m \in U \right\} & if \alpha \in A - B \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{G(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m) \right] \right\} | m \in U \right\} & if \alpha \in A - B \\ \left\{ \left\langle m, \left[\underline{\mu}_{G(\alpha)}(m) \cdot \overline{\mu}_{G(\alpha)}(m) \right] \left[\underline{\nu}_{G(\alpha)}(m) \cdot \overline{\nu}_{G(\alpha)}(m) \right] \right\} | m \in U \right\} & if \alpha \in B - A. \end{cases} \end{cases}$$

$$= (F,A) \bullet (G,B)$$

Hence the proof.

We now define an information measure on IVIFSS.

Definition 3.7. For any IVIFSS(F, A), an information measure to indicate the degree of fuzziness of (F, A) is defined as

$$I_m(F,A) = \frac{1}{n} \sum_{i=1}^n \frac{\min\left\{\underline{\mu}_{F(\alpha)}(x_i), \underline{\nu}_{F(\alpha)}(x_i)\right\} + \min\left\{\overline{\mu}_{F(\alpha)}(x_i), \overline{\nu}_{F(\alpha)}(x_i)\right\}}{\max\left\{\underline{\mu}_{F(\alpha)}(x_i), \underline{\nu}_{F(\alpha)}(x_i)\right\} + \max\left\{\overline{\mu}_{F(\alpha)}(x_i), \overline{\nu}_{F(\alpha)}(x_i)\right\}}.$$

4. Decision making method based on information measure of IVIFSS

In this section, we develop a decision making method based on the information measure of IVIFSS. The algorithm for decision making method is as follows:

Algorithm 4.1.

Step 1. Construct an IVIFSS (F, A) over U based on the previous records of the specific

problem.

Step 2. Calculate the information measure of (F, A_k) .

Step 3. Compare the values of $I_m(F, A_k)$ and conclude.

When $\left|\underline{\mu}_{F(\alpha)}(m), \overline{\mu}_{F(\alpha)}(m)\right| = [0,0], \left[\underline{\nu}_{F(\alpha)}(m), \overline{\nu}_{F(\alpha)}(m)\right] = [0,0] \forall x_i \in U$ and $\alpha \in A$, we get $I_m(F, A) = 1$. Thus the better choice to the decision maker is to choose the alternative for which the information measure is the least.

Example 4.2. A fund manager in a wealth management firm is assessing three potential investment alternatives $E = \{$ Alternative-I, Alternative-II, Alternative-III $\}$ over $U = \{x_1, x_2, x_3, x_4, x_5\}$ where $x_1 = \text{risk}, x_2 = \text{growth},$ $x_3 = \text{socio-political issues},$

 x_4 = environmental impacts and x_5 = cost are the five factors affecting investments.

Step 1. Based on the information obtained from the fund manager pertaining to the factors influencing investment an IVIFSS over U to assess the alternatives are given below:

| U | Alternative-I (F, A_1) | Alternative-II (F, A_2) | Alternative-III (F, A_3) |
|-----------------------|----------------------------|----------------------------|----------------------------|
| <i>x</i> ₁ | [0.77, 0.83], [0.11, 0.15] | [0.42, 0.48], [0.4, 0.5] | [0.41, 0.47], [0.39, 0.51] |
| <i>x</i> ₂ | [0.63, 0.73], [0.21, 0.24] | [0.61, 0.71], [0.16, 0.24] | [0.57, 0.64], [0.17, 0.21] |

"New Operations on Fuzzy Environment"

| <i>x</i> ₃ | [0.67, 0.75], [0.15, 0.24] | [0.42, 0.51], [0.23, 0.35] | [0.53, 0.64], [0.34, 0.43] |
|-----------------------|----------------------------|----------------------------|----------------------------|
| <i>x</i> ₄ | [0.49, 0.55], [0.21, 0.35] | [0.52, 0.73], [0.15, 0.25] | [0.59, 0.69], [0.13, 0.26] |
| <i>x</i> ₅ | [0,62, 0.71], [0.19, 0.24] | [0.62, 0.74], [0.18, 0.22] | [0.72, 0.77], [0.16, 0.22] |

Table 5

Step 2. The $I_m(F, A)$ is estimated as follows:

$$\begin{split} I_m(F,A) &= \frac{1}{5} \sum_{i=1}^{5} \frac{\min\left\{ \frac{\mu}{F(\alpha)}(x_i), \frac{\nu}{F(\alpha)}(x_i) \right\} + \min\left\{ \overline{\mu}_{F(\alpha)}(x_i), \overline{\nu}_{F(\alpha)}(x_i) \right\}}{\max\left\{ \frac{\mu}{F(\alpha)}(x_i), \frac{\nu}{F(\alpha)}(x_i) \right\} + \max\left\{ \overline{\mu}_{F(\alpha)}(x_i), \overline{\nu}_{F(\alpha)}(x_i) \right\}} \\ \text{for } 1 \leq k \leq 3. \\ I_m(F,A_1) &= 0.326, \qquad I_m(F,A_2) = 0.441, \\ I_m(F,A_3) &= 0.493. \end{split}$$

Step 3. The better investment alternative is the one which has least information measure with respect to the five factors.

We have $I_m(F, A_3) < I_m(F, A_2) < I_m(F, A_1)$.

Therefore, ranking of the alternative is

 $I_m(F, A_1) \succ I_m(F, A_2) \succ I_m(F, A_3).$

i.e., Alternative-I is the better investment.

CONCLUSION

In this paper we have introduced the operations Φ and \bullet on interval valued intuitionistic fuzzy soft set and established some properties of these operators. We have developed a decision making method based on information measure of interval valued intuitionistic fuzzy soft set. We have also given an algorithm for the decision making problem using information measure and illustrated the working of the algorithm by means of example.

REFERENCES

- 1. Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1), 87-96, (1986).
- Atanassov, K., and Gargov, G., Interval valued Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 33, 37-46, (1989).
- Atanssov, K., Operations over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 61, 137-142, (1994).
- 4. Atanassov, K., Intuitionistic fuzzy sets, Physicaverlag, Heidelberg, New york, 1999.
- 5. Deschriver, G., and Kerre, E.F., On the relationship between some extensions of fuzzy set theory, Fuzzy Sets and Systems, 133 (2), 227-235, (2003).
- 6. Gorzalczany, M.B., A method of inference in approximate reasoning based on interval valued fuzzy sets, Fuzzy Sets and Systems, 21 (1), 1-17, (1987).
- Jiang, Y., Tang, Y., Chen, Q., Liu, H., Tang, J., Interval valued intuitionistic fuzzy soft

sets and their properties, Journal of Computers and Mathematics with Applications, 60, 906-918, (2010).

- 8. Klir, G.J., and Folger, T.A., Fuzzy sets, uncertainty and information, Prentice-Hall of India, 2005.
- Kong, Z., Goa, L., Wang, L., Comment on A fuzzy soft set theoretic approach to decision making problems, Journal of Computational and Applied Mathematics, 223 (2), 540-542 (2009).
- Maji, P.K., Biswas, R., Roy, A.R., Fuzzy soft sets, Journal of Fuzzy Mathematics, 9 (3), 589-602, (2001).
- Maji, P.K., Biswas, R., Roy, A.R., Intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics, 9 (3) , 677-692, (2001).
- Maji, P.K., Roy, A.R., Biswas, R., Intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics, 12 (3), 669-683, (2004).
- Molodtsov, D., Soft sets theory-first results, Computers and Mathematics with Applications, 37, 19-31, (1999).
- Roy, A.R., Maji, P.K., A fuzzy soft set theoretic approaching decision-making problems, Journal of Computational and Applied Mathematics, 203 (2), 412-418, (2007).
- Yang, X.B., Lin, T.Y., Yang, J.Y., Li, Y., Yu, D., Combination of interval valued fuzzy set and soft set, Computers and Mathematics with Applications, 58 (3), 521-527, (2009).
- Zadeh, L.A., Fuzzy sets, Information and Control, 8, 338-353, (1965).
- 17. Zhang, Q.S., Jiang, S., Jia, B., Luo, S., Some information measures for interval valued intuitionistic fuzzy sets, Information Sciences, 180, 5130-5145, (2010).