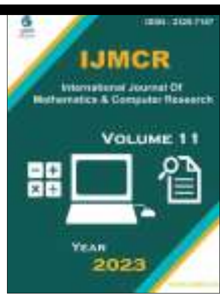
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 11 Issue 01 January 2023, Page no. – 3209-3219

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v11i1.14



Stability Analysis of Dynamical Model Interactions Tea Plants, Pests, and Diseases with Fungicides and Insecticides Controls

Arista Fitri Diana[1], Widowati[2], R H Tjahjana[3], Eka Triyana[4]

[1,2,3,4]Department of Mathematics, Faculty of Sciences and Mathematics, Diponegoro University, Semarang, 50275, Indonesia

ARTICLE INFO

Published Online: 30 January 2023

Corresponding Author: **Widowati**

ABSTRACT

Tea plants are one of the exports commodities in Indonesia. In their development, the plantation ecosystem is heavily influenced by several factors, both internal and external factors. In the field of applied mathematics, mathematical modelling can be used to analyze the development of tea plant growth and their interactions each other in their ecosystem. The mathematical model in this research is combining three main models, there are logistic model, epidemiological model, and predator prey model by adding fungicide and insecticide controls. Furthermore, local and global stability analysis is carried out and the optimal control problem is solved by Pontryagin maximum principle. The results of the analysis obtained five equilibrium points. Local stability analysis was carried out using the Routh Hurwitz criteria which showed the fifth equilibrium point is locally asymptotically stable. The basic reproduction number in the model is 0.99. Because $\mathfrak{R}_0 < 1$, it can be concluded that there is no spread of disease in the tea plantation ecosystem after a period of 5 years. The control provided can reduce pest and disease attacks. After being given control, the population of infected tea plants decreased by 93.21%, Empoasca pests decreased by 99.47%, and leaf roller caterpillars decreased by 99.31% compared to the model that was not given control

KEYWORDS: Tea Plantation, Dynamical Model, Fungicides, Insecticides, Optimal Control.

I. INTRODUCTION

Tea is one of the export commodities in Indonesia. One of the tea plantations in Central Java that is used as the location of this research is the PT RSM (Rumpun Sari Medini) tea plantation located in Kendal. Each tea plantation ecosystem has different problems, both related to pests, diseases, soil fertility conditions, and other factors [1]. The problem of tea plant population growth in this study is related to tea plant pests and diseases. There are three dominant organisms that interfere with the growth of tea plants at PT RSM, namely empoasca pests, leaf rolling caterpillar pests, and blister blight (fungus). In mathematics, the interaction between tea plants and nuisance organisms (pests and diseases) can be modeled as a dynamic model so that the behavior of interactions between individuals can be known as stable conditions. Furthermore, the control model is used to suppress the growth rate of pests and diseases. In this research, fungicide and insecticide are two control that used [2-5].

The important of this research is to explain how the condition of tea plantation ecosystem. The condition from this research describe the population that influence the growth of tea plant. The most population that disturb tea plant can be more concerned to eradicated and otherwise. Furthermore, the optimal control can be used to know the best for dealing the pests and diseases. So, from this condition, PT RSM can choose the step to handle this ecosystem so that the tea plantation can grow optimal.

The dynamic model of predator prey and epidemic has been studied by many researchers, they are Shasi Kant [6] the predator prey model use interaction between three species with migrating and disease, then in Bezabih's research use predator-prey model for epidemiology with diseases. The model of [7] describe that there is interaction between predator and infected prey. In Dipo Aldila's research, the controls are given for covid model. In this paper, we combined [6-8] model, we use mathematical model of predator prey model and SI epidemic model without migrating and interaction beteween pest and disease that applied for tea plantation by adding the optimal control. So, the differences between this model and the research before are this research modified the dynamic model with remove migrating population, interaction between pest and disease, and add the control [9-11]. The dynamic model of the interaction between tea plants with pests and diseases was modelled by following the predator-prey model for the interaction of tea plants and pests, while the interaction model of tea plants and diseases followed the SI epidemic model and then there are two controls applied in this dynamic model with the growth of tea plant followed the logistic model [12-13]. Then we simulated this model that using the data from PT RSM at 2015-2019.

This work is discussed the predator-prey model and epidemic model without migrating population for tea plantation by adding optimal control. Then we analyzed the exsistence and stability of equlibrium point from the dynamic model. The paper proposed five equilibrium points analytical results, there are endemic equilibrium point and non endemic equilibrium point (four equilibrium point otherwise). We also analyzed the Basic Reproduction Number (R_0) that determine the stability analysis of equilibrium points which is derived from Next Generation Matrix (NGM). Using the Routh-Hurwitz stability criteria from Jacobian matrix evaluated the equlibrium point, we analyzed the local stability of the system around the equilibrium point. Furthermore, the global stability analyzed by Lyapunov function. The disease-free equilibrium point or non endemic equilibrium is asymptotically stable if the basic reproduction number is less than one $(R_0 < 1)$ and the endemic equilibrium point is asymptotically stable if $(R_0 > 1)$. Then the optimal control obtained from pontryagin maximum principle [14]. The numerical analysis simulation are also provided to illustrate the analytical results.

II. MATERIALS AND METHODS

The research materials that will be used in this study are data related to production, tea plantation area, types and number of pests, as well as insecticide data taken directly from PT Rumpun Sari Medini tea plantations, at Kendal, Central Java, Indonesia. The data used in this study related to the production of tea plants were taken from daily data from the picking of tea plants and data on the area of the plantation area from 2015 to 2019. In addition, data for the type and number of pests were obtained from the Early Warning Score (EWS) center which carries out routine checks every month to the plantation area.

The methods of the research can be separated into model formulation, positivity and boundedness solution analysis, stability analysis of the equilibrium point by using Routh Hurwitz method for locally stability and Lyapunov method for globally stability, optimal control analysis, laboratory experiment and computational simulation. Next, we proposed the dynamical model formulation, equilibrium point of the dynamical model, and basic reproduction number.

A. Model formulation

Individual groups in dynamic model of interactions is applied to healthy tea plants, Empoasca pests, leaf-rolling caterpillar pests, and infected tea plants. The interaction between tea plant with pests followed predator-prey model and the interaction between healthy tea plant with infected tea plant followed SI epidemic model [15]. The division is grouped into four groups, namely healthy tea plant compartment (S), infected tea plant compartment (I), Empoasca pest compartment (H_1), leaf-rolling cartepillar pest (H_2). Healthy tea plant is healthy prey, infected tea plant is infected prey, Empoasca pest is predator 1, and leaf-rolling cartepillar pest is predator 2.

For mathematical transformation of real situations, we make the following assumption:

a. Predator and prey are closed, this means that neither predators nor prey migrate
b. If there is no interaction between predator and prey, the healthy prey growth follows the logistic model [15]
c. There is no interaction and competition between predators
d. Predators only attack healthy prey
e. The environmental carrying capacity of the healthy prey is constant
f. There is a natural death in each compartment with a death rate of d_i ,$i = 1,2,3,4$
g. There are deaths caused by diseases in the infected prey compartment so that infected prey will decrease with a death rate of c_2
h. There is harvesting in the healthy prey compartment so that healthy prey decrease with harvesting rate of c_1
i. There is an interaction between healthy prey and infected prey which causes healthy prey to become infected, so that the number of healthy prey decrease with an infection rate of β
j. Healthy prey is the only food for predator, then preadator 1 will growth with an teraction rate of healthy prey of q_1 which depends on the rate of maximum consumption of predator 1 on healthy of p_1 with a conversion rate of healthy prey into predator 1 of φ_1 [16].
k. Healthy prey is the only food for predator, then preadator 2 will growth with an teraction rate of healthy prey of q_2 which depends on the rate of maximum consumption of

predator 2 on healthy of p_1 with a conversion rate of healthy prey into predator 1 of φ_2 [16].

l. Growth in healthy prey is assumed to follow a logistic model defined as a function of $G(S,I)$ [7].

m. Effort to prevent pests and diseases are used as optimal control, there are fungisides control (u_1) and insecticides (u_2).

n. The model simulation use maple 12 software and matlab software

Table 1. Descriptions of Parameters

Notation	Interpretations	Notation	Interpretations
K	Carrying capacity	c_1	Rate of healthy tea harvesting
r	Intrinsic growth rate of prey	c_2	Death rate of infected tea due to infection
β	Infection coefficient of healthy tea	d_1	Natural death rate of healthy tea
p_1	Rate of interaction between healthy tea with *empoasca* pest	d_2	Natural death rate of infected tea
p_2	Rate of interaction between healthy tea with leaf-rolling caterpillars	d_3	Natural death rate of *empoasca* pest
q_1	Conversion coefficient from interaction between healthy tea with *empoasca* pest	d_4	Natural death rate of leaf-rolling cartepillar pest
q_2	Conversion coefficient from interaction between healthy tea with leaf-rolling caterpillar pests		

Based on the mathematical model from Kant Shashi's [6], we developed this model to apply the real condition on tea plants ecosystem in PT. Rumpun Sari Medini. The real condition is no migrating in the ecosystem, so we remove the migrating parameter from the model [6]. Then we also remove the interaction between predator's species and the interaction between predator with infected prey. The second that the developed mathematical model as follows the model from the Abayneh Fentie Bezabih's journal [7]. The interaction species can apply to this real conditions but the differences are the model [7] study predator prey model for epidemiology with diseases, but in this paper, we study predator prey model for tea plants ecosystem with disease. From them we have modified mathematical model in tea ecosystem with remove both migrating and interaction between predator's species. So the mathematical model formulation is represented as follows.

$$\begin{aligned}\frac{dS}{dt} &= rS\left(1-\frac{S+I}{K}\right)-\beta SI-p_1SH_1-p_2SH_2-(c_1+d_1)S\\ \frac{dI}{dt} &= \beta SI-(c_2+d_2)I\\ \frac{dH_1}{dt} &= q_1SH_1-d_3H_1\\ \frac{dH_2}{dt} &= q_2SH_2-d_4H_2\end{aligned} \qquad (2.1)$$

With the initial condition $S(0)>0, I(0)\geq 0, H_1(0)\geq 0, H_2(0)\geq 0$ where the parameters are given in Table 1. All parameters are positive.

B. Equilibrium Point of The Dynamic Model

The equilibrium point is a condition where there are no changes in each population over time. System (2.1) can be written:

$$\frac{dS}{dt} = rS\left(1-\frac{S+I}{K}\right)-\beta SI-p_1SH_1-p_2SH_2-(c_1+d_1)S=0$$

$$\frac{dI}{dt} = \beta SI-(c_2+d_2)I=0$$

$$\frac{dH_1}{dt} = q_1SH_1-d_3H_1=0$$

$$\frac{dH_2}{dt} = q_2SH_2-d_4H_2=0$$

We obtained the equilibrium point $E=(E^1,E^2,E^3,E^4,E^5)$ as follows:

$$E^1=(0,0,0,0)$$

$$E^2=\left(\frac{c_2+d_2}{\beta},-\frac{-rK\beta+rc_2+rd_2+c_1K\beta+d_1K\beta}{\beta(r+\beta K)},0,0\right)$$

$$E^3=\left(\frac{d_4}{q_2},0,0,-\frac{-rKq_2+rd_4+c_1Kq_2+d_1Kq_2}{p_2Kq_2}\right)$$

$$E^4=\left(\frac{d_3}{q_1},0,-\frac{-rKq_1+rd_3+c_1Kq_1+d_1Kq_1}{p_1Kq_1},0\right)$$

$$E^5=\left(-\frac{K(-r+c_1+d_1)}{r},0,0,0\right)$$

C. Basic Reproduction Number

The basic reproduction number [17] holds a crucial role in the analysis of an infectious disease. It is used to describe the average number of the new infection due to a sick individual and is denoted by R_0. If $R_0>1$, it means that one

infected tea can produce more than one secondary infection [18-21]. In this case, the disease-free equilibrium is unstable so that it can cause the epidemic outbreaks. But if the $R_0 < 1$, the disease-free equilibrium (DFE) will be locally asymptotically stable and the situation is under control. Since the model (2.1) has DFE $E^5 = \left(S^5 = -\frac{K(-r+c_1+d_1)}{r}, I^5 = 0, H_1^5 = 0, H_2^5 = 0\right)$, so the basic reproduction number can be found analitycally using next generation matrix. The R_0 can be computed by considering the bellow generation matrices FV^{-1}, where $F(E^5) = \left[-\frac{\beta K(-r+c_1+d_1)}{r}\right]$
and $V = [c_2 + d_2]$

Therefore, the spectral radius of FV^{-1} is

$$R_0 = \frac{\beta K(r - c_1 - d_1)}{r(c_2 + d_2)}$$

with $r > c_1 + d_1$

III. RESULTS AND DISCUSSION

A Positivity and Bounded Analysis

In this subsection, we proposed positivity analysis of the model solution, in the Lemma 1 and bounded analysis of the model solution in the Lemma 2.

Lemma 1

if $S(0) \geq 0, I(0) \geq 0, H_1(0) \geq 0, H_2(0) \geq 0,$ the solutions of $S(t), I(t), H_1(t), H_2(t)$ from system (2.1) are positif for all $t \geq 0.$

Proof.

$$S(t) = \frac{\left(e^{\int \frac{rK - rI(t) + \beta I(t)K + p_1 H_1(t)K + p_2 H_2(t)K + c_1 K + d_1 K}{K} dt}\right)}{\frac{\left(e^{\int \frac{rK - rI(t) + \beta I(t)K + p_1 H_1(t)K + p_2 H_2(t)K + c_1 K + d_1 K}{K} dt} r\right)}{K} dt + C} \geq 0$$

$$I(t) = I(0)e^{-\int_0^t (\beta S(u) - c_2 - d_2)t}$$

$$+e^{-\int_0^t (\beta S(u) - c_2 - d_2)t} \int_0^t \left((\beta S(t) - c_2 - d_2) I(t) e^{\int_0^t (\beta S(u) - c_2 - d_2)t} \right) du \geq 0$$

$$H_1(t) = H_1(0)e^{-\int_0^t (q_1 S(u) - d_3)t} + e^{-\int_0^t (q_1 S(u) - d_3)t} \int_0^t \left((q_1 S(t) - d_3) H_1(t) e^{\int_0^t (q_1 S(u) - d_3)t} \right) du \geq 0$$

$$H_2(t) = H_2(0)e^{-\int_0^t (q_2 S(u) - d_4)t} + e^{-\int_0^t (q_2 S(u) - d_4)t} \int_0^t \left((q_2 S(t) - d_4) H_2(t) e^{\int_0^t (q_2 S(u) - d_4)t} \right) du \geq 0$$

It is proven that the solutions $S(t), I(t), H_1(t), H_2(t)$ from system (2.1) are positif for all $t \geq 0$

Lemma 2. The solution of system (2.1) in R_+^4 is bounded

Proof.

The dimensionless from (2.1) is,

$$\frac{dx}{dT} = x(1 - A(x + y) - D_1) - Bxy - P_1 xz - P_2 xu$$

$$\frac{dy}{dT} = Bxy - D_2 y$$

$$\frac{dz}{dT} = Q_1 xz - D_3 z$$

$$\frac{du}{dT} = Q_2 xu - D_4 u$$

With

$x = \frac{S}{r}, y = \frac{I}{r}, z = \frac{H_1}{r}, u = \frac{H_2}{r}, T = rt, x(0) \geq 0, y(0) \geq 0, z(0$

$A = \frac{1}{rK}, D_1 = \frac{(c_1+d_1)}{r}, D_2 = \frac{(c_2+d_2)}{r}, D_3 = \frac{d_3}{r}, D_4 = \frac{d_4}{r}$
$, B = \frac{\beta}{r^2}, P_1 = \frac{p_1}{r^2}, P_2 = \frac{p_2}{r^2}, Q_1 = \frac{q_1}{r^2}, Q_2 = \frac{q_2}{r^2}$

Let the function

$W(T) = e_1 x(T) + e_2 y(T) + e_3 z(T) + e_4 u(T)$, so that

$$W'(T) = e_1 x'(T) + e_2 y'(T) + e_3 z'(T) + e_4 u'(T)$$

$$W'(T) \leq 2e_1 x + e_2 y(Bx + 1) + e_3 z(Q_1 x + 1) + e_4 u(Q_2 x + 1) - \kappa W$$

$$\kappa = \min\{e_1, e_2, e_3, e_4\}$$

$$\frac{dW}{dT} + \kappa W \leq 2e_1 x + e_2 y(Bx + 1) + e_3 z(Q_1 x + 1) + e_4 u(Q_2 x + 1)$$

$$\frac{dW}{dT} + \kappa W \leq \mu$$

$$\mu = 2e_1 x + e_2 y(Bx + 1) + e_3 z(Q_1 x + 1) + e_4 u(Q_2 x + 1)$$

$$W(T) \leq \frac{\mu}{\kappa}(1 - e^{-\kappa T}) + W(x(0), y(0), z(0), u(0))e^{-\kappa T}$$

So for $t \to \infty, 0 < W(T) \leq \frac{\mu}{\kappa}$

The solution fo system (2.1),

$$\Omega = \left\{(x, y, z, u) : 0 < x(T) \leq 1, 0 < y(T) \leq 1, 0 < z(T) \leq 1, 0 < u(T) \leq 1; 0 < W(T) \leq \frac{\mu}{\kappa}\right\}$$

It is proven that the solution of system (2.1) is bounded

B Existence of Equilibrium Point

Next, we established the existence condition of the equilibrium states. The equilibrium points of the model (2.1) are as follow:

$E^1 = (0,0,0,0)$ do always exist. It means in this equilibrium point, there are no healthy tea plant, infected tea plant, *Empoasca* pest, and leaf rolling caterpillar pest in tea plantation ecosystem.

$$E^2 = \left(\frac{c_2 + d_2}{\beta}, -\frac{-rK\beta + rc_2 + rd_2 + c_1 K\beta + d_1 K\beta}{\beta(r + \beta K)}, 0, 0\right)$$

The existence condition for E^2 if,

$rK\beta > rc_2 + rd_2 + c_1 K\beta + d_1 K\beta$

It means that in endemic equilibrium point, there are healthy tea plant and infected tea plant in tea plantation ecosystem.

$$E^3 = \left(\frac{d_4}{q_2}, 0, 0, -\frac{-rKq_2 + rd_4 + c_1 Kq_2 + d_1 Kq_2}{p_2 Kq_2}\right)$$

The existence condition for E^3 if,

$rKq_2 > rd_4 + c_1 Kq_2 + d_1 Kq_2$

It means that in this equilibrium point, there are healthy tea plant and leaf rolling caterpillar pest in tea plantation ecosystem.

$$E^4 = \left(S^4 = \frac{d_3}{q_1}, I^4 = 0, H_1^4 = -\frac{-rKq_1 + rd_3 + c_1 Kq_1 + d_1 Kq_1}{p_1 Kq_1}, H_2^4 = 0\right)$$

The existence condition for E^4 if,

$rKq_1 > rd_3 + c_1 Kq_1 + d_1 Kq_1$

It means that in this equilibrium point, there are healthy tea plant and *Empoasca* pest in tea plantation ecosystem.

$$E^5 = \left(S^5 = -\frac{K(-r + c_1 + d_1)}{r}, I^5 = 0, H_1^5 = 0, H_2^5 = 0\right)$$

The existence condition for E^5 if,

$r > c_1 + d_1$

It means that in disease free equilibrium point, just healthy tea plant in tea plantation ecosystem.

C Local Stability Analysis

In this subsection we will analyze the local stability of endemic point (E^2) and non-endemic point (E^5). We use Routh Hurwitz method to analyze the local stability of endemic and non-endemic equilibrium point.

Theorem 1. Let $R_0 = \frac{\beta K(r-c_1-d_1)}{r(c_2+d_2)}$. *The endemic equilibrium state* $E^2 = (S^2, I^2, H_1^2, H_2^2)$ *is stable if* $R_0 > 1$ *and unstable if* $R_0 < 1$

Proof.

To prove the stability of endemic point is stable if $R_0 > 1$, we use the characteristic polynomial. First step we do the inearization of system (2.1).

$$\frac{d}{dt}\begin{bmatrix} S \\ I \\ H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial H_1} & \frac{\partial f_1}{\partial H_2} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial H_1} & \frac{\partial f_2}{\partial H_2} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial H_1} & \frac{\partial f_3}{\partial H_2} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial H_1} & \frac{\partial f_4}{\partial H_2} \end{bmatrix}_{(S^2, E^2, H_1^2, H_2^2)} \begin{bmatrix} \bar{S} \\ \bar{I} \\ \overline{H_1} \\ \overline{H_2} \end{bmatrix}$$

Then with maple application we obtain :

$$\frac{d}{dt}\begin{bmatrix} S \\ I \\ H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} r\left(1 - \frac{(S+I)}{K}\right) - \frac{rS}{K} - \beta I - p_1 H_1 - p_2 H_2 - c_1 - d_1 & -\frac{rS}{K} - \beta S & -p_1 S & -p_2 S \\ \beta I & \beta S - c_2 - d_2 & 0 & 0 \\ q_1 H_1 & 0 & q_1 S - d_3 & 0 \\ q_2 H_2 & 0 & 0 & q_2 S - d_4 \end{bmatrix} \begin{bmatrix} \bar{S} \\ \bar{I} \\ \overline{H_1} \\ \overline{H_2} \end{bmatrix} \quad (3.1)$$

Then substitute the endemic point to (3.1) and we get the characteristic polynomial as follows:

$$Pol(X) = \frac{1}{Kq_2^3}(X\beta - q_1 c_2 - q_1 d_2 + d_3\beta)(X\beta - q_2 c_2 - q_2 d_2 + d_4\beta)(K\beta X^2 + rK\beta c_2 + rK\beta d_2 - K\beta c_1 c_2 - K\beta c_1 d_2 - K\beta d_1 c_2 - K\beta d_1 d_2 + Xrd_2 + Xrc_2 - rc_2^2 - 2rc_2 d_2 - rd_2^2) \quad (3.2)$$

From (3.2), we use Routh Hurwitz method to analyze the local stability of endemic point. The endemic point will be locally asymptotically stable if :

$$rK\beta c_2 + rK\beta d_2 - K\beta c_1 c_2 - K\beta c_1 d_2 - K\beta d_1 c_2 - K\beta d_1 d_2 - rc_2^2 - rd_2^2 - 2rc_2 d_2 > 0$$

$$rK\beta c_2 + rK\beta d_2 > K\beta c_1 c_2 + K\beta c_1 d_2 + K\beta d_1 c_2 + K\beta d_1 d_2 + rc_2^2 + rd_2^2 + 2rc_2 d_2$$

$$rK\beta c_2 + rK\beta d_2 - K\beta c_1 c_2 - K\beta c_1 d_2 - K\beta d_1 c_2 - K\beta d_1 d_2 > +rc_2^2 + rd_2^2 + 2rc_2 d_2$$

$$K\beta(rc_2 + rd_2 - c_1 c_2 - c_1 d_2 - d_1 c_2 - d_1 d_2) > r(c_2^2 + d_2^2 + 2c_2 d_2)$$

$$K\beta(r - c_1 - d_1)(c_2 + d_2) > r(c_2 + d_2)^2$$

$$K\beta(r - c_1 - d_1) > r(c_2 + d_2)$$

$$\frac{K\beta(r - c_1 - d_1)}{r(c_2 + d_2)} > 1$$

i. $R_0 > 1$

ii. $q_1 c_2 + q_1 d_2 > d_3 \beta$

iii. $q_2 c_2 + q_2 d_2 > d_4 \beta$

It is evident that disease endemic equilibrium point is locally asymptotically stable if $R_0 > 1$

Theorem 2. Let $R_0 = \frac{\beta K(r-c_1-d_1)}{r(c_2+d_2)}$. *The disease free equilibrium will be locally asymptotically stable if* $R_0 < 1$ *and unstable if* $R_0 > 1$.

Proof.

To prove the stability of disese free equilibrium point is stable if $R_0 < 1$, we use the characteristic polynomial. First step we do the liearization of system (2.1).

$$\frac{d}{dt}\begin{bmatrix} S \\ I \\ H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial H_1} & \frac{\partial f_1}{\partial H_2} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial H_1} & \frac{\partial f_2}{\partial H_2} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial H_1} & \frac{\partial f_3}{\partial H_2} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial H_1} & \frac{\partial f_4}{\partial H_2} \end{bmatrix}_{(S^2, E^2, H_1^2, H_2^2)} \begin{bmatrix} \bar{S} \\ \bar{I} \\ \overline{H_1} \\ \overline{H_2} \end{bmatrix}$$

Then with maple application we obtain :

$$\frac{d}{dt}\begin{bmatrix} S \\ I \\ H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} r\left(1 - \frac{(S+I)}{K}\right) - \frac{rS}{K} - \beta I - p_1 H_1 - p_2 H_2 - c_1 - d_1 & -\frac{rS}{K} - \beta S & -p_1 S & -p_2 S \\ \beta I & \beta S - c_2 - d_2 & 0 & 0 \\ q_1 H_1 & 0 & q_1 S - d_3 & 0 \\ q_2 H_2 & 0 & 0 & q_2 S - d_4 \end{bmatrix} \begin{bmatrix} \bar{S} \\ \bar{I} \\ \overline{H_1} \\ \overline{H_2} \end{bmatrix} \quad (3.3)$$

Then substitute the endemic point to (3.3) and we get the characteristic polynomial as follows :

$$P(\lambda) = \frac{1}{r^3}\left(\left(r\left(1 + \frac{-r + c_1 + d_1}{r}\right) - r - \lambda\right)(-rK\beta + c_1 K\beta + d_1 K\beta + rc_2 + rd_2 + \lambda r)(-q_1 Kr + q_1 Kc_1 + q_1 Kd_1 + d_3 r + \lambda r)(-rKq_2 + c_1 Kq_2 + d_1 Kq_2 + rd_4 + \lambda r)\right) \quad (3.4)$$

From (3.4), we use Routh Hurwitz method to analyze the local stability of disease-free equilibrium point. We can use the eigen values to analyze the local stability, we get the eigen values as follows:

$$\lambda_1 = -r + c_1 + d_1$$

$$\lambda_2 = -\frac{-rK\beta + rc_2 + rd_2 + c_1 K\beta + d_1 K\beta}{r}$$

$$\lambda_3 = -\frac{-q_1 Kr + q_1 Kc_1 + q_1 Kd_1 + d_3 r}{r}$$

$$\lambda_4 = -\frac{-rKq_2 + c_1 Kq_2 + d_1 Kq_2 + rd_4}{r}$$

The disease free equilibrium point will be locally asymptotically stable if :

$$rK\beta < rc_2 + rd_2 + c_1K\beta + d_1K\beta$$
$$rK\beta - c_1K\beta - d_1K\beta < rc_2 + rd_2$$
$$K\beta(r - c_1 - d_1) < r(c_2 + d_2)$$
$$\frac{K\beta(r - c_1 - d_1)}{r(c_2 + d_2)} < 1$$
$$R_0 < 1$$

- $r > c_1 + d_1$
- $R_0 < 1$
- $q_1Kr < q_1Kc_1 + q_1Kd_1 + d_3r$
- $rKq_2 < c_1Kq_2 + d_1Kq_2 + rd_4$

It is evident that disease free equilibrium point is locally asymptotically stable if $R_0 < 1$. Furthermore, in this case describe that the spread of disease is under control, so the average of every infected tea plant transmits less disease than one other individual.

D Global Stability Analysis

In this section we will analyze the global stability of endemic equilibrium point. We use the Lyapunov method to analyze the global stability.

Theorem 3. *If $R_0 > 1$, the endemic equilibrium point E^* is asymptotically stable. And unstable if $R_0 < 1$*

Proof.

To prove the global stability, we use the Lyapunov function of ecological and epidemiological model [11] as follows :

$$\sum_{i=1}^{n} a_i\left(x_i - x_i^* - x_i^*\ln\frac{x_i}{x_i^*}\right) \qquad (3.5)$$

Adjust to (3.5), the function $L:\Omega \in R^4 \to R$ is formed with :

$$L(S,I,H_1,H_2) = \left(S - S^* - S^*\ln\frac{S}{S^*}\right) + a_1\left(I - I^* - I^*\ln\frac{I}{I^*}\right) + a_2\left(H_1 - H_1^* - H_1^*\ln\frac{H_1}{H_1^*}\right) + a_3\left(H_2 - H_2^* - H_2^*\ln\frac{H_2}{H_2^*}\right)$$

Where $\forall(S, I, H_1, H_2) \in \Omega$ and a_1, a_2, a_3 are real numbers. Function L is Lyapunov function because by definition of Lyapunov function that will be showns as follows:

The function L continuous on Ω because the function L contains the logarithm and has a partial derivative that is continuous on Ω. For any $E = (S, I, H_1, H_2) \in \Omega$ with $E \neq E^*$, then $L(t) > 0$, furthermore if $E = E^*$ then $L(t) = 0$.

It will show $L(t) > 0$ when $E \neq E^*$

Let $\frac{E}{E^*} = a$ and 1, then:

$$g(a) = E - E^* - E^*\ln\frac{E}{E^*}$$
$$g(a) = E^*\left(\frac{E}{E^*} - 1 - \ln\frac{E}{E^*}\right)$$
$$g(a) = E^*(a - 1 - \ln a)$$

Note that point $a = 1$ is minimum point from $g(a)$ with $g(1) = 0$, because $g'(1) = 0$ and $g''(a) = \frac{1}{a^2} > 0$. So, we get $(a) = E - E^* - E^*\ln\frac{E}{E^*} > 0$, for $E \neq E^*$.

Furthermore, to show the equilibrium point E^* is global minimum point, we use the Hessian matrix in E^* as follows:

$$H(E^*) = \begin{pmatrix} \frac{\partial^2 L}{\partial S^2} & \frac{\partial^2 L}{\partial S\partial I} & \frac{\partial^2 L}{\partial S\partial H_1} & \frac{\partial^2 L}{\partial S\partial H_2} \\ \frac{\partial^2 L}{\partial S\partial I} & \frac{\partial^2 L}{\partial I^2} & \frac{\partial^2 L}{\partial I\partial H_1} & \frac{\partial^2 L}{\partial I\partial H_2} \\ \frac{\partial^2 L}{\partial S\partial H_1} & \frac{\partial^2 L}{\partial I\partial H_1} & \frac{\partial^2 L}{\partial H_1^2} & \frac{\partial^2 L}{\partial H_1\partial H_2} \\ \frac{\partial^2 L}{\partial S\partial H_2} & \frac{\partial^2 L}{\partial I\partial H_2} & \frac{\partial^2 L}{\partial H_1\partial H_2} & \frac{\partial^2 L}{\partial H_2^2} \end{pmatrix}$$

$$H(E^*) = \begin{pmatrix} \frac{1}{S^*} & 0 & 0 & 0 \\ 0 & \frac{a_1}{I^*} & 0 & 0 \\ 0 & 0 & \frac{a_2}{H_1^*} & 0 \\ 0 & 0 & 0 & \frac{a_3}{H_2^*} \end{pmatrix}$$

Matrix $H(E^*)$ definite positif because

$$\det(H(E^*)) = \frac{a_1a_2a_3}{S^*I^*H_1^*H_2^*} > 0$$

Then, when $E = E^*$ obtained $L(t)$ as follows:

$$L(S^*,I^*,H_1^*,H_2^*) = \left(S - S^* - S^*\ln\frac{S}{S^*}\right) + a_1\left(I^* - I^* - I^*\ln\frac{I}{I^*}\right) + a_2\left(H_1 - H_1^* - H_1^*\ln\frac{H_1}{H_1^*}\right) + a_3\left(H_2 - H_2^* - H_2^*\ln\frac{H_2}{H_2^*}\right)$$

$$L(S^*,I^*,H_1^*,H_2^*) = (S^*\ln 1) + a_1(I^*\ln 1) + a_2(H_1^*\ln 1) + a_3(H_2^*\ln 1)$$
$$L(S^*,I^*,H_1^*,H_2^*) = 0$$

It is proven that $L(t) > 0$ when $E \neq E^*$ with $E = (S, I, H_1, H_2)\epsilon\Omega$, $L(t) = 0$ when $E = E^*$ and E^* is global minimum.

Then the derivative of $L(t)$ as follows:

$$\frac{dL}{dt}=S\left(r\left(1-\frac{S+I}{K}\right)-(c_1+d_1)\right)+\beta SI(-1+a_1)+SH_1(-p_1+a_2q_1)$$
$$+SH_2(-p_2+a_3q_2)$$
$$-rS^*\left(1-\frac{S+I}{K}\right)+\beta S^*I+p_1S^*H_1+p_2S^*H_2+(c_1+d_1)S^*-a_1(c_2+d_2)I$$
$$-a_1\beta SI^*$$
$$+a_1(c_2+d_2)I^*-a_2d_3H_1-a_2q_1SH_1^*+a_2d_3H_1^*-a_3d_4H_2-a_3q_2SH_2^*$$
$$+a_3d_4H_2^* \quad (3.6)$$

By aljabar manipulation from (3.6) and $\left(\frac{S^*}{S},\frac{I^*}{I},\frac{H_1^*}{H_1},\frac{H_2^*}{H_2}\right)=(y_0,y_1,y_2,y_3)$ we obtain,

$$\frac{dL}{dt}=\beta S^*I^*\left(-\frac{1}{y_0}-1\right)+a_2q_1S^*H_1^*\left(-\frac{1}{y_0}-1\right)+a_3q_2S^*H_2^*\left(-\frac{1}{y_0}-1\right)$$
$$\frac{dL}{dt}=\left(-\frac{1}{y_0}-1\right)(\beta S^*I^*+a_2q_1S^*H_1^*+a_3q_2S^*H_2^*)$$

Based on the principle of inequalities arithmetical and geometrical means obtained as folllows :

$$\frac{\left(-\frac{1}{y_0}-1\right)}{2}\geq\sqrt{-1\times\left(-\frac{1}{y_0}\right)}$$
$$\left(\frac{\left(-\frac{1}{y_0}-1\right)}{2}\right)^2\geq\frac{1}{y_0}$$
$$\frac{\left(-\frac{1}{y_0^2}-\frac{2}{y_0}-1\right)}{4}\geq\frac{1}{y_0}$$
$$\left(-\frac{1}{y_0^2}-\frac{2}{y_0}-1\right)\geq\frac{1}{y_0}$$
$$\left(-\frac{2}{y_0}-1\right)\geq\frac{1}{y_0}$$
$$0\geq\frac{1}{y_0}+\frac{2}{y_0}+1$$
$$0\geq\frac{3}{y_0}+1$$
$$0\geq\frac{1}{y_0}+1$$
$$\frac{1}{y_0}+1\leq 0$$
$$\frac{1}{y_0}-1\leq 0$$
$$-\frac{1}{y_0}-1\leq 0$$

It is proven that $\frac{dL}{dt}\leq 0$ Since the Lyapunov function can be formed, it is evident that the endemic equilibrium point is globally asymptotically stable if $R_0>1$.

E Analysis of Optimal Control

Model (2.1) was modified by reducing the rate of disease transmission by applying of fungicides (u_1) and reducing the attack of empoasca pests and leaf rolling caterpillars pests by applying of insecticides (u_2). Due to these actions, the system of equations (2.1) becomes:

$$\frac{dS}{dt}=rS\left(1-\frac{S+I}{K}\right)-\beta SI-p_1SH_1-p_2SH_2-(c_1+d_1)S$$
$$\frac{dI}{dt}=\beta SI-(c_2+d_2)I-u_1I$$
$$\frac{dH_1}{dt}=q_1SH_1-d_3H_1-u_2H_1$$
$$\frac{dH_2}{dt}=q_2SH_2-d_4H_2-u_2H_2$$

With the initial condition,

$$S(0)>0, I(0)>0, H_1(0)>0, H_2(0)>0$$

Functional objetive J formulates optimization problems to identify effective strategies. The optimal control strategy has the aim of controlling costs related Infected tea compartment, *Empoasca* pests, leaf rolling caterpillar pests, and the cost of applying insecticides and fungisides. The objective functional of (4.1) is defined as:

$$J(U)=\int_0^{tf}\left(\omega_1I(t)+\omega_2H_1(t)+\omega_3H_2(t)+\frac{v_1}{2}u_1^2(t)+\frac{v_2}{2}u_2^2(t)\right)dt$$

Where tf is the final time and the coefficients of $\omega_1,\omega_2,\omega_3,v_1,v_2$ balance the costs factor caused by the scale and importance of the five parts of the objective function. To find the optimal control on u_1^*,u_2^* use,

$J(u_1^*,u_2^*)=\min\{J(u_1,u_2)\ |u_1,u_2\in U\}$

The Hamiltonian function of (4.1) as :

$$H=\omega_1I+\omega_2H_1+\omega_3H_2+\frac{v_1}{2}u_1^2+\frac{v_2}{2}u_2^2$$
$$+\lambda_1\left(rS\left(1-\frac{S+I}{K}\right)-\beta SI-p_1SH_1-p_2SH_2-(c_1+d_1)S\right)$$
$$+\lambda_2(\beta SI-(c_2+d_2)I-u_1I)+\lambda_3(q_1SH_1-d_3H_1-u_2H_1)$$
$$+\lambda_4(q_2SH_2-d_4H_2$$
$$-u_2H_2)$$

Theorem 4. There exist optimal controls u_1^*,u_2^* and the solution S^*,I^*,H_1^*,H_2^* on system (4.1) that minimize $J(u_1,u_2)$ on $U=\{u_1,u_2\}$, when there are adjoint variable $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ that satisfied:

$$-\frac{d\lambda_1}{dt}=-\lambda_1\left(r-\frac{r(S+I)}{K}-\frac{rS}{K}-\beta I-p_1H_1-p_2H_2-c_1-d_1\right)-\lambda_2\beta I-\lambda_3q_1H_1-\lambda_4q_2H_2$$
$$-\frac{d\lambda_2}{dt}=-\omega_1-\lambda_1\left(-\frac{rS}{K}-\beta S\right)-\lambda_2(\beta S-c_2-d_2-u_1)$$
$$-\frac{d\lambda_3}{dt}=-\omega_2-\lambda_1p_1S-\lambda_3(q_1S-d_3-u_2)$$

$$-\frac{d\lambda_4}{dt} = -\omega_3 + \lambda_1 p_2 S - \lambda_4(q_2 S - d_4 - u_2)$$

where $\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t) = \lambda_4(t_f) = 0$ are trenversality conditions and the optimal control $(u_1^*(t), u_2^*(t))$ satistfied the optimalitas conditions.

Proof.

We use the Pontryagin Maximum Principle to get the optimal control solutions. Differentiate the Hamiltonian equation (6.3) to u_1, u_2 and evaluated the optimal control variable as:

$$0 = \frac{\partial H}{\partial u_1} = -\lambda_2 I + v_1 u_1$$

$$0 = \frac{\partial H}{\partial u_2} = v_2 u_2 - \lambda_3 H_1 - \lambda_4 H_2$$

From this, we get the optimal control u_1^*, u_2^* as:

$$u_1^* = \frac{\lambda_1 I}{v_1}$$

$$u_2^* = \frac{\lambda_3 H_1 + \lambda_4 H_2}{v_2}$$

There for optimal control variables of u_1^*, u_2^* characterize by:

$$u_1^* = \begin{cases} 0 & jika & \psi_1^* \leq 0 \\ \psi_1^* & jika & 0 \leq \psi_1^* \leq 1 \\ 1 & jika & \psi_1^* \geq 1 \end{cases}$$

$$u_2^* = \begin{cases} 0 & jika & \psi_2^* \leq 0 \\ \psi_2^* & jika & 0 \leq \psi_2^* \leq 1 \\ 1 & jika & \psi_2^* \geq 1 \end{cases}$$

where

$$\psi_1^* = \frac{\lambda_1 I}{v_1}$$

$$\psi_2^* = \frac{\lambda_3 H_1 + \lambda_4 H_2}{v_2}$$

IV. NUMERICAL SIMULATION

In this section, numerical results are presented for the model with control and the model without control. Using Maple software version "Maple 12" and data from PT RSM at 2015-2019, we simulated the model (2.1) and (4.1). The values of parameters are given in the Table 2.

Table 2. Parameter Values

Notation	Value	Unit	References
K	3,43	individual	estimated
r	0,344	semester^{-1}	estimated
β	0.019	semester^{-1}	estimated
p_1	0.0022	(individual.year)$^{-1}$	estimated
p_2	0.006	(individual.year)$^{-1}$	estimated
q_1	0.0017	(individual.year)$^{-1}$	estimated
q_2	0.004	semester^{-1}	estimated
c_1	0.00001	semester^{-1}	estimated
d_1	0,00099	semester^{-1}	estimated
c_2	0.063	semester^{-1}	estimated
d_2	0.003	semester^{-1}	estimated
d_3	0.061	semester^{-1}	estimated
d_4	0.016`1		estimated

From the values of the parameters contained in the table 2 we obtained numerical simulation results, namely the model with control will use the Runge-Kutta Order-4 algorithm, where to solve the system state using the forward Runge-Kutta algorithm, while the backward Runge-Kutta is used to solve the system state. complete the co-state system. The period of time used is 5 years.

We use the initial condition of system (2.1) as follows. $S(0) = 3448191$, $I(0) = 223, H_1(0) = 986, H_2(0) = 746$.

Further, we obtained the graph of the solution $S(t), I(t), H_1(t), H_2(t)$ for 5 years, as given in the Figure 1.

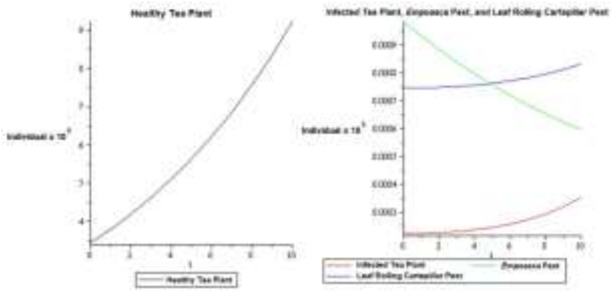


Figure 1. Solution of Dynamic Model Tea plant, Pest, and Disease

Figure 1. shows the changes in the population for 5 years. The population of healthy tea plant, infected tea plant, and leaf rolling caterpillar pest have increased, while the

population of *Empoasca* pest has decreased. But the graph have not shown the stability. Furthermore, from the data, the 5 equilibrium point (disease free equilibrium point) is locally asymptotically stable. The basic reproduction number is 0,99. So because the $\Re_0 < 1$, there is no spread of disease in the tea plantation ecosystem.

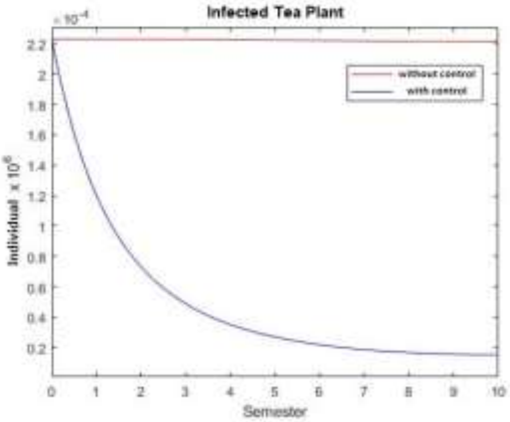


Figure 2. Infected Tea Plants with control and without control

The simulation model in Figure 2. shows the effect after giving of a fungicide control (u_1) on tea plants infected with the disease. It can be seen that after giving of control, the graph of infected tea (Infected) is lower than before giving of control so that *Blister Blight* disease can be reduced. From the graph above, it is shown that the population of infected tea plants decreased from 221 individuals to 15 individuals.

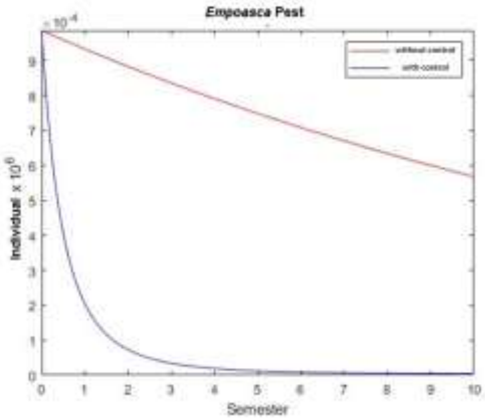


Figure 3. *Empoasca* Pest Plants with control and without control

The simulation model in Figure 3. shows the effect after giving insecticide control (u_2) on *Empoasca* pests. It can be seen that after giving a control, the *Empoasca* pest graph was lower than before giving a control so that the attack of the *Empoasca* pest could be reduced. From the graph above, it is shown that the *Empoasca* pest population was reduced from 576 individuals to 3 individuals.

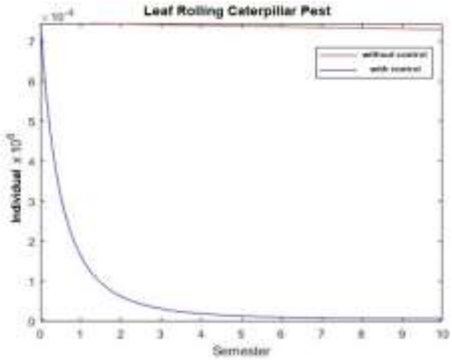


Figure 4. Leaf Roller Caterpillar Pests when with control and without control

The simulation model in Figure 4. shows the effect after giving insecticide control (u_2) on leaf rolling caterpillar pest. It can be seen that after giving a control, the leaf roller caterpillar pest graph was lower than before giving a control so that the leaf rolling caterpillar pest attack could be reduced. From the graph above, it is shown that the leaf rolling caterpillar pest population decreased from 729 individuals to 5 individuals.

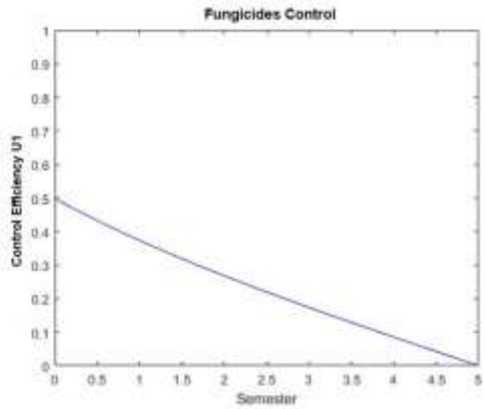


Figure 5. Control Conditions u_1

The value of the controller u1 in Figure 5. ranges from $0 \leq u_1 \leq 1$. The percentage of fungicides given to fungi from $t = 0$ to $t = 5$ decreased gradually from a dose of $0{,}4$ to 0. Giving control u_1 which was not maximal at 100% still had an impact on increasing the compartment of healthy tea plants and decreasing the tea compartment infected with the disease caused by fungus.

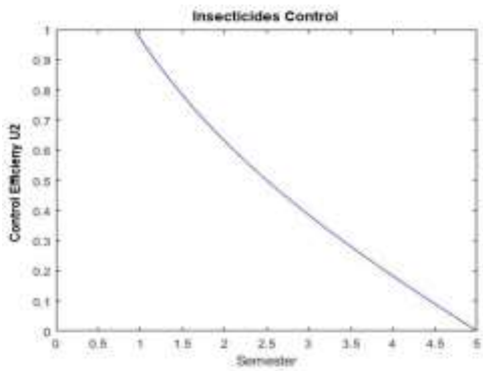


Figure 6. Control Conditions u_2

The value of the controller u_2 in Figure 6. ranges from $0 \leq u_2 \leq 1$. Giving insecticides to *Empoasca* pests and leaf-rolling caterpillars pests was carried out maximally and maintained until $t = 1{,}25$, then decreased gradually until $t = 5$ reached 0. Giving control of u_2 which is not maximal at 100% still has an impact on increasing the compartment of healthy tea plants and decreasing the compartments of *Empoasca* pests and leaf rolling caterpillars pests.

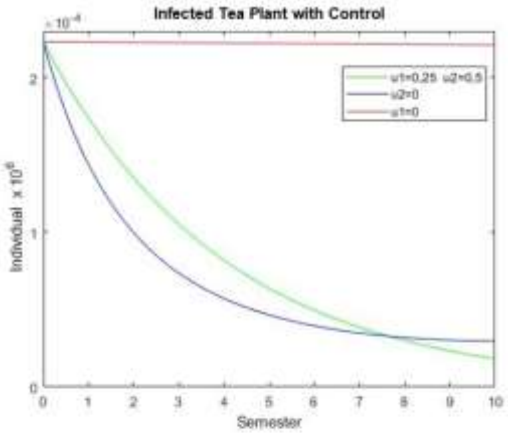


Figure 7. Comparison of the effectiveness of the control performance u_1 , u_2

Figure 7. shows a single comparison of giving the fungicide control (u_1), insecticide control (u_2), and the combination control of the two controls. In this case, the overall efficiency is defined as $\bar{\epsilon} = 1 - (1 - u_1)(1 - u_2) = 0{,}6$. This means that giving a combination control of fungicide and insecticide is more effective than giving a single control of fungicide and insecticide control.

From the overall results obtained, the simultaneous giving control of u_1 , u_2 will have an impact on decreasing infected tea compartments, *Empoasca* pests, and leaf rolling caterpillar pests. The value of weight from objectif function (4.2) are $0 \leq \omega_i, v_j \leq 0$, $i = 1,2,3$, $j = 1,2$ dan $\omega_1 = 0{,}25$, $\omega_2 = 0{,}1$, $\omega_3 = 0{,}15$, $v_1 = 0{,}25$, $v_2 = 0{,}25$ with the resulting objective functional value is 29.7326.

V. CONCLUSION

Based on the result of this research, it can be obtained the development of a dynamic model of interaction between tea plant, pest, and disease by adding two controls. The model was developed by combining the logistic growth model, the SI (Susceptible Infected) epidemic model, and the predator prey model by ading fungicides and insecticides controls. From the development of the model in this research, we get five equilibrium point. The we analyze the local and global stability of endemik and non-endemik point. Based on the data, we get that non-endemic point (the equilibrium point 5) is locally asymptotically stable with the basic reproduction number is 0.99, this indicates that $\Re_0 < 1$, so that after a period of 5 years there has been no spread of disease in the tea plantation ecosystem at PT RSM.

Furthermore, for the control model, it is obtained optimal control solution using Pontryagin’s maximum principle which can provide an overview of pest and disease control by choosing the optimal fungicides and insecticides controls strategy. From the numerical solution results can be obtained comparison of each compartment when given control and when whitout control. The giving of fungicides and insecticides control can reduce the popupation of infected tea, Empoasca pest, and leaf rolling caterpillar pest. After being given contro,the population of infected tea plant decreased by 93.21%, Empoasca pest decreased by 99.47%, and leaf rolling caterpillar pest decreased by 99.31% compared to the model that was not given control. So that the provision of optimal control can simultaneously reduce the spread of disease and control pest attacks.

VI. ACKNOWLEDGMENT

This work is supported by Diponegoro University, Indonesia, under PNBP FSM Research Grant with contract number 3094F/UN7.5.8/PP/2022. This research was supported by the Laboratory of Computer Modelling Mathematics Department, Faculty of Science and Mathematics, Diponegoro University, Semarang, Indonesia

REFERENCES

1. Eskundari, R. D. (2020): Budidaya jaringan tanaman teh di Indonesia, Bioma : Jurnal Biologi Makassar, Vol. 5(2) : 121-130, ISSN : 2528-7168 (printed) ; 2548-6659 (online).
2. Strzelecka, A., Kujawska, K. A., and Zawadzka, D. (2020): Application of logistic regression models to assess household financial decisions regarding debt, 24th International Conference on Knowledge-Based and Intelligent Information & Engineering System, Procedia Computer Science 176, 3418-3427.
3. Naji R. K. and Hussien, R. M., The Dynamics of Epidemic Model with Two Types of Infectious Diseases and Vertical Transmission, Journal of applied mathematics, 2016, (2016).
4. Hao, P., Wei, J. and Fan, D. Analysis of dynamics in an eco-epidemiological model with stage structure, Advances in Difference Equations 2016(1),(2016).
5. Shabbir, M. S., Din, Q , Muhammad Safeer, M., Khan, M. A., and Ahmad, K. A dynamically consistent nonstandard finite difference scheme for a predator–prey model. Advances in Difference Equations (2019) 2019:381
6. Kant, S. and Kumar, V. (2017): Stability analysis of predator-prey system with migrating prey and disease infection in both species, Applied Matehmatical Modelling, 42, 509-539

7. Bezabih, A. F., Edessa, G. K., and Rao, K. P. (2021): Ecoepidemiological model and analysis of prey-predator system, Journal of Applied Mathematics, Article ID 6679686, 17 pages
8. Becker, O. and Wildburger, U. L. (2020): Optimal dynamic control of predator-prey models, Central European Journal of Operations Research, 28, 425-440.
9. Djilali, S and Ghanbar, B. Dynamical behavior of two predators–one prey model with generalized functional response and time-fractional derivative. Advances in Difference Equations Advances in Difference Equations (2021) 2021:235.
10. Bai, D., Zhang, X. Dynamics of a Predator–Prey Model with the Additive Predation in Prey. Mathematics 2022, 10, 655. https://doi.org/10.3390/math10040655.
11. Meng, X.; Li, J. Dynamical Behavior of a Delayed Prey-Predator-Scavenger System with Fear Effect and Linear Harvesting. Int. J. Biomath. 2020, 14, 2150024
12. Aldila, D., Ndii, M. Z., and Samiadji, B. M. (2020): Optimal control on Covid-19 eradication program in Indonesia under teh effect of community awareness, Matehmatical Biosciences and Engineering, Research Article MBE, 17(6): 6355-6389.
13. Bessonov, N., Bocharov, G., Touaoula, T.M., Trofimchuk, S., Volpert, V.: Delay reaction–diffusion equation for infection dynamics. Discrete Contin. Dyn. Syst., Ser. B 24(5), 2073–2091 (2019)
14. Syamsir. (2020): Analisis Kestabilan dan Kontrol Optimal Model Matematika Dinamik Perceraian, Tesis Program Studi Magister, Fakultas Matematika dan Ilmu Pengetahuan Alam Universitas Hasanuddin Makassar.
15. Chasnov, J. R. (2016): Matehmatical biology, Lecture notes for MATH 4333, 1-14.
16. Mansur, Ansar, A., dan Fardinah. (2020): Analisis model predator prey dengan adanya penyakit pada prey dan pemanenan pada predator, Journal of Mathematics: Theory and Applications, Vol. 2, No. 2, P-ISSN 2685-9653 e ISSN 2722-2705.
17. Naji, R. K., and Majeed, S. J. The Dynamical Analysis of a Delayed prey-Predator Model with a Refuge-Stage Structure Prey Population. Iranian Journal of Mathematical Sciences and Informatics Vol. 15, No. 1 (2020), pp 135-159
18. Selvam, A. G. M., Janagaraj, R and Jacintha, M. Dynamical analysis of a discrete fractional order prey-predator system incorporating a prey refuge with Holling type II response. Journal of Physics: Conference Series 1597 (2020) 012008
19. Castillo-Chavez, C., & Song, B. (2004): Dynamical models of tuberculosis and their applications, Mathematical Biosciences and Engineering, MBE, 1(2), 361–404. https://doi.org/10.3934/mbe.2004.1.361).
20. Pal, A.K., Bhattacharyya, A., and Mondal, A. (2022). Qualitative analysis and control of predator switching on an eco-epidemiological model with prey refuge and hasvesting. Science Direct. Results In Control and Optimization 7 100099.
21. Das, N., and Bhattacharya, R. (2020): Eigen value analysis in lower bounding uncertainty of kalman filter estimated, Conference Paper Archive, IFAC PaperOnline 53-2 5022-5027.
22. Li, J., 2011. An algebraic approach to proving the global stability of a class of epidemic models. Nonlinear Analysis: Real World Applications. 13(5):2006–2016.
23. Kuniya, T., Touaoula, T.M.: Global stability for a class of functional differential equations with distributed delay and non-monotone bistable nonlinearity. Math. Biosci. Eng. 17(6), 7332–7352 (2020)