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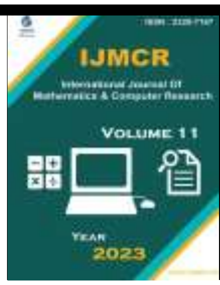

Optimal Inventory Decisions for a Vendor with Perishable Items

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ARTICLE INFO	ABSTRACT
Published Online: 25 January 2023 Corresponding Author: **Nathaniel Kayode Oladejo**	Retailers of perishable goods are usually confronted with the inventory decision problems. The growth of retail business of this nature depends largely on the quality of the inventory decisions. In this paper the optimal price and trade cycle length is determined and used to determined the optimal order quantity for each trade cycle. This will provide the retailer with guidelines to make efficient decisions for maximizing the gains of business operations.

KEYWORDS: Perishable, Deterioration, Inventory, Stock, Shelf life, Retail.

1. INTRODUCTION

Retailers are confronted with inventory decision problems such as how long to keep the goods in store, what quantity of goods should be ordered for each trade cycle and how much to sell the goods.They normally wish to make decisions that will result in maximum profit or minimum loss. Many researchers have developed models to ease the process of inventory decision. The study of inventory optimization dates back to the19th century when [1] developed a model to optimize cash reserves to satisfy random withdrawals. A single period model also known as the news vendor model was formulated by [2] to optimize operations with products that loose value after a specific period. Man extension has been made to this classical model due change in the suitability of the original assumptions. These extension can be seen in research papers such as [3],[4] and [5]. These extension aim to reduce the difficulty decision making process by including new factors that affect the retail businesses. There are a lot of factors that affect retailer business especially when dealing with perishable items. There is the risk of deterioration when the goods are not sold on time. When demand exceed the ordered quantity there will be opportunity loss due to shortage. Business with perishable items is characterized by a short-selling season and highly volatile demand. Perishable goods are very sensitive to time, as time pass b the goods loose their value and after a specific period(life span), they are either salvaged or discarded. Several Mathematical models has propounded by researchers for determining the Economic Order Quantity(EOQ). Thus, the order quantity that minimizes the holding and other related costs to maximize profit. Management of the inventory of goods with expiry date, pharmaceuticals, vegetables and any other goods with a specific life span after which they cannot be sold need to be done cautiously to avoid unbearable losses. There are three main types of expenses which should be considered in developing effective inventory model for perishable gods. These are;

i. Order Cost, thus the cost incurred to acquire the goods.

ii. Holding Cost, thus the expenses incurred to store the goods until they are sold.

iii. Shortage Cost is the cost or loss as a result of the retailer's inability to service some demand.

In this paper we seek a Mathematical structure and procedure to obtain the optimal selling price, cycle length and order quantity for retailer by reviewing existing literature and exploring other techniques.

2. LITERATURE REVIEW

Maximizing gains of operations is a crucial part of business survival and growth. This reason lead to lots of research in developing efficient models for profit maximization. In the quest to provide retailers with efficient decision making mechanism, [6] developed an inventory model for perishable items with price and stock-level dependent demand rate,taking into consideration the shelflife and a non-linear holding cost. Six different price dependent function where tested on the model and showed that the profit function of the retailer business is concave meaning there is a unique maximum. [7] presented a simple method for finding solutions to inventory problems. It showed that the description of ordering policy can be done with a single equation irrespective of the sign of the covariance term. [8] proposed a model for retailers' ordering policies in EOQ model when trade in conducted on credit bases. An Economic Order quantity Model(EOQ) for optimal pricing and replenishment policies considering backlogging and shortages was developed in [9] to determine the unique optimal solution. Optimal price and inventory control of products was studied in [10] considering price and stock-level dependent demand to determine the impact of physical deterioration and freshness degradation of perishables on retailer's optimal decisions. An inventory model for perishable goods with a stock-level dependent demand rate was examined in [11] to determine the optimal decisions of retailer considering back-logging and rate of deterioration. Also an integrated inventory system for deteriorating items with back-order was examined by [12] considering the effect of marketing strategies on the profitability of the inventory system. Profit optimization in an inventory system with time-varying demand partial back-ordering and the discrete stock cycle presented by [13] derived an inventory model for profit maximization. This resulted in a non-linear problem which can be solved for optimal inventory decisions. A mathematical for retailer-to-individual customer for optimal supply methods under irregular demand patterns was examined by [14]. Irregular demand patterns was considered in determining the optimal supply strategy. [15] presented an optimization model considering the retailer's sale stimulating strategies. An inventory model for perishable items with exponential demand considering shortages was developed by [16]. In this model considered partial back-logging and deterioration to minimize inventory cost. An Economic Order Quantity(EOQ) model for items with constant shelf-life and falling demand based on time-to expiry technical note was examined and presented by [17]. This provided information on the impact of the expiry date on demand of good as the expiry date draw nearer.

The existing literature has provided great deal of information on retailer of perishable goods but in most of these literature complex calculations need to be done by the retailer when the decide to use the models provided. Due the stated reason intend to provide a simpler model and procedure for retailers in this line of business.

3. ASSUMPTIONS AND NOTATIONS

The following are the assumptions used;

1. Items deteriorate at a constant rate.
2. Items have specific shelf life.
3. Items cannot be sold or repaired at the end of the shelf life.
4. Salvage value and deterioration expenses are considered for items that deteriorate throughout the trade cycle.
5. Stock period does not surpass the items shelf life.
6. Holding cost per unit is constant.

Notations in the model

Q is the order quantity at the beginning of the trade cycle.

p is the retail price. c is the cost per unit item. s is the salvage value.

θ is the deterioration rate.

ϕ is the salvage ratio.

H is the holding cost per unit. c_d is the cost of deterioration.

T is the replenishment cycle length.

$d(p)$ is the price dependent function.

$I(t)$ is inventory level at time t.

4. FORMULATION OF THE LOSS FUNCTION

The retailer is at loss when the total cost of operation exceeds the total gain. On the other hand the retailer makes profit. That means when the result of the loss function is negative there is profit from the operation. In order to optimize the loss function, the factors that affect the business in terms of cost and revenue considered are; the purchase cost, the holding cost, deterioration cost, salvage value and sales revenue. Consider a change in inventory level depend on deterioration and demand which is dependent on price and stock-level. The rate of change in inventory level can be expressed in the form;

$$\frac{dI(t)}{dt} = -\frac{n-t}{n}d(p) - \omega I(t) - \theta I(t) \qquad (1)$$

When equation (1) is solved in the definite interval $0 \le t \ge T$ the stock-level at an given time t is given by

$$I(t) = \frac{d(p)}{n}\left|\frac{T-t}{\theta+\omega}\right| \qquad (2)$$

The Order quantity Q occurs at the beginning of the trade cycle. Thus at $t = 0$. Therefore

$$Q = \frac{d(p)}{n}\left|\frac{T}{\theta+\omega}\right| \qquad (3)$$

4.1. Purchase cost(PC)

This is cost involved in the acquisition of the items. It is given as

$$PC = Q \times c = \frac{d(p)T}{n(\theta+\omega)}c \qquad (4)$$

4.2. Holding Cost(HC)

The cost incurred in keeping goods in favourable conditions before sale is the holding cost. The longer the goods are kept the higher the holding cost.

$$HC = h\int_0^T I(t)dt = h\int_0^T \frac{d(p)(T-t)}{n(\theta+\omega)}dt \qquad (5)$$

$$\Rightarrow HC = \frac{hd(p)T^2}{2n(\theta+\omega)} \qquad (6)$$

4.3. Deterioration Cost (DC)

Lost in value of perishable goods as time goes by lead to the cost known as deterioration cost. It is given as;

$$DC = c_d\theta\left(Q - \int_0^T \frac{n-t}{n}d(p) + \omega I(t)dt\right.$$!

(7)

From (7)

$$DC = c_d\theta d(p)\left(\frac{T - n\omega T^2}{n(\theta+\omega)} - \frac{2nT - T^2}{2n}\right.$$!

(8)

4.4. Sales Revenue(SR)

The product of total quantity of goods sold and the selling price gives the sales revenue. It can be obtained from the expression

$$SR = p\int_0^T \frac{n-t}{n}d(p) + \omega I(t)dt \qquad (9)$$

From equation (9) sales revenue is obtained as;

$$SR = \frac{d(p)p}{2n}\left(T(2n-T) + \frac{\omega T^2}{\theta+\omega}\right.$$!

(10)

4.5. Salvage Value.(SV)

The quantity of goods that are not sold at the end of the trade cycle has to be discarded or given out at a lower price. The proceeds from given out goods for scrap value is the salvage value. It is in the form;

$$SV = s\phi(Q - \int_0^T \frac{n-t}{n}d(p) + \omega I(t)dt\ ) \qquad (11)$$!

$$\Rightarrow SV = s\phi d(p)\left(\frac{T - n\omega T^2}{n(\theta\omega)} - \frac{2nT - T^2}{2n}\right.$$

(12)

With both costs and revenue expressed in the equations above, the retailer loss can be calculated using the function,

$\pi(p,T) = (PC + DC + HC) - (PR + SV)$ (13)

$$\pi(p,T) = d(p)\left(\frac{c + c_d\theta - s\phi}{n(\theta+\omega)} + s\phi - c_d\theta - p\right)T + d(p)\left(\frac{h - 2np\omega - 2nc_d\theta\omega + 2s\phi n\omega}{2n(\theta+\omega)} + \frac{c_d\theta + p - s\phi}{2n}\right)T^2$$

(14)

4.6. Retailer’s optimal price and length.

The loss function $\pi(p,T)$ is convex in terms of p and T. Therefore there is a
unique minimum solution for the loss function. Considering the time constraint a non linear optimization below can be generated as follows;

$$Manimize\pi(p,T) = d(p)\left(\frac{c + c_d\theta - s\phi}{n(\theta+\omega)} + s\phi - c_d\theta - p\right)T + d(p)\left(\frac{h - 2np\omega - 2nc_d\theta\omega + 2s\phi n\omega}{2n(\theta+\omega)} + \frac{c_d\theta + p - s\phi}{2n}\right)T^2$$

(15)

Subject to:

$T - 1 \le 0$

5. NUMERICAL ANALYSIS OF RESULTS

In order to obtain the optimal values of p and T the values that produce the least value of the loss function is sort by minimizing the loss function with the time constraint. The optimization problem can be solved using the Karush Kuhn Tucker conditions. The non-linear optimization problem can also be solved using the Matlab function fmincon.

5.1. Illustration 1

Consider a retailer dealing with perishable goods has the following information; Cost price is $GHc5.00$, holding cost per unit is $GHc0.90$, deterioration rate 0.05,Deterioration cost $GHc2.00$ salvage price $GHc4.00$, Salvage ratio of 0.8, Shelf life is 1 week, inventory level dependent demand rate of 0.5 and price dependent demand of$600-20p$. with this information the retailer’s optimal quantity , price and cycle length is determined b solving the non-linear optimization problem below;

$$Min\Pi(p,T) = (600 - 20p)\left[\left(\frac{459}{220} + \frac{p}{22}\right)T^2\left(\frac{721}{110} - p\right)T\right]$$

(16)

Subject to:

$T - 1 \le 0\ p \ge 0, T \ge 0$

The optimal results obtained from solving (16) under the time constraint are as follows; Optimal selling price is $GHc19,53$ and the optimal cycle length is 1 week which is equal to the shelf life of the goods.This decision produces a profit of $GHc2094.30$ and an optimal order quantity of 231 units per trade cycle. The results are obtained using the Karush Kuhn Tucker conditions or the Matlab function *fmincon*.

5.2. Illustration 2

Given that the Cost of perishable goods is $GHc7.00$ deteriorating at the rate of 0.1 at the cost of $GHc2.00$. If the remaining goods can be salvaged at $GHc4.00$ with an inventory level dependent rate of 0.5 and a holding cost $GHc2.00$ per unit. If price dependent demand of the items is exponential in the form $d(p) = 2000e^{-0.2p}$ Then the non-linear optimization problem takes the form;

$$Min\pi(p,T) = (2000e^{-0.2p})\left[\left(\frac{16}{3} + \frac{7p}{12}\right)T^2 + (13 - p)T\right]$$

(17)

Subject to;

$T - 1 \le 0, p \ge 0, T \ge 0$

From the optimization problem (17) the optimal solution is obtained as; $p^* =$
$GHc24.65, T^* = 0.2783$ of the shelf life and this decision leads to profit of 130 $GHc188.26$ dealing with 59 units per trade cycle.

6. CONCLUSION

Trading with perishable is a risky business since after sometime the items become worthless. The non-linear optimization problem in this paper provides the retailer with guidance to make decisions that would not lead to losses that may be unbearable for the vendor. The non-linear optimization can be solved manually with the KKT conditions or using matlab and other Mathematical programmes. The efficiency of business operation can be lifted when the vendor knows the right quantity to order, the most efficient price and the best time to salvage goods. Waste will be minimized and returns of business will be maximised. It is best for vendors to survey the business environment and make proper calculation before starting operation. In future research the risk-averse vendor can be considered to provide insights into how the risk resistance affect the business when dealing with perishable goods.

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