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Atom Bond Connectivity E-Banhatti Indices

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ARTICLE INFO	ABSTRACT
Published Online: 28 January 2023 Corresponding Author: **V.R.Kulli**	In this paper, we introduce the atom bond connectivity E-Banhatti index and the sum atom bond connectivity E-Banhatti index of a graph. Also we compute these newly defined atom bond connectivity E-Banhatti indices for wheel graphs, friendship graphs, chain silicate networks, honeycomb networks and nanotubes.

KEYWORDS: atom bond connectivity E-Banhatti index, sum atom bond connectivity E-Banhatti index, chain silicate network, honeycomb network, nanotube.

I. INTRODUCTION

We consider only simple, finite, connected graphs. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The edge e connecting the vertices u and v is denoted by uv. If $e=uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e=uv$. For term and concept not given here, we refer [1].

Mathematical Chemistry has an important effect on the development of Chemical Sciences. Several graph indices [2] have found some applications in Chemistry, especially in QSPR/QSAR research [3].

In [4], Kulli defined the Banhatti degree of a vertex u of a graph G as

$$B(u) = \frac{d_G(e)}{n - d_G(u)},$$

where $|V(G)| = n$ and the vertex u and edge e are incident in G.

In [4], Kulli proposed the first and second E-Banhatti indices of a graph G and they are defined as

$$EB_1(G) = \sum_{uv \in E(G)} \left[B(u) + B(v)\right],$$

$$EB_2(G) = \sum_{uv \in E(G)} B(u)B(v).$$

Recently, some E-Banhatti indices were introduced and studied in [5, 6, 7, 8, 9, 10].

We put forward the atom bond connectivity E-Banhatti index of a graph G as follows:

The atom bond connectivity E-Banhatti index of a graph G is defined as

$$AEB(G) = \sum_{uv \in E(G)} \sqrt{\frac{B(u) + B(v) - 2}{B(u)B(v)}}.$$

In view of the atom bond connectivity E-Banhatti index, we define the sum atom bond connectivity E-Banhatti of a graph G as

$$SAEB(G) = \sum_{uv \in E(G)} \sqrt{\frac{B(u) + B(v) - 2}{B(u) + B(v)}}.$$

Recently, some atom bond connectivity indices were studied in [11, 12, 13].

In Chemical Graph Theory, several graph indices were introduced and studied such as the Zagreb indices [14, 15, 16], the Gourava indices [17, 18, 19, 20], the Revan indices [21, 22, 23, 24].

In this paper, we compute the atom bond connectivity E-Banhatti index and sum atom bond connectivity E-Banhatti index for wheel graphs, friendship graphs, some important molecular structures such as chain silicate networks, honeycomb networks and nanotubes.

II. RESULTS FOR SOME STANDARD GRAPHS

2.1. Atom Bond connectivity E-Banhatti Index

Proposition 1. If G is an r-regular graph with n vertices and $r \geq 2$, then

$$AEB(G) == \frac{nr\sqrt{(6r-2n-4)(n-r)}}{2(2r-2)}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$. Then $|E(G)| = \frac{nr}{2}$. For any edge $uv=e$ in G, $d_G(e)=2r-2$. Then

$$AEB(G) = \sum_{uv\in E(G)} \sqrt{\frac{B(u)+B(v)-2}{B(u)B(v)}}$$

$$= \frac{nr}{2}\left[\left(\frac{2r-2}{n-r}+\frac{2r-2}{n-r}-2\right)\div\left(\frac{2r-2}{n-r}\times\frac{2r-2}{n-r}\right)\right]^{\frac{1}{2}}$$

$$= \frac{nr\sqrt{(6r-2n-4)(n-r)}}{2(2r-2)}.$$

Corollary 1.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$AEB(C_n) = \frac{n\sqrt{(8-2n)(n-2)}}{2}.$$

Corollary 1.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$AEB(K_n) = \frac{n(n-1)\sqrt{4n-10}}{4(n-2)}.$$

Proposition 2. Let P_n be a path with $n \geq 3$ vertices. Then

$$AEB(P_n) = \frac{2\left(\frac{1}{n-1}+\frac{2}{n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{1}{n-1}\times\frac{2}{n-2}\right)^{\frac{1}{2}}} + \frac{(n-3)\left(\frac{2}{n-2}+\frac{2}{n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{2}{n-2}\times\frac{2}{n-2}\right)^{\frac{1}{2}}}$$

$$= \sqrt{2}\sqrt{9n-2n^2-6} + \frac{1}{2}(n-3)\sqrt{8-2n}\sqrt{n-2}.$$

Proposition 3. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ *and* $n \geq 2$. Then

$$AEB(K_{m,n}) = \frac{mn\sqrt{m^2+n^2-2(m+n)}}{m+n-2}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices and mn *edges* such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Then $d_G(e)= d_G(u)+ d_G(v) -2= m + n -2$.

$$AEB(K_{m,n}) = \sum_{uv\in E(G)} \sqrt{\frac{B(u)+B(v)-2}{B(u)B(v)}}$$

$$= \frac{mn\left(\frac{m+n-2}{m+n-n}+\frac{m+n-2}{m+n-m}-2\right)^{\frac{1}{2}}}{\left(\frac{m+n-2}{m+n-m}+\frac{m+n-2}{m+n-m}\right)^{\frac{1}{2}}}$$

$$= \frac{mn\sqrt{m^2+n^2-2(m+n)}}{m+n-2}.$$

Corollary 3.1. Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$AEB(K_{n,n}) = \frac{n^2\sqrt{2n(n-2)}}{2(n-1)}.$$

Corollary 3.2. Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$AEB(K_{1,n}) = \frac{n\sqrt{n^2-2n-1}}{n-1}.$$

2.1. Sum Atom Bond Connectivity E-Banhatti Index

Proposition 4. If G is an r-regular graph with n vertices and $r \geq 2$, then

$$SAEB(G) = \frac{nr\sqrt{(3r-n-2)}}{2\sqrt{2(r-1)}}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$. Then $|E(G)| = \frac{nr}{2}$. For any edge $uv=e$ in G, $d_G(e)=2r-2$. Then

$$SAEB(G) = \sum_{uv\in E(G)} \sqrt{\frac{B(u)+B(v)-2}{B(u)+B(v)}}$$

$$= \frac{nr}{2}\left[\left(\frac{2r-2}{n-r}+\frac{2r-2}{n-r}-2\right)\div\left(\frac{2r-2}{n-r}+\frac{2r-2}{n-r}\right)\right]^{\frac{1}{2}}$$

$$= \frac{nr\sqrt{(3r-n-2)}}{2\sqrt{2(r-1)}}.$$

Corollary 4.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$SAEB(C_n) = \frac{n\sqrt{(4-n)}}{\sqrt{2}}.$$

Corollary 4.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$SAEB(K_n) = \frac{n(n-1)\sqrt{2n-5}}{4\sqrt{2}\sqrt{n-2}}.$$

Proposition 5. Let P_n be a path with $n \geq 3$ vertices. Then

$$SAEB(P_n) = \frac{2\left(\frac{1}{n-1}+\frac{2}{n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{1}{n-1}+\frac{2}{n-2}\right)^{\frac{1}{2}}} + \frac{(n-3)\left(\frac{2}{n-2}+\frac{2}{n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{2}{n-2}+\frac{2}{n-2}\right)^{\frac{1}{2}}}$$

$$= \frac{2\sqrt{9n-2n^2-6}}{\sqrt{3n-4}} + \frac{1}{2}(n-3)\sqrt{8-2n}.$$

Proposition 6. Let $K_{m,n}$ be a complete bipartite graph with $1 \le m \le n$ *and* $n \ge 2$. Then

$$SAEB(K_{m,n}) = \frac{mn\sqrt{m^2+n^2-2(m+n)}}{m+n-2}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices and mn *edges* such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \le m \le n$, and $n \ge 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Then $d_G(e) = d_G(u) + d_G(v) - 2 = m + n - 2$.

$$SAEB(K_{m,n}) = \sum_{uv\in E(G)} \sqrt{\frac{B(u)+B(v)-2}{B(u)+B(v)}}$$

$$= \frac{mn\left(\frac{m+n-2}{m+n-n}+\frac{m+n-2}{m+n-m}-2\right)^{\frac{1}{2}}}{\left(\frac{m+n-2}{m+n-m}+\frac{m+n-2}{m+n-m}\right)^{\frac{1}{2}}}$$

$$= \frac{mn\sqrt{m^2+n^2-2(m+n)}}{m+n-2}.$$

Corollary 6.1. Let $K_{n,n}$ be a complete bipartite graph with $n \ge 2$. Then

$$SAEB(K_{n,n}) = \frac{n^2\sqrt{2n(n-2)}}{2(n-1)}.$$

Corollary 6.2. Let $K_{1,n}$ be a star with $n \ge 2$. Then

$$SAEB(K_{1,n}) = \frac{n\sqrt{n^2-2n-1}}{n-1}.$$

III. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F_4 is shown in Figure 1. A friendship graph F_n is a graph with $2n+1$ vertices and $3n$ edges.

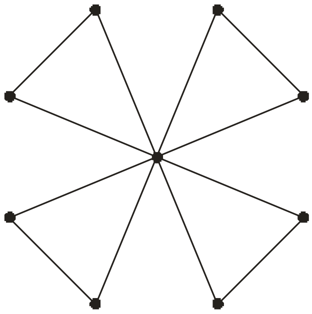

Figure 1. Friendship graph F_4

In F_n , there are two types of edges as follows:

$$E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}, \qquad |E_1| = n.$$

$$E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}, \quad |E_2| = 2n.$$

Therefore, in F_n, we obtain that $\{B(u), B(v): uv$ □ $E(F_n)\}$ has two Banhatti edge set partitions.

$BE_1 = \{uv \in E(F_n) \mid B(u) = B(v) = \frac{2}{2n-1}\}$, $\qquad |BE_1| = n$.

$BE_2 = \{uv \in E(F_n) \mid B(u) = \frac{2n}{2n-1}, B(v) = 2n\}$, $\qquad |BE_2| = 2n$.

We calculate the atom bond connectivity E-Banhatti index and the sum atom bond connectivity E-Banhatti index of a friendship graph F_n as follows:

Theorem 1. Let F_n be a friendship graph. Then

(i) $$AEB(F_n) = \frac{1}{2}n\sqrt{6-4n}\sqrt{2n-1} + \sqrt{n}\sqrt{4n^2-6n+4}.$$

(ii) $$SAEB(F_n) = \frac{1}{2}n\sqrt{6-4n} + \frac{2n\sqrt{4n^2-6n+4}}{\sqrt{4n^2-2n+2}}.$$

Proof: Applying definition and Banhatti edge partition of F_n, we conclude

(i) $$AEB(F_n) = \sum_{uv\in E(F_n)} \sqrt{\frac{B(u)+B(v)-2}{B(u)B(v)}}$$

$$= \frac{n\left(\frac{2}{2n-1}+\frac{2}{2n-1}-2\right)^{\frac{1}{2}}}{\left(\frac{2}{2n-1}\times\frac{2}{2n-1}\right)^{\frac{1}{2}}} + \frac{2n\left(\frac{2}{2n-1}+2n-2\right)^{\frac{1}{2}}}{\left(\frac{2}{2n-1}\times 2n\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$SAEB(F_n) = \sum_{uv\in E(F_n)} \sqrt{\frac{B(u)+B(v)-2}{B(u)+B(v)}}$$

$$=\frac{n\left(\frac{2}{2n-1}+\frac{2}{2n-1}-2\right)^{\frac{1}{2}}}{\left(\frac{2}{2n-1}+\frac{2}{2n-1}\right)^{\frac{1}{2}}}+\frac{2n\left(\frac{2}{2n-1}+2n-2\right)^{\frac{1}{2}}}{\left(\frac{2}{2n-1}+2n\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

IV. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of C_n and K_1. Then W_n has n+1vertices and 2n edges. A graph W_n is presented in Figure 2.

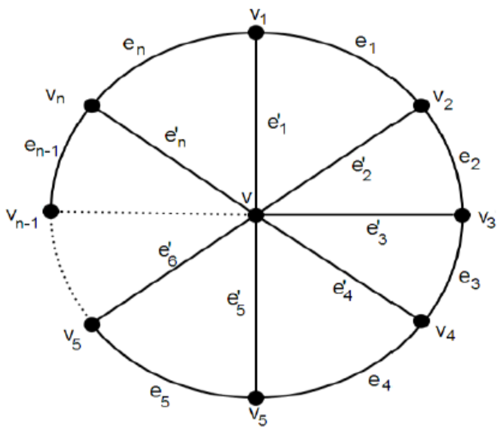

Figure 2. Wheel graph W_n

In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}$, $|E_2| = n$.

Therefore, in W_n, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$BE_1 = \{uv \in E(W_n) \mid B(u) = B(v) = \frac{4}{(n-2)}\}$, $|BE_1| = n$.

$BE_2 = \{uv \in E(W_n) \mid B(u) = \frac{n+1}{n-2}, B(v) = n+1\}$, $|BE_2| = n$.

We calculate the atom bond connectivity E-Banhatti index and the sum atom bond connectivity E-Banhatti index of a wheel graph W_n as follows:

Theorem 2. Let W_n be a wheel graph. Then

(i) $$AEB(W_n)=\frac{1}{4}n\sqrt{12-2n}\sqrt{n-2}+\frac{n\sqrt{n^2-2n+3}}{n+1}.$$

(ii) $$SAEB(W_n)=\frac{1}{2\sqrt{2}}n\sqrt{12-2n}+\frac{n\sqrt{n^2-2n+3}}{\sqrt{n^2-1}}.$$

Proof: Applying definition and Banhatti edge partition of W_n, we conclude

(i) $$AEB(W_n)=\sum_{uv\in E(W_n)}\sqrt{\frac{B(u)+B(v)-2}{B(u)B(v)}}$$

$$=\frac{n\left(\frac{4}{n-2}+\frac{4}{n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{4}{n-2}\times\frac{4}{n-2}\right)^{\frac{1}{2}}}+\frac{n\left(\frac{n+1}{n-2}+(n+1)-2\right)^{\frac{1}{2}}}{\left(\frac{n+1}{n-2}\times(n+1)\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$SAEB(W_n)=\sum_{uv\in E(W_n)}\sqrt{\frac{B(u)+B(v)-2}{B(u)+B(v)}}$$

$$=\frac{n\left(\frac{4}{n-2}+\frac{4}{n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{4}{n-2}+\frac{4}{n-2}\right)^{\frac{1}{2}}}+\frac{n\left(\frac{n+1}{n-2}+(n+1)-2\right)^{\frac{1}{2}}}{\left(\frac{n+1}{n-2}+(n+1)\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

V. RESULTS FOR CHAIN SILICATE NETWORKS

Silicates are very important elements of Earth's crust. Sand and several minerals are constituted by silicates. A family of chain silicate network is symbolized by CS_n and is obtained by arranging $n\geq 2$ tetrahedral linearly, see Figure 3.

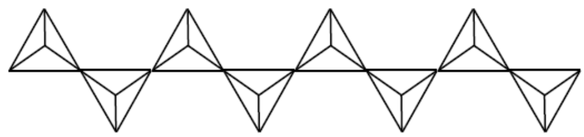

Figure 3. Chain silicate network

Let G be the graph of a chain silicate network CS_n with 3n+1 vertices and 6n edges. In G, by calculation, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(CS_n) \mid d_G(u) = d_G(v) = 3\}$, $|E_1| = n + 4$.

$E_2 = \{uv \in E(CS_n) \mid d_G(u) = 3, d_G(v) = 6\}$,$|E_2| = 4n - 2$.

$E_3 = \{uv \in E(CS_n) \mid d_G(u) = d_G(v) = 6\}$, $|E_3| = n - 2$.

Therefore, in CS_n, there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$BE_1 = \{uv \in E(CS_n) \mid B(u) = B(v) = \frac{4}{3n-2}\}$,

$|BE_1| = n+4$.

$BE_2 = \{uv\in E(CS_n) \mid B(u) = \frac{7}{3n-2}, B(v) = \frac{7}{3n-5}\}$,

$|BE_2| = 4n - 2$.

$BE_3 = \{uv \in E(CS_n) \mid B(u) = \frac{10}{3n-5},\ B(v) = \frac{10}{3n-5}\}$,

$|BE_3| = n - 2$.

We calculate the atom bond connectivity E-Banhatti index and the sum atom bond connectivity E-Banhatti index of a graph of CS_n as follows:

Theorem 3. Let CS_n be a chain silicate network. Then

(i) $$AEB(CS_n) = \frac{1}{4}(n+4)\sqrt{12-3n}\sqrt{3n-2} + \frac{1}{7}(4n-2)\sqrt{82n-18n^2-69} + \frac{1}{10}(n-2)\sqrt{30-6n}\sqrt{3n-5}.$$

(ii) $$SAEB(CS_n) = \frac{1}{2\sqrt{2}}(n+4)\sqrt{12-3n} + \frac{(4n-2)\sqrt{82n-18n^2-69}}{\sqrt{42n-49}} + \frac{1}{2\sqrt{5}}(n-2)\sqrt{30-6n}.$$

Proof: Applying definition and Banhatti edge partition of CS_n, we conclude

(i) $$AEB(CS_n) = \sum_{uv \in E(CS_n)} \sqrt{\frac{B(u)+B(v)-2}{B(u)B(v)}}$$

$$= \frac{(n+4)\left(\frac{4}{3n-2}+\frac{4}{3n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{4}{3n-2}\times\frac{4}{3n-2}\right)^{\frac{1}{2}}} + \frac{(4n-2)\left(\frac{7}{3n-2}+\frac{7}{3n-5}-2\right)^{\frac{1}{2}}}{\left(\frac{7}{3n-2}\times\frac{7}{3n-5}\right)^{\frac{1}{2}}} + \frac{(n-2)\left(\frac{10}{3n-5}+\frac{10}{3n-5}-2\right)^{\frac{1}{2}}}{\left(\frac{10}{3n-5}\times\frac{10}{3n-5}\right)^{\frac{1}{2}}}$$

gives the desired result after simplification.

(ii) $$SAEB(CS_n) = \sum_{uv \in E(CS_n)} \sqrt{\frac{B(u)+B(v)-2}{B(u)+B(v)}}$$

$$= \frac{(n+4)\left(\frac{4}{3n-2}+\frac{4}{3n-2}-2\right)^{\frac{1}{2}}}{\left(\frac{4}{3n-2}+\frac{4}{3n-2}\right)^{\frac{1}{2}}} + \frac{(4n-2)\left(\frac{7}{3n-2}+\frac{7}{3n-5}-2\right)^{\frac{1}{2}}}{\left(\frac{7}{3n-2}+\frac{7}{3n-5}\right)^{\frac{1}{2}}} + \frac{(n-2)\left(\frac{10}{3n-5}+\frac{10}{3n-5}-2\right)^{\frac{1}{2}}}{\left(\frac{10}{3n-5}+\frac{10}{3n-5}\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

VI. RESULTS FOR HONEYCOMB NETWORKS

Honeycomb networks are useful in Computer Graphics and Chemistry. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 4.

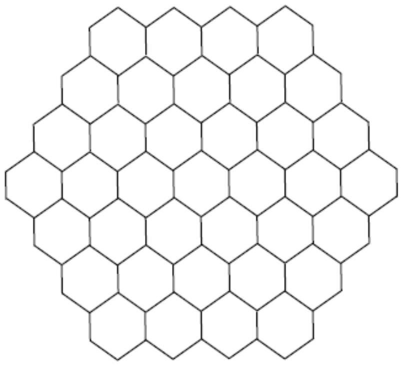

Figure 4. A 4-dimensional honeycomb network

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices and $9n^2–3n$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 2\}$,
$|E_1| = 6$.
$E_2 = \{uv \in E(HC_n) \mid d_G(u) = 2, d_G(v) = 3\}$,
$|E_2| = 12n – 12$.
$E_3 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 3\}$,
$|E_3| = 9n^2 – 15n + 6$.

Therefore, in HC_n, there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge as follow:

$BE_1 = \{uv \in E(HC_n) \mid B(u) = B(v) = \frac{2}{6n^2-2}\}$,

$|BE_1| = 6$.

$BE_2 = \{uv \in E(HC_n) \mid B(u) = \frac{3}{6n^2-2}, B(v) = \frac{3}{6n^2-3}\}$,

$|BE_2| = 12n - 12$.

$BE_3 = \{uv \in E(HC_n) \mid B(u) = B(v) = \frac{4}{6n^2-3}\}$,

$|BE_3| = 9n^2 - 15n + 6$.

We calculate the atom bond connectivity E-Banhatti index and the sum atom bond connectivity E-Banhatti index of a graph of HC_n as follows:

Theorem 4. Let HC_n be a honeycomb network. Then

(i)

$$AEB(HC_n) = 3\sqrt{8-12n^2}\sqrt{6n^2-2} + \frac{1}{3}(12n-12)\sqrt{96n^2-72n^4-27} + \frac{1}{4}(9n^2-15n+6)\sqrt{14-12n^2}\sqrt{6n^2-3}.$$

(ii)

$$SAEB(HC_n) = 3\sqrt{8-12n^2} + \frac{(12n-12)\sqrt{96n^2-72n^4-27}}{\sqrt{36n^2-15}} + \frac{1}{2\sqrt{2}}(9n^2-15n+6)\sqrt{14-12n^2}.$$

Proof: Applying definition and Banhatti edge partition of HC_n, we conclude

(i) $$AEB(HC_n) = \sum_{uv \in E(HC_n)} \sqrt{\frac{B(u)+B(v)-2}{B(u)B(v)}}$$

$$= \frac{6\left(\frac{2}{6n^2-2}+\frac{2}{6n^2-2}-2\right)^{\frac{1}{2}}}{\left(\frac{2}{6n^2-2}\times\frac{2}{6n^2-2}\right)^{\frac{1}{2}}} + \frac{(12n-12)\left(\frac{3}{6n^2-2}+\frac{3}{6n^2-3}-2\right)^{\frac{1}{2}}}{\left(\frac{3}{6n^2-2}\times\frac{3}{6n^2-3}\right)^{\frac{1}{2}}} + \frac{(9n^2-15n+6)\left(\frac{4}{6n^2-3}+\frac{4}{6n^2-3}-2\right)^{\frac{1}{2}}}{\left(\frac{4}{6n^2-3}\times\frac{4}{6n^2-3}\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$SAEB(HC_n) = \sum_{uv \in E(HC_n)} \sqrt{\frac{B(u)+B(v)-2}{B(u)+B(v)}}$$

$$= \frac{6\left(\frac{2}{6n^2-2}+\frac{2}{6n^2-2}-2\right)^{\frac{1}{2}}}{\left(\frac{2}{6n^2-2}+\frac{2}{6n^2-2}\right)^{\frac{1}{2}}} + \frac{(12n-12)\left(\frac{3}{6n^2-2}+\frac{3}{6n^2-3}-2\right)^{\frac{1}{2}}}{\left(\frac{3}{6n^2-2}+\frac{3}{6n^2-3}\right)^{\frac{1}{2}}} + \frac{(9n^2-15n+6)\left(\frac{4}{6n^2-3}+\frac{4}{6n^2-3}-2\right)^{\frac{1}{2}}}{\left(\frac{4}{6n^2-3}+\frac{4}{6n^2-3}\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

VII. RESULTS FOR HC_5C_7 [p, q] NANOTUBES

We consider $HC_5C_7[p, q]$ nanotubes, see Figure 5.

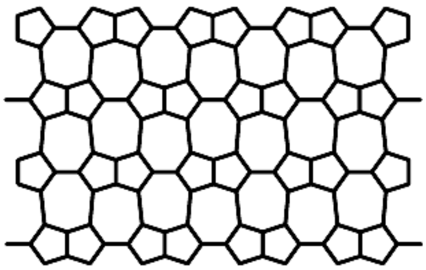

Figure 5. 2-*D* lattice of HC_5C_7 [8, 4] nanotube

The graphs of a nanotube HC_5C_7 [p, q] have $4pq$ vertices and $6pq - p$ edges are shown in above graph. Let $G = HC_5C_7[p, q]$.

In G, there are two types of edges as follows:

$E_1 = \{uv \in E(G) \mid d(u)=2, d(v) = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv \in E(G) \mid d(u)= d(v) = 3\}$, $|E_2| = 6pq - 5p$.

Therefore, in G, we obtain that $\{B(u), B(v): uv$ □ $E(NHPX[m, n])\}$ has two Banhatti edge set partitions.

$BE_1 = \{uv \in E(G) \mid B(u) = \frac{3}{4pq-2}, B(v) = \frac{3}{4pq-3}\}$,

$|BE_1| = 4p$.

$BE_2 = \{uv \in E(G) \mid B(u) = B(v) = \frac{4}{4pq-3}\}$,

$|BE_2| = 6pq-5p$.

We calculate the atom bond connectivity E-Banhatti index and the sum atom bond connectivity E-Banhatti index of a nanotube $HC_5C_7[p, q]$ as follows:

Theorem 5. Let $HC_5C_7[p, q]$ be a nanotube. Then

(i) $$AEB(G) = \frac{4}{3}p\sqrt{16p^2q^2 + 4pq - 9} + \frac{2}{3}(6pq - 5p)\sqrt{3 - 2pq}\sqrt{4pq - 3}.$$

(ii) $$SAEB(G) = \frac{4p\sqrt{16p^2q^2 + 4pq - 9}}{\sqrt{24pq - 15}} + \frac{2}{\sqrt{6}}(6pq - 5p)\sqrt{3 - 2pq}.$$

Proof: Applying definition and Banhatti edge partition of G, we conclude

(i) $$AEB(G) = \sum_{uv \in E(G)} \sqrt{\frac{B(u) + B(v) - 2}{B(u)B(v)}}$$

$$= \frac{4p\left(\frac{3}{4pq-2} + \frac{3}{4pq-3} - 2\right)^{\frac{1}{2}}}{\left(\frac{3}{4pq-2} \times \frac{3}{4pq-3}\right)^{\frac{1}{2}}} + \frac{(6pq-5p)\left(\frac{3}{4pq-3} + \frac{3}{4pq-3} - 2\right)^{\frac{1}{2}}}{\left(\frac{3}{4pq-3} \times \frac{3}{4pq-3}\right)^{\frac{1}{2}}}$$

gives the desired result after simplification.

(ii) $$SAEB(G) = \sum_{uv \in E(G)} \sqrt{\frac{B(u) + B(v) - 2}{B(u) + B(v)}}$$

$$= \frac{4p\left(\frac{3}{4pq-2} + \frac{3}{4pq-3} - 2\right)^{\frac{1}{2}}}{\left(\frac{3}{4pq-2} + \frac{3}{4pq-3}\right)^{\frac{1}{2}}} + \frac{(6pq-5p)\left(\frac{3}{4pq-3} + \frac{3}{4pq-3} - 2\right)^{\frac{1}{2}}}{\left(\frac{3}{4pq-3} + \frac{3}{4pq-3}\right)^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

VIII. CONCLUSION

In this study, we have introduced the atom bond connectivity E-Banhatti index and the sum atom bond connectivity E-Banhatti index of a graph. Also we have determined these newly defined E-Banhatti indices for some standard graphs, wheel graphs, friendship graphs, certain networks and nanotubes.

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