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Derivative Free Three-Step Iterative Method to solve Nonlinear Equations

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ARTICLE INFO	ABSTRACT
Published online: 27 February 2023 Corresponding Author: **Dyah Alfa Sa'adah Al-adawiyyah**	This article discusses a derivative free three-step iterative method to solve a nonlinear equation using Steffensen method, after approximating the derivative in the method proposed by Abro et al. [Appl. Math. Comput.,55(2019),516-536] by a divided difference method. We show analytically that the method is of order sixth under a condition and for each iteration it requires three function evaluations. Numerical experiments show that the new method is comparable with other discussed method.

KEYWORDS: Steffensen Method, derivative-free method, nonlinear equation, order of convergence, sixth order.

I. INTRODUCTION

The study of efficient methods for solving a nonlinear problem of the form

$$f(x) = 0,$$

has become one of the most important in mathematics and engineering.

One way to solve these nonlinear equations is to use the iteration method. Gerald and Wheatley [7, p.42] stated that one of the most widely used iterative methods for solving nonlinear equations is Newton's method.

Recently, some researchers have modified Newton's method to obtain sixth-order convergence such as [8], [9], [12], [13], [14], and [15]. One of the modified methods is the sixth-order method which requires two evaluations of the function and two evaluations of the first derivative of each iteration proposed by Parhi and Gupta [6]. The method is denoted by M6 and can be written as

$$\left.\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= x_n - \frac{2f(x_n)}{f'(x_n)+f'(y_n)} \\ x_{n+1} &= z_n - \frac{f(z_n)}{f'(x_n)}\frac{f'(x_n)+f'(y_n)}{3f'(y_n)-f'(x_n)} \end{aligned}\right\}, n = 0,1,2,\ldots$$

Shah et al. [1] proposes two sixth-order methods denoted by SA1 and SA2 as described in equations (1) and (2) which use three evaluations of functions and two evaluations of the first derivative of each iteration as follows:

$$\left.\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{f(y_n)}{f'(y_n)} \\ x_{n+1} &= z_n - \frac{f(z_n)}{f'(y_n)-f(y_n)} \end{aligned}\right\}, n = 0,1,2,\ldots \quad (1)$$

and

$$\left.\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{f(y_n)}{f'(y_n)} \\ x_{n+1} &= z_n - \frac{f(z_n)f(y_n)}{f'(y_n)f(y_n)-2f(z_n)} \end{aligned}\right\}, n = 0,1,2,\ldots \quad (2)$$

Abro et al. [5] combined the modified third order newton method [4] with the second order classical newton method [2]. The proposed method is of order six which is abbreviated as P6 and can be written in the form

$$\left.\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{f(y_n)}{f'(y_n)} \\ x_{n+1} &= y_n - \frac{f(y_n)+f(z_n)}{f'(y_n)} \end{aligned}\right\}, n = 0,1,2,\ldots \quad (3)$$

The main objective of this paper is to propose a three-step iteration method for solving nonlinear equations having sixth-order convergence. The proposed method is intended to perform better in some cases similar to other methods but with less time.

II. PROPOSED METHOD

If the first derivative, $f'(x_n)$ on equation (3) is approximated by using a divided difference of the form

$$f'(x_n) = \frac{f\big(x_n + f(x_n)\big) - f(x_n)}{f(x_n)}$$

Then by substituting $f'(x_n)$ into (3), we obtain a new derivative free three-step iteration method (MIBT) of type Steffensen as follows

$$y_n = x_n - \frac{(f(x_n))^2}{f(x_n+f(x_n))-f(x_n)}, \quad (4)$$

$$z_n = y_n - \frac{(f(y_n))^2}{f(y_n+f(y_n))-f(y_n)}, \quad (5)$$

$$x_{n+1} = z_n - \frac{f(y_n)f(z_n)}{f(y_n+f(y_n))-f(y_n)}, \quad (6)$$

where $n = 0,1,2, \ldots$.
The convergence analysis of MIBT is given in Theorem 1

Theorem 1 (Order of Convergence):
Suppose that $\alpha \epsilon I$ is a simple zero of a function $f: I \subset \mathbb{R} \to \mathbb{R}$ sufficiently differentiable at open interval I. If x_0 is closed enough to α, the iterative method defined in equations (4), (5), and (6) has a sixth order convergence with condition $c_2 = \frac{F_1^{(2)}(\alpha)}{n!F_1(\alpha)} = 0$ and satisfies the error equation

$$\begin{aligned} e_{n+1} = \frac{1}{(1+F_1)^2}&(8c_2(936 + 3600F_1 - 336c_2 \\ &-1704c_2F_1 - 3723F_1^2c_2 + 6906c_2^2F_1^2 \\ &+6858F_1^2 + 735c_2^2 - 5526c_2^3F_1^3 \\ &+7974F_1^3c_2^2 - 4824F_1^3c_2 - 594c_2^3 \\ &+8712F_1^3 + 5895F_1^4c_2^2 - 4218F_1^4c_2 \\ &+ 1324c_2^4F_1^4 - 3810F_1^4c_2^3 + 7470F_1^4 \\ &+216c_2^4 - 1698c_2^3F_1^5 - 2490F_1^5c_2 \\ &+2805F_1^5c_2^2 + 4014F_1^5 + 1224F_1^6 \\ &+804c_2^2F_1^6 - 879F_1^6c_2 - 138c_2F_1^7 \\ &+1864c_2^4F_1^2 + 3453F_1c_2^2 - 5130c_2^3F_1^2 \\ &- 2730c_2^3F_1 - 450c_2^3F_1^6 + 568c_2^4F_1^5 \\ &+108c_2^2F_1^7 + 1976c_2^4F_1^3 + 144c_2^4F_1^6 \\ &-54c_2^3F_1^7 + 16c_2^4F_1^7 + 996F_1c_2^4 \\ &+162F_1^7))e_n^6 + O(e_n^7), \end{aligned}$$

where $F_1 = D(f)(\alpha)$, and $e_n = x_n - \alpha$, n=1,2,3,4,5,6.

Proof: Suppose that α is the simple root of $f(x) = 0$, hence $f'(x) = 0$ and let $x_n = \alpha + e_n$. Then by Taylor's expansion of $f(x_n)$ and $f(x_n + f(x_n))$ about $x_n = \alpha$ we have consecutively

$$\begin{aligned} f(x_n) = &F_1e_n + 2c_2F_1e_n^2 + 6c_2F_1e_n^3 + 24c_2F_1e_n^4 \\ &+120c_2F_1e_n^5 + 720c_2F_1e_n^6 + O(e_n^7), \end{aligned} \quad (7)$$

and

$$\begin{aligned} f(x_n + f(x_n)) = &(2688c_2^2F_1^2 + 7872F_1^3c_2^2 + 1272c_2^3F_1^3 \\ &+10800F_1^3c_2 + 10800F_1^5c_2 \\ &+576c_2^3F_1^5 + 48c_2^4F_1^4 + 4320F_1^6c_2 \\ &+720c_2F_1 + 1200c_2^2F_1^6 + 1584F_1^4c_2^3 \\ &+5376F_1^5c_2^2 + 720c_2F_1^7 + 9360F_1^4c_2^2 \\ &+14400F_1^4c_2 + 5040F_1^2c_2)e_n^6 \\ &+(192F_1^5c_2^2 + 684F_1^4c_2^2 + 120F_1^6c_2 \\ &+720F_1^2c_2 + 72F_1^4c_2^3 + 120c_2F_1 \\ &+1200F_1^3c_2 + 888F1^3c_2^2 + 1200F_1^4c_2 \\ &+120c_2^3F_1^3 + 396c_2^2F_1^2 + 600F_1^5c_2)\, e_n^5 \\ &+(60c_2^2F_1^2 + 96F_1^4c_2 + 8c_2^3F_1^3 \\ &+24c_2F_1 + 24c_2F_1^5 + 96F_1^3c_2^2 \\ &+36F_1^4c_2^2 + 120F_1^2c_2 + 14F_1^3c_2)e_n^4 \\ &+(6c_2F_1 + 24F_1^2c_2 + 6F_1^4c_2 + 8c_2^2F_1^2 \\ &+18F_1^3c_2 + 8F_1^3c_2^2)e_n^3 + (2c_2F_1 \\ &+2F_1^3c_2 + 6F_1^2c_2)e_n^2 + (F_1 + F_1^2)e_n \\ &+O(e_n^7). \end{aligned} \quad (8)$$

By using equation (7) and equation (8), we get

$$\begin{aligned} \frac{(f(x_n))^2}{f(x_n + f(x_n)) - f(x_n)} = &(-14400F_1^2c_2 + 14832c_2^2F_1^2 \\ &+4800c_2^2 + 768F_1^5c_2^2 - 504c_2^3F_1^5 \\ &-720F_1^5c_2 - 10416c_2^3F_1 + 4512c_2^2F_1^4 \\ &+12768F_1c_2^2 - 10872F_1^2c_2^3 - 10800F_1^3c_2 \\ &-6984F_1^3c_2^3 + 10272F_1^3c_2^2 - 5064c_2^3 \\ &-4320F_1^4c_2 + 4176F_1^4c_2^2 - 2712F_1^4c_2^3 \\ &+1008c_2^4F_1^4 + 2496c_2^4 - 512c_2^5 \\ &+2688c_2^4F_1^3 + 192c_2^4F_1^5 - 160c_2^5F_1^4 \\ &-32c_2^5F_1^5 - 448c_2^5F_1^3 + 4704c_2^4F_1 \\ &-800c_2^5F_1^2 - 896c_2^5F_1 - 5040c_2 \\ &-10800c_2F_1)e_n^6 + (128c_2^4 - 648F_1^2c_2^3 \\ &+64c_2^4F_1^3 + 1296F_1c_2^2 + 696c_2^2 + 144c_2^4F_1^2 \\ &+132F_1^4c_2^2 - 120F_1^4c_2 + 16c_2^4F_1^4 - 792c_2^3F_1 \\ &-72F_1^4c_2^3 - 312c_2^3F_1^3 + 600F_1^3c_2^2 - 480c_2^3 \\ &+1200c_2^3F_1^2 - 600F_1^3c_2 - 1200F_1^2c_2 \\ &+192c_2^4F_1 - 1200c_2F_1 - 480c_2)\, e_n^5 \\ &+(84c_2^2 - 40c_2^3F_1 - 24F_1^2c_2^3 - 24F_1^3c_2 \\ &-72c_2 - 144c_2F_1 - 96F_1^2c_2 - 24F_1^3c_2^2 \\ &-8c_2^3F_1^3 + 120F_1c_2^2 + 84c_2^2F_1^2 - 32c_2^3)\, e_n^4 \\ &+(-12c_2 - 18c_2F_1 - 8c_2^2 + 8F_1c_2^2 + 4c_2^2F_1^2 \\ &\quad -6F_1^2c_2)e_n^3 + (-2c_2 - 2c_2F_1)e_n^2 + e_n \\ &+O(e_n^7) \end{aligned} \quad (9)$$

On substituting equation (9) into equation (4), we have

$$\begin{aligned} y_n = &(5040c_2 + 10800c_2F_1 + 14400F_1^2c_2 - 14832c_2^2F_1^2 \\ &- 4800c_2^2 + 6984c_2^3F_1^3 - 10272F_1^3c_2^2 \\ &+ 10800F_1^3c_2 + 5064c_2^3 - 4176F_1^4c_2^2 \\ &+ 4320F_1^4c_2 - 1008c_2^4F_1^4 + 2712F_1^4c_2^3 \\ &- 2496c_2^4 + 504c_2^3F_1^5 + 720F_1^5c_2 \\ &- 768F_1^5c_2^2 - 4512c_2^4F_1^2 - 12768F_1c_2^2 \\ &+ 10872F_1^2c_2^3 + 10416c_2^3F_1 + 512c_2^5 \\ &- 192c_2^5F_1^5 + 160c_2^5F_1^4 - 2688c_2^4F_1^4 \\ &+ 32c_2^5F_1^5 + 448c_2^5F_1^3 - 4704F_1c_2^4 \\ &+ 800F_1^5c_2^5 + 896c_2^5F_1)e_n^6 + (312c_2^3F_1^3 \\ &+ 648F_1^2c_2^3 + 480c_2^3 - 1296F_1c_2^2 \\ &- 16c_2^4F_1^4 + 600F_1^3c_2 + 1200F_1^2c_2 \\ &+ 480c_2 - 696c_2^2 - 144c_2^4F_1^2 - 600F_1^3c_2^2 \\ &+ 120F_1^4c_2 - 1200c_2^2F_1^2 - 132F_1^4c_2^2 \\ &+ 1200c_2F_1 - 192F_1c_2^4 + 792c_2^3F_1 \\ &- 64c_2^4F_1^3 - 128c_2^4 + 72F_1^4c_2^3)e_n^5 \\ &+ (40c_2^3F_1^3 - 120F_1c_2^2 + 24F_1^2c_2^3 \\ &+ 8c_2^3F_1^3 + 96F_1^2c_2 - 84c_2^2 + 24F_1^3c_2 \\ &- 84c_2^2F_1^2 + 32c_2^3 + 144c_2F_1 + 72c_2 \\ &- 24F_1^3c_2^2)e_n^4 + (-4c_2^2F_1^2 + 6F_1^2c_2 \\ &+ 12c_2 - 8c_2^2 - 8F_1c_2^2 + 18c_2F_1)e_n^3 \\ &+ (2c_2F_1 + 2c_2)e_n^2 + \alpha + O(e_n^7). \end{aligned}$$

Then do the expansion $f(y_n)$ at $y_n = \alpha$, after that calculate $\frac{(f(y_n))^2}{f(y_n+f(y_n))-f(y_n)}$ and substitute into equation (5) we get

$$z_n = \alpha + 8c_2^3(3F_1^2 + F_1^3 + 1 + 3F_1)e_n^4 + \frac{1}{(1+F_1)^2}\Big((1824c_2 + 8208c_2F_1 + 15360F_1^2c_2 - 19536c_2^2F_1^2 - 2976c_2^2 + 13536c_2^3F_1^3 - 18528F_1^3c_2^2 + 15456F_1^3c_2 + 2400c_2^3 - 10632F_1^4c_2^2 + 8928F_1^4c_2 - 2144c_2^4F_1^4 + 7440F_1^4c_2^3 - 768c_2^4 + 2352c_2^3F_1^5 + 2832F_1^5c_2 - 3480F_1^5c_2^2 - 504c_2^2F_1^6 + 384F_1^6c_2 - 4448c_2^4F_1^2 - 11640F_1c_2^2 + 14784F_1^2c_2^3 + 9072c_2^3F_1 + 336c_2^3F_1^6 - 672c_2^4F_1^5 - 3968c_2^4F_1^3 - 96c_2^4F_1^6 - 2816F_1c_2^4)e_n^5\Big) + \frac{1}{(1+F_1)^2}\Big((7488c_2 + 28800c_2F_1 + 54864F_1^2c_2 - 29784c_2^2F_1^2 - 2688c_2^2 + 63672c_2^3F_1^3 - 38592F_1^3c_2^2 + 69696F_1^3c_2 + 5880c_2^3 - 33744F_1^4c_2^2 + 59760F_1^4c_2 - 31200c_2^4F_1^4 + 46920F_1^4 - 4800c_2^4 + 22200c_2^3F_1^5 + 32112F_1^5c_2 - 19920F_1^5c_2^2 - 7032c_2^2F_1^6 + 9792F_1^6c_2 + 1296F_1^7c_2 - 41760c_2^4F_1^2 - 13632F_1c_2^2 + 55224F_1^2c_2^3 + 27624c_2^3F_1 + 1632c_2^5 + 6312c_2^3F_1^6 - 13872c_2^4F_1^5 - 1104c_2^2F_1^7 + 8480c_2^5F_1^4 - 45168c_2^4F_1^3 + 96c_2^5F_1^7 - 3648c_2^4F_1^6 + 840c_2^3F_1^7 - 432c_2^4F_1^7 + 864c_2^5F_1^6 + 3488c_2^5F_1^5 + 13312c_2^5F_1^3 - 22128F_1c_2^4 + 13184F_1^2c_2^5 + 7328c_2^5F_1)e_n^6\Big) + O(e_n^7).$$

Then, by calculating $\frac{f(y_n)-f(z_n)}{f(y_n+f(y_n))-f(y_n)}$ and substituting into equation (6) we get

$$e_{n+1} = \frac{1}{(1+F_1)^2}(8c_2(936 + 3600F_1 - 336c_2 - 1704c_2F_1 - 3723F_1^2c_2 + 6906c_2^2F_1^2 + 6858F_1^2 + 735c_2^2 - 5526c_2^3F_1^3 + 7974F_1^3c_2^2 - 4824F_1^3c_2 - 594c_2^3 + 8712F_1^3 + 5895F_1^4c_2^2 - 4218F_1^4c_2 + 1324c_2^4F_1^4 - 3810F_1^4c_2^3 + 7470F_1^4 + 216c_2^4 - 1698c_2^3F_1^5 - 2490F_1^5c_2 + 2805F_1^5c_2^2 + 4014F_1^5 + 1224F_1^6 + 804c_2^2F_1^6 - 879F_1^6c_2 - 138c_2F_1^7 + 1864c_2^4F_1^2 + 3453F_1c_2^2 - 5130c_2^3F_1^2 - 2730c_2^3F_1 - 450c_2^3F_1^6 + 568c_2^4F_1^5 + 108c_2^2F_1^7 + 1976c_2^4F_1^3 + 144c_2^4F_1^6 - 54c_2^3F_1^7 + 16c_2^4F_1^7 + 996F_1c_2^4 + 162F_1^7))e_n^6 + O(e_n^7),$$

From the definition order of convergence, it can be seen that the new derivative-free iterative method (MIBT) has a convergence order of six, so Theorem 1 is proven. □

III. NUMERICAL EXAMPLES

In this section, we present a computational test using Lotfi's method (M6), Shah's method (SA1) and (SA2), Abro's method (P6) and the derivative-free iterative method (MIBT). In this comparison, see [10], [11] and the cited references for some of the nonlinear equations used.

1. $f_1(x) = \cos(x) - xe^x + x^2$, $\alpha_1 \in [0.0, 2.0]$
2. $f_2(x) = x - \cos(x)$, $\alpha_2 \in [0.0, 2.0]$
3. $f_3(x) = e^x - 1.5 + \arctan(x)$, $\alpha_3 \in [0.0, 1.5]$

The computational test was performed using Maple 13 software, with a tolerance of 1.0×10^{-100} and a maximum iteration is 100. The order of convergence computationally can be approximated using the formula [3]

$$p \approx \frac{\ln|x_{n+1} - x_n|/|x_n - x_{n-1}|}{\ln|x_n - x_{n-1}|/|x_{n-1} - x_{n-2}|}$$

In Table 1, $f_n(x)$ represents the nonlinear function, n represents the number of iterations, p is the convergence order, $|f(x_n)|$ is the absolute value of the function and $|x_n - x_{n-1}|$ is the absolute value of the difference between two consecutive approximations of the root.

Table 1: Comparison of several iteration methods

$f_n(x)$	x_0	Method	n	p	$\lvert f(x_n)\rvert$	$\lvert x_n - x_{n-1}\rvert$
f_1	1.5	M6	4	6	1.5705e-450	1.5705e-450
		SA1	4	6	2.3954e-416	2.3954e-416
		SA2	4	6	2.9071e-299	2.9071e-299
		P6	4	6	5.7400e-316	5.7400e-316
		MIBT	4	6	4.9128e-566	4.9128e-566
f_1	0.7	M6	3	6	8.6880e-288	8.6880e-288
		SA1	3	6	2.1209e-293	2.1209e-293
		SA2	3	6	4.4864e-268	4.4864e-268
		P6	3	6	1.7876e-274	1.7876e-274
		MIBT	3	6	1.2302e-271	1.2302e-271
f_2	1.5	M6	3	6	2.4647e-168	2.4647e-168
		SA1	3	6	2.1606e-198	2.1606e-198
		SA2	3	6	1.3285e-220	1.3285e-220
		P6	3	6	1.7395e-204	1.7395e-204
		MIBT	3	6	5.2046e-145	5.2046e-145
f_2	0.9	M6	3	6	4.0305e-329	4.0305e-329
		SA1	3	6	6.4724e-304	6.4724e-304
		SA2	3	6	2.2419e-325	2.2419e-325
		P6	3	6	1.7715e-308	1.7715e-308
		MIBT	3	6	1.1209e-230	1.1209e-230
f_3	1.0	M6	3	6	1.0543e-137	1.0543e-137
		SA1	3	6	3.1307e-146	3.1307e-146
		SA2	3	6	1.6081e-168	1.6081e-168
		P6	3	6	2.3620e-158	2.3620e-158
		MIBT	4	6	4.5265e-225	4.5265e-225
f_3	0.3	M6	3	6	1.2463e-411	1.2463e-411
		SA1	3	6	5.4145e-394	5.4145e-394
		SA2	3	6	2.5137e-414	2.5137e-414
		P6	3	6	4.1055e-403	4.1055e-403
		MIBT	3	6	5.1644e-302	5.1644e-302

In Table 1 it can be seen that a numerical comparison was made by taking different initial guesses for each function to see how was the influence of the initial guesses to the number of iterations. These different initial guesses are also used to see whether the methods, which are of order six, are sensitive to the initial guesses to reach the roots of the nonlinear equations. At $f_1(x)$ for a large initial guess, all methods only require four iterations with the smallest $|x_n - x_{n-1}|$, namely the MIBT method. a small initial guess, all methods require three iterations with the smallest $|x_n - x_{n-1}|$, occurs in the method P6. At $f_2(x)$ for each initial guess only three iterations are needed. for a large initial guess with the smallest $|x_n - x_{n-1}|$ value occurs the SA2 method and for a small initial guess with the smallest $|x_n - x_{n-1}|$ value raises in the M6 method. At $f_3(x)$ for a large initial guess, the MIBT method requires four iterations while the other comparison methods only require three iterations but the MIBT method has a very small $|x_n - x_{n-1}|$, compared to the other methods. For small initial guesses, all methods require three iterations with the smallest $|x_n - x_{n-1}|$ happens in the SA2 method. Hence, the MIBS method can be used as an alternative method for solving nonlinear equation problems. Also this method is very efficient for all types of functions because it does not calculate the derivative of the fuctions.

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