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Some Fixed Point Theorems for Occasionally Weakly Compatible Mappings Related with Fuzzy-2 and Fuzzy-3 Metric spaces

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ARTICLE INFO	ABSTRACT
Published Online: 20 February 2023 Corresponding Author: **Rasik M.Patel**	In this paper, we give some definitions of occasionally weakly compatible maps in fuzzy-2 metric and fuzzy-3 metric spaces and some common fixed point theorems for six mappings under the condition of occasionally weakly compatible mappings in complete Fuzzy-2 and Fuzzy-3 matric spaces. 2010 Mathematics Subject Classification: Primary 47H10, 54H25

KEYWORDS: Fuzzy metric space, Fuzzy-2 & 3 metric space, Fixed point theorem, Occasionally weakly Compatible mappings

I. INTRODUCTION

Impact of fixed point theory in different branches of mathematics and its applications is immense. The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [19] in 1922. In the general setting of complete metric space, this theorem runs as the follows, Theorem 1.1 (Banach's contraction principle) Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y ∈ X, d (fx, fy) ≤ c d(x, y).Then f has a unique fixed point a∈X, such that for each x ∈ X, $\lim_{n\to\infty} f^n x = a$. After the classical result, R.Kannan [16] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. In 2002, A. Branciari [1] analyzed the existence of fixed point for mapping f defined on a complete metric space (X , d) satisfying a general contractive condition of integral type.

Theorem 1.2 (A.Branciari) Let (X, d) be a complete metric space, c∈ (0, 1) and let f: X→X be a mapping such that for each x, y ∈ X, $\int_0^{d(fx,fy)} \varphi(t)dt \le c\int_0^{d(x,y)} \varphi(t)dt$.

Where φ: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non negative, and such that for each ε >o, $\int_0^{\varepsilon} \varphi(t)dt$, then f has a unique fixed point a ∈ X such that for each x ∈X, $\lim_{n\to\infty} f^n x = a$ After the paper of Branciari, a lot of a research works have been carried out on generalizing contractive conditions of integral type for a different contractive mapping satisfying various known properties. A fine work has been done by Rhoades [3] extending the result of Branciari by replacing the condition by the following

$$\int_0^{d(fx,fy)} \varphi(t)dt \le \int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt$$

The aim of this paper is to generalize some mixed type of contractive conditions to the mapping and then a pair of mappings, satisfying a general contractive mapping such as R. Kannan type [16], S.K. Chatrterjee type [20], T. Zamfirescu type [25], Schweizer and A.Sklar [21]etc.

The concept of Fuzzy sets was introduced initially by Zadeh [27]. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets. Both George and Veermani [4], Kramosil [8] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have obtained common fixed point theorems for mappings satisfying different types of commutativity conditions. Vasuki [17] proved fixed point theorems for R-weakly commutating mappings. R.P. Pant and Jha [13, 14, 15] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam et al [12] have shown that B.E.Rhoades [3] open problem on the existence of contractive definition which generates a fixed point but does not force the mappings to be continuous at the fixed point, posses an affirmative answer. Pant and Jha obtained some analogous results proved by Balasubramaniam. Recently many authors [9, 22, 23, 24] have also studied the fixed point theory in fuzzy metric spaces.

2. PRELIMINARIES

Definition2.1: A binary operation *: [0, 1] $\times$ [0, 1] $\to$ [0, 1] is a continuous t-norm if it satisfies the following conditions:

(1) * is associative and commutative,

(2) * is continuous,

(3) a * 1 = a for all a $\in$ [0, 1],

(4) a * b $\leq$ c * d whenever a $\leq$ c and b $\leq$ d for all a, b, c, d $\in$ [0, 1],

Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition2.2: A 3-tuple (X, M,*) is called a fuzzy metric space if X is an arbitrary (Non-empty) set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$satisfying the following conditions: for all x, y, z $\in$ X and t, s > 0,

(1) M(x, y, t) > 0.

(2) M(x, y, t) $= 1$ if and only if x = y,

(3) M(x, y, t) $= M(y,x,t)$.

(4) M(x, y, t) $* M(y,z,s) \leq M(x,z,t+s)$.

(5) M(x, y, .) : $(0,\infty) \to [0,1]$ is continuous.

Let M(x, y, t) be a fuzzy metric space. For any t > 0, the open ball B(x, r, t) with center x$\in$ X and radius 0 < r < 1 is defined by B(x, r, t) = {y $\in$ X: M(x, y, t) $> 1-r$}. Let (X, M,*) be a fuzzy metric space. Let s be the set of all $A \subset S$ with x $\in$ A if and only if there exist t > 0 and 0 < r < 1 such that B(x, r, t) $\subset$ A. Then s is a topology on X (induced by the fuzzy metric M). This topology is Hausdorff and first countable. A sequence $\{x_n\}$ in X converges to x if and only if M $(x_n, x,$ t) $\to 1$ as n $\to \infty$ for all t > 0. It is called a Cauchy sequence if, for any 0 < e < 1 and t > 0, there exits $n_0 \in$ N such that M $(x_n, x_m,$ t) $> 1-\varepsilon$ for any n, m $\geq n_0$ The fuzzy metric space (X, M,*) is said to be complete if every Cauchy sequence is convergent. A subset A of X is said to be F-bounded if there exists t > 0 and 0 < r < 1 such that M(x, y, t) $> 1-r$ for all x, y $\in$ A.

Example 2.3 [10]: Let X = R and denote a * b = ab for all a, b $\in$ [0, 1]. For any t $\in$ (0,∞), define

M(x, y, t) $= \frac{t}{t+|x-y|}$ for all x, y $\in$ X. Then M is a fuzzy metric in X.

Definition 2.4: Let f and g be mappings from a fuzzy metric space (X, M,*) into itself. Then the mappings are said to be compatible if, for all t > 0, $\lim_{n\to\infty}$ M $(fgx_n, gfx_n,$t$) = 1$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x \in X.$

Definition 2.5: Let f and g be mappings from a fuzzy metric space (X, M,*) into itself. Then the mappings are said to be

(1) Weakly compatible if $M(fgx, gfx, t) \geq M(fx, gx, t)$ for all $x \in X$ and t > 0,

(2) R- Weakly compatible if there exists some $R > 0$ such that $M(fgx, gfx, t) \geq M(fx, gx, \frac{t}{R})$ for all $x \in X$ and t > 0.

Lemma 2.6: Let (X, M,*) be a fuzzy metric space. If there exists q $\in$ (0, 1) such that M(x, y, qt) $\geq$ M(x, y, t) for all x, y $\in$ X and t > 0, then x = y.

Definition 2.7: Let X be a set, f and g self maps of X. A point x $\in$ X is called a coincidence point of f and g iff $fx = gx$. We shall call $w = fx = gx$ a point of coincidence of f and g.

Definition 2.8: A pair of maps S and T is called weakly compatible pair if they commute at coincidence points. The concept of occasionally weakly compatible is introduced by A. Al-Thagafi and Naseer Shahzad [2]. It is stated as follows.

Example 2.9 [2]: Let R be the usual metric space. Define S, T: R$\to$R by $Sx = 2x$ and $Tx = x^2$ for all x $\in$ R. Then $Sx = Tx$ for x =0, 2 but ST0 = TS0 and ST2 $\neq$ TS2. S and T are occasionally weakly compatible self maps but not weakly compatible.

Lemma 2.10 [5-7]: Let X be a set, f and g occasionally weakly compatible self maps of X. If f and g have a unique point of coincidence,$w = fx = gx$, then w is the unique common fixed point of f and g.

Definition 2.11: The 3-tuple (X, M,*) is called a fuzzy 2-metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X^3 \times [0,\infty)$ satisfying the following conditions: for all x, y, z, u $\in$ X and t_1, t_2, t_3 > 0,

(1) M(x, y, z, 0) = 0,

(2) M(x, y, z, t) = 1 for all t > 0 (only when the three simplex x, y, z degenerate)

(3) M(x, y, z, t) = M(x, z, y, t) = M (y, z, x, t) =……

(4) M(x, y, z, w,$t_1 + t_2 + t_3$) $\geq$ *(M(x, y, u, t_1)*M(x, u, z,t_2)*M (u, y, z,t_3)

(5) M(x, y, z, .): [0, 1) $\times$ [0, 1] is left continuous.

Definition 2.12: Let (X, M,*) be a fuzzy- 2 metric space.

(1) A sequence $\{x_n\}$ in fuzzy -2 metric space X is said to be convergent to a point x∈X (denoted by $\lim_{n\to\infty} x_n = x \;\; or \;\; x_n \to x$
if for any $\lambda \in (0,1)$ and t > 0, there exists $n_0 \in$ N such that for all n $\geq n_0$ and $a \in$ X, M (x_n, x, a, t) $> 1 - \lambda$.
That is
$$\lim_{n\to\infty} \mathrm{M}\,(x_n,x,a,t) = 1 \;\text{ for all } a \in \mathrm{X} \text{ and } t > 0.$$

(2) A sequence $\{x_n\}$ in fuzzy- 2 metric space X is called a Cauchy sequence, if for any $\lambda \in (0,1)$ and t > 0, there exists $n_0 \in$ N such that for all m, n $\geq n_0$ and a $\in$ X, M (x_n, x_m, a,t) $> 1 - \lambda$.

(3) A fuzzy- 2 metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.13: Self A function M is continuous in fuzzy 2-metric space if $x_n \to x$, $y_n \to y$, then
$$\lim_{n\to\infty} \mathrm{M}\,(x_n,y_n,a,t) = \mathrm{M}(x,y,a,t) \; for\ all\ a \in X\ and\ t > 0.$$

Definition 2.14: Two mappings f and g on fuzzy 2-metric space X are weakly commuting iff

M(fgu,gfu, a,t)≥M(fu,gu,a,t) $for\ all\ a, u \in X\ and\ t > 0.$

Definition 2.15[10]: The 3- tuple (X, M,*) is called a fuzzy-3 metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X^4 \times [0,\infty)$ satisfying the following conditions, for all x, y, z, w, u ∈ X and $t_1, t_2, t_3, t_4 > 0.$

(1) M(x, y, z, w, 0) = 0,

(2) M(x, y, z, w, t) =1 for all t > 0, [only when the three simplex (x, y, z, w) degenerate]

(3) M(x, y, z, t) = M(x,w, z, y, t) = M(y, z,w, x, t) = M(z,w, x, y, t) = ...

(4) M(x, y, z, w,$t_1 + t_2 + t_3 + t_4$) ≥ M(x, y, z, w, t_1) * M(x, y, z, w,t_2) * M(x, y, z, w,t_3) * M(x, y, z, w,t_4)

(5) M(x, y, z, w): [0, 1) → [0, 1] is left continuous.

Definition 2.16[10]: Let (X, M,*) be a fuzzy 3-metric space, then

(1) A sequence $\{x_n\}$ in fuzzy 3-metric space X is said to be convergent to a point x∈ X if $\lim_{n\to\infty} \mathrm{M}\,(x_n,x,a,b,t) = 1\; for\ all\ a,b \in X\ and\ t > 0,$

(2) A sequence $\{x_n\}$ in fuzzy 3-metric space X is called a Cauchy sequence, if
$\lim_{n\to\infty} \mathrm{M}\,(x_{n+p},x_n,a,b,t) = 1\; for\ all\ a,b \in X\ and\ t > 0, p > 0.$

(3) A fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.17[10]: A function M is continuous in fuzzy 3-metric space iff whenever $x_n \to x, y_n \to y,$ then
$$\lim_{n\to\infty} \mathrm{M}\,(x_n,y_n,a,b,t) = \mathrm{M}\,(x,y,a,b,t)\; for\ all\ a,b \in X\ and\ t$$

Definition 2.18[10]: Two mappings A and S on fuzzy 3-metric space are weakly commuting iff
M(ASu, SAu,a,b,t) ≥ M(Au, Su,a,b,t) for all u, a, b∈ X and t> 0 .

3. MAIN RESULT

Theorem 3.1: Let (X, M, *) be a complete fuzzy 2-metric space and let A, B, S,T,P and Q be self mappings of X. Let the pairs $\{P, ST\}$ and $\{Q, AB\}$ be occasionally weakly compatible. If there exist $q \in (0,1)$ such that
$$\int_0^{M(Px,Qy,a,qt)} \xi(t)dt \geq \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(STx,ABy,a,t)*M(STx,Px,a,t)* \\ M(Qy,ABy,a,t)*M(Px,ABy,a,t)* \\ M(Qy,STx,a,t)*\left(\frac{M(STx,ABy,a,t)}{M(Px,ABy,a,t)}\right)* \\ \left(\frac{M(Qy,STx,a,t)}{M(STx,Px,a,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$
…(3.3.1)

For all x, y ,a∈ X, $t > 0$ and $\emptyset : [0,1]^7 \to [0,1]$ such that $\emptyset(t,1,1,t,t,1,t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A ,B, S ,T, P and Q.

Proof:

Let the pairs $\{P, ST\}$ and $\{Q, AB\}$ be occasionally weakly compatible. So there are points x, y ∈ X such that $Px = STx$ and $Qy = ABy$. we claim that $Px = Qy$, if not, by inequality (3.3.1)
$$\int_0^{M(Px,Qy,a,qt)} \xi(t)dt \geq \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(STx,ABy,a,t)*M(STx,Px,a,t)* \\ M(Qy,ABy,a,t)*M(Px,ABy,a,t)* \\ M(Qy,STx,a,t)*\left(\frac{M(STx,ABy,a,t)}{M(Px,ABy,a,t)}\right)* \\ \left(\frac{M(Qy,STx,a,t)}{M(STx,Px,a,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Px,Qy,a,t)*M(Px,Px,a,t)* \\ M(Qy,Qy,a,t)*M(Px,Qy,a,t)* \\ M(Qy,Px,a,t)*\left(\frac{M(Px,Qy,a,t)}{M(Px,Qy,a,t)}\right)* \\ \left(\frac{M(Qy,Px,a,t)}{M(Px,Px,a,t)}\right) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\left\{\min\left\{\begin{matrix} M(Px,Qy,a,t)*1*1*M(Px,Qy,a,t)* \\ M(Qy,Px,a,t)*1*M(Qy,Px,a,t) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi(M(Px,Qy,a,t))} \xi(t)dt$$

$$\geq \int_0^{M(Px,Qy,a,t)} \xi(t)dt$$

Therefore $Px = Qy$, i.e. $Px = STx \text{ and } Qy = ABy$.

Suppose that there is another point z such that $Pz = STz$ then by inequality (3.1.1) we have $Pz = STz = Qy = AB$, so $Px = Pz$ and $w = Px = STx$ is the unique point of coincidence of P and ST. Similarly there is a unique point z ∈ X such that $z = Qz = ABz$.

Assume that $w \neq z$. We have by inequality (3.1.1)

$$\int_0^{M(w,z,a,qt)} \xi(t)dt = \int_0^{M(Pw,Qz,a,qt)} \xi(t)dt$$

$$\geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(STw,ABz,a,t)*M(STw,Pw,a,t)* \\ M(Qz,ABz,a,t)*M(Pw,ABz,a,t)* \\ M(Qz,STw,a,t)*\left(\frac{M(STw,ABz,a,t)}{M(Pw,ABz,a,t)}\right)* \\ \left(\frac{M(Qz,STw,a,t)}{M(STw,Pw,a,t)}\right) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\left\{\min\left\{\begin{matrix} M(w,z,a,t)*M(w,w,a,t)* \\ M(z,z,a,t)*M(w,z,a,t)* \\ M(z,w,a,t)*\left(\frac{M(w,z,a,t)}{M(w,z,a,t)}\right)* \\ \left(\frac{M(z,w,a,t)}{M(w,w,a,t)}\right) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\left\{\min\left\{\begin{matrix} M(w,z,a,t)*1*1*M(w,z,a,t)* \\ M(z,w,a,t)*1*M(z,w,a,t) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\{M(w,z,a,t)\}} \xi(t)dt$$

$$\geq \int_0^{M(w,z,a,t)} \xi(t)dt$$

Therefore we have $w = z$, by Lemma 2.10, z is a common fixed point of A, B, S ,T ,P and Q.

To prove uniqueness let u be another common fixed point of A, B, S, T, P and Q. Then

$$\int_0^{M(z,u,a,qt)} \xi(t)dt = \int_0^{M(Pz,Qu,a,qt)} \xi(t)dt$$

$$\geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(STz,ABu,a,t)*M(STz,Pz,a,t)* \\ M(Qu,ABu,a,t)*M(Pz,ABu,a,t)* \\ M(Qu,STz,a,t)*\left(\frac{M(STz,ABu,a,t)}{M(Pz,ABu,a,t)}\right)* \\ \left(\frac{M(Qu,STz,a,t)}{M(STz,Pz,a,t)}\right) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\left\{\min\left\{\begin{matrix} M(z,u,a,t)*M(z,z,a,t)* \\ M(u,u,a,t)*M(z,u,a,t)* \\ M(u,z,a,t)*\left(\frac{M(z,u,a,t)}{M(z,u,a,t)}\right)* \\ \left(\frac{M(u,z,a,t)}{M(z,z,a,t)}\right) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\left\{\min\left\{\begin{matrix} M(z,u,a,t)*1*1*M(z,u,a,t)* \\ M(u,z,a,t)*1*M(u,z,a,t) \end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\phi\{M(z,u,a,t)\}} \xi(t)dt$$

$$\geq \int_0^{M(z,u,a,t)} \xi(t)dt$$

Therefore we have $w = z$, by Lemma 2.10

Thus, u is a common fixed point of A, B, S, T, P and Q.

Theorem 3.2: Let (X, M, *) be a complete fuzzy 3-metric space and let A, B, S,T,P and Q be self mappings of X. Let the pairs $\{P, ST\}$ and $\{Q, AB\}$ be occasionally weakly compatible. If there exist $q \in (0,1)$ such that

$$\int_0^{M(Px,Qy,a,b,qt)} \xi(t)dt \geq \int_0^{\phi\left\{\min\left\{\begin{matrix} M(STx,ABy,a,b,t)*M(STx,Px,a,b,t)* \\ M(Qy,ABy,a,b,t)*M(Px,ABy,a,b,t)* \\ M(Qy,STx,a,b,t)*\left(\frac{M(STx,ABy,a,b,t)}{M(Px,ABy,a,b,t)}\right)* \\ \left(\frac{M(Qy,STx,a,b,t)}{M(STx,Px,a,b,t)}\right) \end{matrix}\right\}\right\}} \xi(t)dt$$

…(3.2.1)

For all x, y , a , $b\in$ X, $t > 0$ and $\emptyset : [0,1]^7 \to [0,1]$ such that $\emptyset(t,1,1,t,t,1,t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A ,B, S ,T, P and Q.

Proof:

Let the pairs $\{P, ST\}$ and $\{Q, AB\}$ be occasionally weakly compatible. So there are points x, y ∈ X such that $Px = STx$ and $Qy = ABy$. we claim that $Px = Qy$, if not, by inequality (3.2.1)

$$\int_0^{M(Px,Qy,a,b,qt)} \xi(t)dt \geq \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(STx,ABy,a,b,t)*M(STx,Px,a,b,t)* \\ M(Qy,ABy,a,b,t)*M(Px,ABy,a,b,t)* \\ M(Qy,STx,a,b,t)*\left(\frac{M(STx,ABy,a,b,t)}{M(Px,ABy,a,b,t)}\right)* \\ \left(\frac{M(Qy,STx,a,b,t)}{M(STx,Px,a,b,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(Px,Qy,a,b,t)*M(Px,Px,a,b,t)* \\ M(Qy,Qy,a,b,t)*M(Px,Qy,a,b,t)* \\ M(Qy,Px,a,b,t)*\left(\frac{M(Px,Qy,a,b,t)}{M(Px,Qy,a,b,t)}\right)* \\ \left(\frac{M(Qy,Px,a,b,t)}{M(Px,Px,a,b,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(Px,Qy,a,b,t)*1*1*M(Px,Qy,a,b,t)* \\ M(Qy,Px,a,b,t)*1*M(Qy,Px,a,b,t)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset(M(Px,Qy,a,b,t))} \xi(t)dt$$

$$\geq \int_0^{M(Px,Qy,a,b,t)} \xi(t)dt$$

Therefore $Px = Qy$, i.e. $Px = STx \text{ and } Qy = ABy$.

Suppose that there is another point z such that $Pz = STz$ then by inequality (3.2.1) we have $Pz = STz = Qy = AB$, so $Px = Pz$ and $w = Px = STx$ is the unique point of coincidence of P and ST. Similarly there is a unique point z ∈ X such that $z = Qz = ABz$.

Assume that $w \neq z$. We have by inequality (3.2.1)

$$\int_0^{M(w,z,a,b,qt)} \xi(t)dt = \int_0^{M(Pw,Qz,a,b,qt)} \xi(t)dt$$

$$\geq \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(STw,ABz,a,b,t)*M(STw,Pw,a,b,t)* \\ M(Qz,ABz,a,b,t)*M(Pw,ABz,a,b,t)* \\ M(Qz,STw,a,b,t)*\left(\frac{M(STw,ABz,a,b,t)}{M(Pw,ABz,a,b,t)}\right)* \\ \left(\frac{M(Qz,STw,a,b,t)}{M(STw,Pw,a,b,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(w,z,a,b,t)*M(w,w,a,b,t)* \\ M(z,z,a,b,t)*M(w,z,a,b,t)* \\ M(z,w,a,b,t)*\left(\frac{M(w,z,a,b,t)}{M(w,z,a,b,t)}\right)* \\ \left(\frac{M(z,w,a,b,t)}{M(w,w,a,b,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(w,z,a,b,t)*1*1*M(w,z,a,b,t)* \\ M(z,w,a,b,t)*1*M(z,w,a,b,t)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\{M(w,z,a,b,t)\}} \xi(t)dt$$

$$\geq \int_0^{M(w,z,a,b,t)} \xi(t)dt$$

Therefore we have $w = z$, by Lemma 2.10 z is a common fixed point of A, B, S ,T ,P and Q.

To prove uniqueness let u be another common fixed point of A, B, S, T, P and Q. Then

$$\int_0^{M(z,u\ a,b,qt)} \xi(t)dt = \int_0^{M(Pz,Qu,a,b,qt)} \xi(t)dt$$

$$\geq \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(STz,ABu,a,b,t)*M(STz,Pz\ a,b,t)* \\ M(Qu,ABu,a,b,t)*M(Pz,ABu,a,b,t)* \\ M(Qu,STz,a,b,t)*\left(\frac{M(STz,ABu,a,b,t)}{M(Pz,ABu,a,b,t)}\right)* \\ \left(\frac{M(Qu,STz,a,b,t)}{M(STz,Pz,a,b,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(z,u,a,b,t)*M(z,z\ a,b,t)* \\ M(u,u,a,b,t)*M(z,u,a,b,t)* \\ M(u,z,a,b,t)*\left(\frac{M(z,u,a,b,t)}{M(z,u,a,b,t)}\right)* \\ \left(\frac{M(u,z,a,b,t)}{M(z,z,a,b,t)}\right)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\left\{min\left\{\begin{matrix} M(z,u\ a,b,t)*1*1*M(z,u,a,b,t)* \\ M(u,z,a,b,t)*1*M(u,z,a,b,t)\end{matrix}\right\}\right\}} \xi(t)dt$$

$$= \int_0^{\emptyset\{M(z,u,a,b,t)\}} \xi(t)dt$$

$$\geq \int_0^{M(z,u,a,b,t)} \xi(t)dt$$

Therefore we have $w = z$, by Lemma 2.10, Thus, u is a common fixed point of A, B, S, T, P and Q.

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