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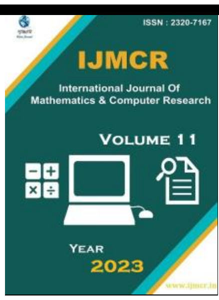

Fractional Order Distribution on Heat Flux for Crystalline Concrete Material

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ARTICLE INFO	ABSTRACT
Published Online: 23 February 2023 Corresponding Author: **Ninny Verma**	The present work devoted to analyse the impact of fractional order one dimensional heat equation on heat transfer in concrete substances, during conduction process. Throughout the evolution the thermal properties of substances are considered as constant parameters. Investigations were done through statistical phenomena with linear representation of curve.

KEYWORDS: Fractional order heat equation, Laplace transform, Fourier transform, Wright function, Heat flux.

1. INTRODUCTION

In thermal physics heat transfer is thermal energy between two physical systems. Heat flux is the rate of thermal energy flow per unit surface area of heat transfer on surface. A semi- infinite solid is an idealized body that has single plane surface and extend to infinity in all directions. The idealized body is used to indicate that the temperature changes in the part of body due to the thermal conditions on a single surface [1]. For example, the Earth, thick wall, steel piece of any shaped quenched rapidly etc. in determining variation of temperature near its surface and other surface being too far to have any impact on the region in short period of time since heat doesn't have sufficient time to penetrate deep into body thus thickness can be neglected. Beyond the critical distance heat transfer through conduction is not significant for the specific time duration. A good understanding of temperature and heat flux is important for the fire test in building structures and aerospace industries [2]. The thermal properties of solid materials have been studied widely in recent years [3-6]. Different materials give the application of heat differently, but their response can be measured according to the functional characteristics of temperature [7]. The small change in domain of thermal conductivity shows resistive change in heat transfer [8]. Therefore, many studies have been done which are focused on the modification of the heating and cooling system.

2. PRELIMINARIES

Initially at temperature T_0 consider a semi-infinite solid. Suddenly on applying external heat source the temperature of the one face of the solid is raised up to temperature T_s at time zero. Defining $V = \frac{T-T_0}{T_s-T_0}$. If thermal conditions (conductivity, diffusivity and specific heat) of solid are constant, no internal heat generation and negligible temperature variations in the y and z directions, then appropriate classical heat conduction equation is defined by [9]

$$\frac{\partial V}{\partial t} = \lambda \frac{\partial^2 V}{\partial x^2} \quad \dots (1)$$

Where λ is the thermal diffusivity, subject to suitable boundary conditions given below

$$\left.\begin{aligned} &Time\ t = 0; temperature\ V = 0 \\ &At\ the\ surface\ x = 0; temperature\ V = 1, and \\ &When\ surface\ x \to \infty;\ temperature\ V = 0 \end{aligned}\right\} \quad \dots (2)$$

The following facts are considered to study the heat flux and temperature distribution in the semi infinite solid medium.

The Laplace transform for the function $f(x)$, for all real numbers $x \geq 0$ defined by [10]

$$L\{f(x), S\} = \int_0^\infty e^{-st} f(t)dt = F(s)\,; Re(s) > 0 \quad \dots (3)$$

Where $'s'$ is a complex frequency parameter.

The Fourier sine transform is given by [11]

$$F_s(n,t) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x,t) sinnx\, dx \quad \dots (4)$$

The Mittage-Leffler function $E_{\alpha,\beta}(z)$ is defined by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta+\alpha k)}, \alpha \in C, z \in C, \mathrm{R}(\alpha) > 0, \beta > 0 \quad \dots (5)$$

In series form the Wright function $W(\alpha,\beta;z)$ is represented by [12]

$$W(\alpha,\beta;z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(\beta+\alpha m)m!} \quad \dots (6)$$

The Laplace transform of fractional deferential operator ${}_0D_x^{\alpha,\beta} f(x)$ is given by [13]

$$L[\,{}_0D_x^{\alpha,\beta} f(x); s] = s^{\alpha}\tilde{f}(s) - s^{\beta(\alpha-1)} I_{0+}^{(1-\alpha)(1-\beta)} f(0+); 0 < \alpha < 1 \quad \dots (7)$$

The error function $erfc(x)$ is given by [14]

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt \quad \dots (8)$$

The complementary error function $erfc(x)$ *is* given by

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2} dt \quad \dots (9)$$

The relation between complementary error function and Wright function is defined by

$$W(-{}^1\!/_2, 1; z) = erfc({}^z\!/_2) \quad \dots (10)$$

The following integral within limit 0 to ∞ is required for solution [15]

$$\int_0^{\infty} n \sin nx E_{\alpha,\alpha+1}(-n^2 K t^2) dn = W\left(-{}^{\alpha}\!/_2\,; 1; \frac{-x}{\sqrt{Kt^{\alpha}}}\right) \quad \dots (11)$$

For the flow of heat energy through the solid materials the thermal conditions play an important key role. According to second law of thermo-dynamics the conduction process in solid be done through hot surface to cold surface on the basic of their thermal properties of materials.

To get the agreement between two variables we use the phenomena called correlation. To measure the quantitative dependence of concern random variables X and Y we find coefficient of correlation .

3. SHAPE OF THE MATHEMATICAL MODELLING

The one dimensional heat equation with fractional order is given by

$$D_t^{\alpha,\beta} V(x,t) = \lambda \frac{\partial^2 V}{\partial x^2}\,;\ 0 < \alpha < 1, t > 0, x \in \mathbb{R} \quad \dots (13)$$

In engineering field, we generally had seen that many engineering devices are used to perform the mechanical experimentation in various laboratories. During the execution it was observed that lot of heat manumit to the surroundings in the form of energy loss. The capability of any device can be maximizing by minimizing thermal loss due to heat transfer through it. To solve real-life heat loss issues lot of methods had been proposed. The most common numerical method to calculate the heat loss is finite difference method in which difference operator forward in time and central in space, also spectral method with boundary conditions (specified temperature, specified heat flux and specified transfer coefficient) used to calculate the loss energy to the environment.

Now in this section we are presenting theorems based on heat flux and n^th^ order of accelerating rate of flow energy per unit surface of semi-infinite solid.

Theorem-1: Let $D_t^{\alpha,\beta} V(x,t) = \lambda V''(x), (\alpha > 0, \beta > 0, \lambda > 0, x \in R)$ with boundary conditions $V(x,t=0) = 0, V(x=0,t) = 1, x \in R, \lim_{x\to 0} V(x,t) = 0$, and if temperature is

$$V(x,t) = 1 - \frac{\frac{x}{\sqrt{\lambda t^{\alpha}}}}{\Gamma\left(\frac{-\alpha}{2}+1\right)1!} + \frac{\left(\frac{x}{\sqrt{\lambda t^{\alpha}}}\right)^2}{\Gamma(-\alpha+1)2!} - \dots = W\left(-{}^{\alpha}\!/_2, 1; \frac{-x}{\sqrt{\lambda t^{\alpha}}}\right)$$

Where $W(.)$ is wright function. The heat flux at a certain point x=0 at the surface of solid is given by

$$\Phi_q = \frac{C(T_s - T_0)}{\sqrt{\lambda t^{\alpha}}\,\Gamma\left(1-\frac{\alpha}{2}\right)}; c > 0, \alpha > 0, \lambda > 0.$$

Proof: The surface heat flux of the solid is

$$\Phi_q = -C\left[\frac{\partial T}{\partial X}\right]$$

By using the considered boundary conditions, we get

$$\Phi_q = -C\frac{\partial}{\partial X}\left\{T_0 + (T_s - T_0) W\left(-{}^{\alpha}\!/_2, 1; \frac{-X}{\sqrt{\lambda t^{\alpha}}}\right)\right\}$$

On expending the function in right hand side and by differentiating partially with respect to $'t'$, we get

$$\Phi_q = -C(T_s - T_0)\left\{0 - \frac{\frac{1}{\sqrt{\lambda t^{\alpha}}}}{\Gamma\left(\frac{-\alpha}{2}+1\right)1!} + \frac{2\frac{x}{\sqrt{\lambda t^{\alpha}}}}{\Gamma(-\alpha+1)2!} - \dots\right\}$$

On the surface of solid by considering, $X = 0$

$$\Phi_q = \frac{C(T_s - T_0)}{\sqrt{\lambda t^{\alpha}}\,\Gamma\left(1-\frac{\alpha}{2}\right)}; c > 0, \alpha > 0, \lambda > 0.$$

Finally, we get the desired result of theorem 1.

Note: With the help of equation (10), the above obtained result can also have expressed in the form of complement error function.

Theorem 2: For fractional order heat equation $D_t^{\alpha,\beta} V(x,t) = \lambda V''(x), (\alpha,\beta > 0,, \lambda > 0, x \in R)$ if $V(x,t) = 1 - \frac{\frac{x}{\sqrt{\lambda t^{\alpha}}}}{\Gamma\left(\frac{-\alpha}{2}+1\right)1!} + \frac{\left(\frac{x}{\sqrt{\lambda t^{\alpha}}}\right)^2}{\Gamma(-\alpha+1)2!} - \dots = W\left(-{}^{\alpha}\!/_2, 1; \frac{-x}{\sqrt{\lambda t^{\alpha}}}\right)$,with boundary conditions $V(x,t=0) = 0$ and $V(x=0,t) = 1, x \in R, \lim_{x\to 0} V(x,t) = 0$, Where $W(.)$ is wright function.At a certain point $x = 0$ the heat flux on the surface of solid

$\Phi_q = \frac{C(T_s - T_0)}{\sqrt{\lambda t^{\alpha}}\,\Gamma\left(1-\frac{\alpha}{2}\right)}; c > 0, \alpha > 0, \lambda > 0,$ then accelerating heat flux of n^{th} order is given by

$$\Phi_q^{(n)}\Big|_{\{Time=t\}} = \frac{(-1)^n C(T_s - T_0)\left({}^{\alpha}/_2\right)_n}{\sqrt{(\lambda)}t^{\left({}^{\alpha}/_2+n\right)}\Gamma\left(1-\frac{\alpha}{2}\right)}$$

Where $C > 0, \alpha > 0, \lambda > 0, T_s > 0$.

Proof: By using the result of theorem (1), with the help of boundary conditions the zero order derivative of heat flux with respect to time is

$\Phi_q = \frac{C(T_s - T_0)}{\sqrt{\lambda t^{\alpha}}\Gamma\left(1-\frac{\alpha}{2}\right)}; c > 0, \alpha > 0, \lambda > 0,$

On differentiation the equation above with time $'t'$, the first derivative is

$$\Phi_q' = \frac{-C\frac{\alpha}{2}(T_s - T_0)}{\sqrt{\lambda}\Gamma\left(1-\frac{\alpha}{2}\right)t^{\left(\frac{\alpha}{2}+1\right)}}; c > 0, \alpha > 0, \lambda > 0,$$

On performing the successive differentiation, we observe that the fractional order $'\alpha'$ appear in multiplicative series of respective terms $\frac{\alpha}{2}\left(\frac{\alpha}{2}+1\right)\left(\frac{\alpha}{2}+2\right)\left(\frac{\alpha}{2}+3\right)\ldots, \alpha > 0$, which is the standard structure of pochhammer symbol $(\gamma)_n$.

By n^{th} order derivative of $'\Phi_q'$ we obtained the result at any time $t > 0$ as

$$\Phi_q^{(n)}\Big|_{\{Time=t\}} = \frac{(-1)^n C(T_s - T_0)\left({}^{\alpha}/_2\right)_n}{\sqrt{(\lambda)}t^{\left({}^{\alpha}/_2+n\right)}\Gamma\left(1-\frac{\alpha}{2}\right)} c > 0, \alpha > 0, \lambda > 0;$$

which is the required proof of the theorem.

Corollary: If we use the relationship between the pochhammer symbol and gamma function

$$\left({}^{\alpha}/_2\right)_n = \frac{\Gamma\left(\frac{\alpha}{2}+n\right)}{\Gamma\left(\frac{\alpha}{2}\right)}; \alpha > 0, n > 0,$$

The result of above theorem turns into the following manner

$$\Phi_q^{(n)}\Big|_{\{Time=t\}} = \frac{(-1)^n C(T_s - T_0)}{\sqrt{(\lambda)}t^{\left({}^{\alpha}/_2+n\right)}\Gamma\left(1-\frac{\alpha}{2}\right)} * \frac{\Gamma\left(\frac{\alpha}{2}+n\right)}{\Gamma\left(\frac{\alpha}{2}\right)} c > 0, \alpha > 0, n > 0, \lambda > 0;$$

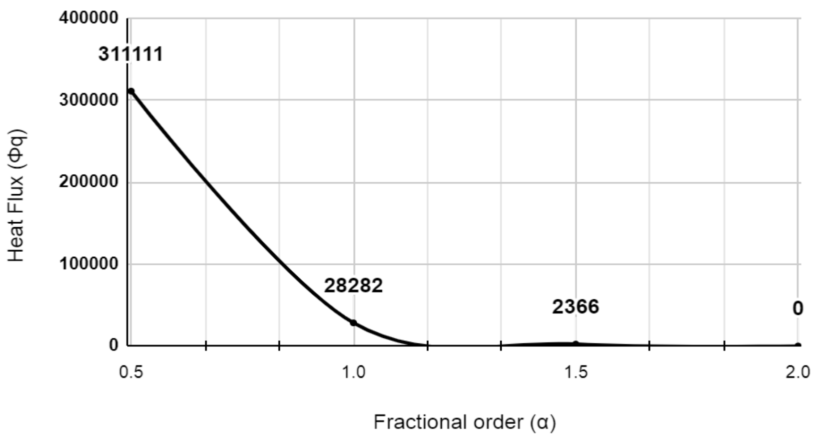
4. NUMERICAL IMPLEMENTATIONS OF STUDY ON CONCRETE WALL

The proposed result of theorem (1) is concern with the heat transfer as heat flux in the solid material and fractional order of heat equation. For the numerical implementation of the above study, we consider the concrete wall. At the critical length and specific allowable time period they can be consider as semi-infinite solid. By plugging the initial and appropriate boundary values of respective variables and thermal parameters we analyse mathematically the dependency of heat flux on time and other variables. At time $t = 0$, the initial temperature of the solid be T_0, and at the finite length of the solid substance temperature is T_s, $(T_s > T_0)$. Depending on the thermal diffusivity and thermal conductivity solids are allowing the amount of heat passes through it.

Heat Flux on semi-infinite concrete wall:

We consider the 15cm thick concrete wall which is coated with black silicone paint for the experimental study of flowing thermal energy. Since the black silicone paint is the less amount heat transferring substance, therefore to perform the uninterrupted experiment on concrete wall, we consider the coating of silicone paint on concrete wall as heat insulator. At 1000K the concrete wall is approximated as a black body. The initial temperature of concrete wall at time $t = 0$ is $T_{0\,(concrete)} = 300K$. After experimental time of 2 minutes the temperature of concrete surface reached at $T_{s\,(concrete)} = 500K$. The resultant heat flux for the fractional order $\alpha = 1.5$, on the semi-infinite wall was observed by $2366\ W/m^2$. The thermal diffusivity of concrete was $\lambda = 0.75 \times 10^{-6} m^2/s$ and thermal conductivity was $C = 1.4\ W/mK$. The geometrical representation of fractional order over the heat flux is shown in the figure 2.1

Figure 2.1:Heat Flux v/s Fractional order on concrete wall



It can be observed that after experimental time of 2 minutes, the flow of thermal energy through concrete wall decreases as we increase the value of fractional order $\alpha > 0$.

5. Correlativity Between Heat Flux and Fractional Order Derivative

We introducing the new model to know about the concrete relationship between fractional orders derivatives with amount of heat flux generate on the semi-infinite body. For this we consider the range of fractional order α varies in the interval $0 \leq \alpha \leq 0.5$

To find the distribution of fractional order derivatives for the time $t > 0$, the difference in temperature at the particular point of body over the heat transfer we use some statistical techniques. For the analysis first variable is fractional order α, which is considered as $variable_1$ (X). The second variable is calculated heat flux Φ_q, which is considered as $variable_2$ (Y) . The following table 2.2 give the respective values of heat flux on the respective values of fractional order α which lies in the range of $0 \leq \alpha \leq 0.5$ .The degree of relationship between these two variables under the consideration we evaluates the coefficient of correlation which is denoted by $'\mathbf{r}'$.

For Concrete Wall

Table 2.2: Calculated Heat Flux with Fractional Order for Concrete wall

S.No.	1	2	3	4	5	6	7	8	9	10
Fractional order(α) (X)	0.09	0.10	0.11	0.12	0.14	0.16	0.20	0.25	0.3	0.5
Heat Flux(Φ_q)KW/m^2 (Y)	255	255	255	233	215	200	187	165	133	77

Generally concrete wall is made from various types of aggregates, water and mixture of Portland cement, blended cement. Our investigation was done with the thermal conditions and temperature variation on the surface of concrete wall. Here we are neglecting the consideration of thermal properties such as heat capacity, conductivity, diffusivity etc. of ingredients included in the manufacturing of wall. The correlativity among the considered variables is calculated by the following data

$$n = 10, \sum XY = 324, \sum X = 1.9700, \sum Y = 1975, \sum X^2 = 0.5323, \sum Y^2 = 421401$$

We found that the coefficient of correlation is -0.9783

The quantitative value of coefficient of correlation lies between $-1 \leq r \leq 0$. We conclude that the thermal variable and fractional order containing perfect negative correlation.

6.CONCLUSION

The behaviour of concrete substance in sense of heat transfer with respect to fractional order was pointed out. An attempt was done to explain the role of fractional domain in thermo-physics. The correlativity of heat flux with non-integer order of concrete was well explained. We investigate that within the range $0.09 \leq \alpha \leq 0.5$ the authenticity of bonding between fractional distribution on the theoretically proposed heat flux were established in the form of negative correlation.

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