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Cellular structure of wreath product $\overrightarrow{S_n}$ using signed Brauer diagrams

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Abstract: In this paper we prove that the wreath product $\overrightarrow{S_n} = Z_2 \wr S_n$ with symmetric group S_n, is cellular for algebra $Z_2(x)$. We obtain simple cell modules which satisfy semi-simplicity conditions. We make use of method of iterated inflations for this purpose.

Keywords: cellular algebra, signed Brauer algebras, symmetric groups, wreath product.

1. Introduction:

The wreath product $\overrightarrow{S_n} = Z_2 \wr S_n$ is a semi-direct product of group algebras Z_2 and S_n whose elements are represented by using signed Brauer diagrams. A new class of algebras namely signed Brauer's algebras denoted by $\overrightarrow{B_n}(x)$ for a positive integer n and a complex number x was first introduced by Parvathi and Kamaraj [2]. $\overrightarrow{B_n}(x)$ contain the Brauer algebras $B_n(x)$ and the group algebras $C\overrightarrow{S_n}$ where $\overrightarrow{S_n} = Z_2 \wr S_n$ is a wreath product of Z_2 by S_n. In a graph, if every edge is labelled by either a plus sign or a minus sign then it is called the signed diagram. These edges are called signed edges. An edge labelled by plus sign is called a *positive* edge, denoted by $\downarrow (\rightarrow)$ and an edge labelled by negative sign is called a *negative* edge denoted by $\uparrow (\leftarrow)$. $\overrightarrow{B_n}(x)$ is an associative algebra with a basis of signed diagrams and multiplication defined in it. Elements of $\overrightarrow{S_n}$ are represented by the signed diagrams with no horizontal edges. Thus the structure of semi-simple algebras $C\overrightarrow{S_n}$ helps to understand the structure of signed Brauer algebra. $\overrightarrow{S_n}$ is a wreath product of group algebras Z_2^n and S_n where $Z_2^n = \{f | f : \{1, 2, ..., n\} \rightarrow Z_2\}$ is an associative unital algebra and S_n is a symmetric group of order n. For $n \geq 2$, $\overrightarrow{S_n}$ becomes a subgroup of the symmetric group S_{2n}. Graham Leherer in [1] introduced the cellular algebra and since then it has its wide applications in many fields like the representation theory of wreath product algebra. R. Green[3, 4] has proved that for any cellular algebra A, the wreath product $A \wr S_n$ is cellular if it is an iterated inflation of tensor products of group algebras of symmetric groups. Konig and Xi [5, 6] were the first to introduce the concept of iterated inflation of tensor products and R. Green [4] has applied it for cellular algebras. Sharma R.P., Parmar R. and Kapil V.S. [7] have constructed a complete set of inequivalent irreducible $\overrightarrow{S_n}-$modules and used it to understand cellular structure of $\overrightarrow{B_n}(x)$. T.Geetha and F.M. Goodman [8] have proved that the wreath product algebras $A \wr G_n$ and $A-$Brauer algebras $D_n(A)$ both are cyclic cellular algebras if A is a cyclic cellular algebra. In this paper we prove that wreath product algebra $\overrightarrow{S_n} = Z_2 \wr S_n$ cyclic cellular algebra provided the group algebra $Z_2^n(x)$ with symmetric group S_n is a cyclic cellular algebra. We obtain the simple modules and cell modules which satisfy a semi-simplicity condition for $\overrightarrow{S_n}$. We make use the method of iterated inflations to acheive this.

2. Priliminaries:

For a field R of characteristic p, let $Z_2^n(x)$ be an unital associative $R-$algebra whose dimension is finte. We consider the right modules of finite $R-$ dimension and denote $\otimes_R$ by $\otimes$. For all $f, g \in Z_2^n$, a self-inverse $R-$linear isomorphism $g \to g^*$ such that $(fg)^* = g^* f^*$ is an anti-involution on $R-$algebra Z_2^n.

Definition 2.1.[7] Let n be a non-negative integer. An ordered sequence $a = (a_1, a_2, ..., a_s)$ such that $\sum_j a_j = n$ is called a composition or tuple of non-negative integers of order n. If two compositions $a = (a_1, a_2, ..., a_s)$ and $b = (b_1, b_2, ..., b_s)$ are such that $\sum_j a_j \geq \sum_j b_j$ for each $j = 1, 2, ..., s$, then $a \trianglerighteq b$. Also if $a_j > b_j$, then $a \triangleright b$. Thus for $a_1 \geq a_2 \geq ... \geq a_s$, a composition of non-negative integers $a = (a_1, a_2, ..., a_s)$ represents a partition. In particular, when $n = 0$, there is only one partition $()$.

Definition 2.1. [3] Let R be commutative ring with unit 1 and $Z_2^n(x)$ be an associative, unital $R-$algebra. Let Λ be a finite set with partial order $\leq$ and for each $\lambda \in \Lambda$, let $M(\lambda)$ be a finite right indexing set. Then for all $(s, t) \in M(\lambda) \times M(\lambda)$, there is an element $C_{s,t}^{\lambda} \in Z_2^n$ such that there is an injective map $(\lambda, s, t) \to C_{s,t}^{\lambda}$ and

$$C_{s,t}a = \sum_{p \in M(\lambda)} R(t,p) C_{s,p}^{\lambda} \text{ ......(1)}$$

is a free $R-$basis for Z_2^n. The action of Z_2^n on the right cell module Δ^{λ} is

$$C_t a = \sum_{p \in M(\lambda)} R_a(t,p) C_p \text{.......(2)}$$

Definition 2.2. [7] The wreath product $\overrightarrow{S_n}$ can be redefined as

$$\overrightarrow{S_n} = \{\alpha \in S_{2n} | \text{ if } \alpha(r) = s \text{ then } \theta(r^*) = s^*\}$$

where for each $r \in (1, 2, ..., 2n)$, r^*, is given by :

$$r^* = \begin{cases} r + n, & 1 \leq r \leq n \\ r - n, & n < r \leq 2n \end{cases}$$

2.3. Elements of $\overrightarrow{S_4}$ using signed Brauer diagrams: Basis of the natural permutations of $\overrightarrow{S_4}$ are identified with the set of all signed Brauer diagrams consisting $2n$ dots. There will be two rows each having n dots and there are n signed edges connecting pairs of dots taking one from each row. This can be illustrated with an example:

For $n = 4$ and $2n = 8$, the anti-involution element $(17)(53)(28)(64)$ of $\overrightarrow{S_4}$ is identified with the signed Brauer diagram

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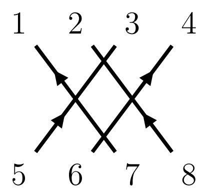
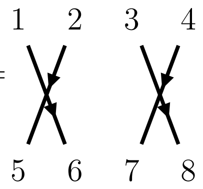
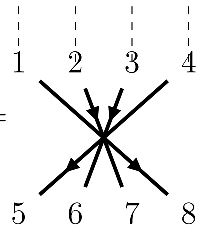
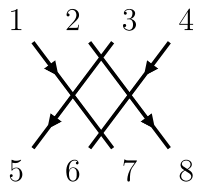


$=(1\ 7)(5\ 3)(2\ 8)(6\ 4)$

Let $\sigma, \alpha \in \overrightarrow{S_n}$ be any two anti-involution elements. The conjugate of these two anti-involution elements is again an anti-involution element of $\overrightarrow{S_n}$.

For example: for $n = 4, 2n = 8$, if $\sigma = (12)(56)(34)(78), \alpha = (14)(58)(23)(67)$ are two anti-involution elements of $\overrightarrow{S_4}$ then the conjugate $\sigma^{\alpha} = (13)(57)(24)(68)$ in $\overrightarrow{S_4}$ is again an anti-involution element. This can be well understood through the signed Brauer diagrams:

$\sigma =$  $=(1\ 2)(5\ 6)(3\ 4)(7\ 8)$

$\alpha =$  $=(1\ 4)(5\ 8)(2\ 3)(6\ 7)$ $\sigma^{\alpha} =$  $=(1\ 3)(5\ 7)(2\ 4)(6\ 8)$

3. Cellular algebras of wreath product $\overrightarrow{S_n} = Z_2 \wr S_n$:

Let Z_2^n be a finite dimensional unital assocaitive R-algebra. Consider the R-vector space ${Z_2}^{\otimes n} \otimes RS_n$, A pure tensor $\alpha \otimes f_1 \otimes f_2 \otimes ... \otimes f_n$ in this vector space be written as $(\alpha; f_1, f_2, ..., f_n)$. Then a well defined multiplication is given by

$$(\alpha; f_1, f_2, ..., f_n)(\beta; g_1, g_2, ..., g_n) = (\alpha\beta; f_{1\beta^{-1}}g_1, f_{2\beta^{-1}}g_2, ..., f_{n\beta^{-1}}g_n)$$

for $\alpha, \beta \in S_n$ and $f_i, g_i \in Z_2^n$. A pure tensor $(\alpha; f_1, f_2, ..., f_n)$ in $\overrightarrow{S_n}$ where $\alpha \in S_n$ and $f_i \in Z_2^n$, can be represented by using signed Brauer diagram.

For example:Take $n = 6$, Let $\alpha, \beta \in S_n$ such that

$$(\alpha; f_1, f_2, f_3, f_4, f_5, f_6) = (1, 3, 6, 10, 5, 8)(7, 9, 12, 4, 11, 2)$$

and

$$(\beta; g_1, g_2, g_3, g_4, g_5, g_6) = (1, 9, 4, 6,)(7, 3, 10, 12)$$

are the elements of $\overrightarrow{S_n}$.

Then their product

$$(\alpha; f_1, f_2, f_3, f_4, f_5, f_6)(\beta; g_1, g_2, g_3, g_4, g_5, g_6)$$

$$=(\gamma; h_1, h_2, h_3, h_4, h_5, h_6)$$

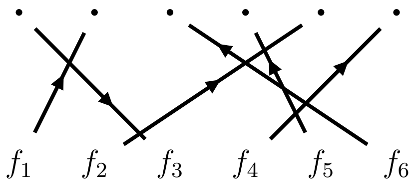
$$= (1, 10, 5, 8, 9, 7, 4, 11, 2, 3)(6, 12).$$

is an element in $\overrightarrow{S_n}$.

The same is illustrated using signed Brauer diagram where the product is obtained by resolving the two connected edges and the resulting element is an element of $\overrightarrow{S_n}$ given by

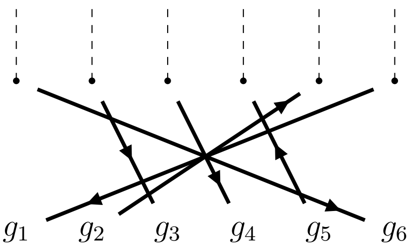
$$(\gamma; f_6g_1, f_2g_2, f_1g_3, f_3g_4, f_5g_5, f_4g_6) = (1, 10, 5, 8, 9, 7, 4, 11, 2, 3)(6, 12).$$

$\alpha =$



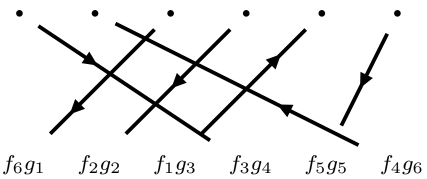
$=(1, 3, 12, 4, 11, 2, 7, 9, 6, 10, 5, 8)$

$\beta =$



$=(1, 6)(7, 12)(2, 3, 4, 11)(8, 9, 10, 5)$

$\alpha^\beta =$



$=(1, 4, 2, 12, 11, 3, 7, 10, 8, 6, 5, 9)$

Anti-involution $*$ of $\overrightarrow{S_n}$ is given by

$$(\alpha; f_1, ..., f_n)^* = (\alpha^{-1}; f^*_{(1)\alpha}, ..., f^*_{(n)\alpha})$$

where $\alpha \in S_n$ and $f_1, ..., f_n \in Z_2^n$. The mapping of anti-involution elements takes place through the diagram by replacing each element f_i with its image f_i^* under the anti-involution on Z_2^n, and then sliding each element f_i^* to the bottom of the row.

3.1.Construction of modules for $\overrightarrow{S_n}$: Let μ be the s-part composition of n, $A_1, ..., A_s$ be Z_2^n modules and for each $j = 1, ..., s$ let B_j be kS_μ module. $\overrightarrow{S_\mu} = Z_2^n \wr S_\mu$ be the subalgebra of $\overrightarrow{S_n} = Z_2^n \wr S_n$ spanned by all elements $(\alpha; f_1, ..., f_n)$ where $f_j \in Z_2^n$ and $\alpha \in S_\mu$. Then

$A_1^{\otimes\mu_1} \otimes ... \otimes A_s^{\otimes\mu_s} \otimes B_1^{\otimes} ... \otimes B_s$ is $\overrightarrow{S'_\mu}$-module through the action

$$(a_1 \otimes ... \otimes a_n \otimes b_1 \otimes ... \otimes b_s)(\alpha; f_1, ..., f_n) = a_{(1)\alpha^{-1}} f_1 \otimes ... \otimes a_{(n)\alpha^{-1}} f_n \otimes b_1\alpha_1 \otimes ... \otimes_s \alpha_s,$$

where each $\alpha_i \in S_\mu$, are such that whenever S_μ is identified naturally with $S_{\mu_1} \times ... \times S_{\mu_s}$ and α is identified with $(\alpha_1, ..., \alpha_s)$.
Therefore by induction, from $\overrightarrow{S'_\mu} = Z_2^n \wr S_\mu$ to $\overrightarrow{S'_n} = Z_2^n \wr S_n$, we get a module isomorphic to

$$A_1^{\otimes\mu_1} \otimes ... \otimes A_s^{\otimes\mu_s} \otimes B_1 \otimes ... \otimes B_s \otimes kR_\mu,$$

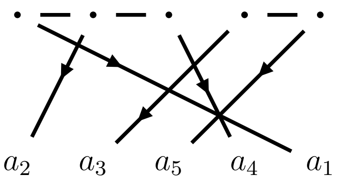
where the basis of vector space kR_μ is R_μ which contains coset representations of minimal length. Let $\gamma, \delta \in R_\mu$, and $\theta, \alpha \in S_\mu$ such that $\gamma\alpha = \theta\delta$, then the action of $\overrightarrow{S'_n}$ on kR_μ is given by

$$(a_1 \otimes ... \otimes a_s \otimes b_1 \otimes ... \otimes b_s \otimes \delta)(\alpha; f_1, ..., f_n) = a_{(1)\theta^{-1}} f_1\delta \otimes ... \otimes a_{(n)\theta^{-1}} f_n\delta \otimes b_1\theta_1 \otimes ... \otimes b_s\theta_s,$$

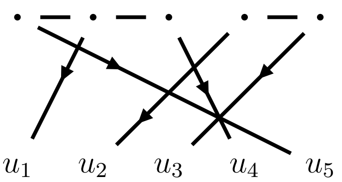
Let $\overline{A} = (A_1, ..., A_s)$ and $\overline{B} = (B_1, ..., B_s)$ be the tuples and the modules obtained be $\Theta^\mu(\overline{A}, \overline{B})$. A pure tensor $a_1 \otimes ...a_n \otimes b_1 \otimes ... \otimes b_s \otimes \delta$ is taken as a pure tensor for $\delta \in R_\mu$. To obtain the signed Brauer diagram of this tensor, first label the edges in the lower row from left to right with the elements $a_{(1)\delta^{-1}}, ...a_{(n)\delta^{-1}}$, then link them to the first μ_1 edges on the top row and label the linked edges with b_1 Similarly link the next μ_2 edges on the top row and label the linked edges as b_2, and so on.
For example, take $n = 5, s = 2, \mu = (3, 2)$ and $\delta = (1, 5, 3, 4, 2)(6, 10, 8, 9, 7)$ of $\overrightarrow{S'_5}$, then the tensor

$$a_1 \otimes a_2 \otimes a_3 \otimes a_4 \otimes a_5 \otimes b_1 \otimes b_2 \otimes \delta$$

can be represented by the diagram



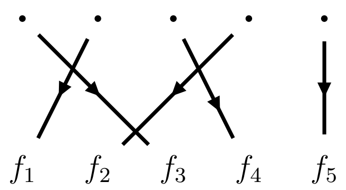
The elements of A_j are associated with the edges b_j and hence $\Theta^\mu(\overline{A}, \overline{B})$ can be identified with the k-vector space by diagram of some element in R_μ where we see that for each $j = 1, 2, ...s$, $(\mu_1 + ... + \mu_{j-1} + 1)$th to $(\mu_1 + ... + \mu_j)$th edges are connected so as to form a single block which is labelled by an element of B_j. For $\delta \in R_\mu$, $b_1, ..., b_s$ labels in the top row and $u_1, ..., u_n$ labels in bottom row and the permutation diagram is $\delta \in R_\mu$. This represents pure tensor $u_{(1)\delta} \otimes ... \otimes u_{(n)\delta} \otimes b_1 \otimes ... \otimes b_s \otimes \delta$. Set of these elements span $\Theta^\mu(\overline{A}, \overline{B})$ but not linearly independent. The diagram



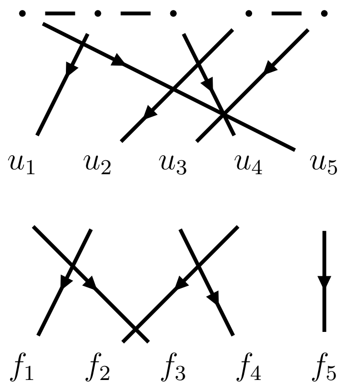
represents the pure tensor

$$u_2 \otimes u_3 \otimes u_5 \otimes u_4 \otimes u_1 \times b_1 \otimes b_2 \otimes (1, 5, 3, 4, 2)(6, 10, 8, 9, 7)$$

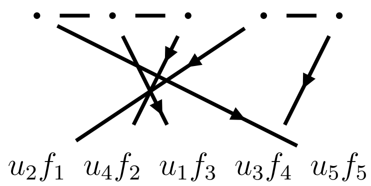
Consider another element $((1,3,4,2)(6,8,9,7); f_1, f_2, f_3, f_4, f_5)$ of $\overrightarrow{S_{10}} = Z_2 \wr S_5$,



Action of element of $\overrightarrow{S_{10}}$ on the element of $\Theta^{\mu}(\overline{A}, \overline{B})$ is calculated as in the diagram

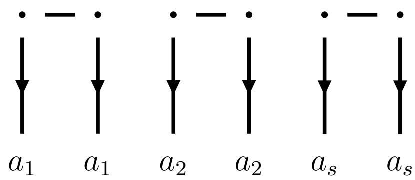


After this action, the element of $\overrightarrow{S_{10}}$ from this diagram is $(1,5,4)(6,10,9)(2,3)$.



This element is the product of elements from S_μ and R_μ.
That is $(1,5,3,4,2)(6,10,8,9,7) = (1,5,4)(2,3).(6,10,9)(7,8)$ where $(1,5,4)(2,3) \in S_\mu$ and $(6,10,9)(7,8) \in R_\mu$.

Proposition 3.2. If $A_1, ..., A_s$ are Z_2^n- modules, and $B_1, ..., B_s$ are $kS_\mu-$modules, then for any composition μ which is s part of n with a_k and b_k as generators of A_k and B_k, $\Theta^{\mu}(\overline{A}, \overline{B})$ is a cyclic $\overrightarrow{S_n}-$ module. The signed Brauer diagram given by



generates $\Theta^{\mu}(\overline{A}, \overline{B})$.

Proof: Let us denote the diagram in the proposition D . For each $a \in S_\mu$, apply $(a; j, ..., j)$ of $\overrightarrow{S_n}$ to replace each element b_k in D with an arbitrary element of B_k. For some $\delta \in R_\mu$, apply $(\delta; j, ..., j)$ to arrange the strings of the diagram D. Then replace each element a_k with an arbitrary element of A_k by applying an element $(e; f_1, ..., f_n)$. These diagrams span $\Theta^{\mu}(\overline{A}, \overline{B})$. This completes the proof.

Let Δ^λ be cell module of the cellular algebra Z_2^n. Then for $\alpha \in S_n$,

$$\alpha_\lambda : C^{\lambda}_{s,t} + \overline{Z}_2^{\,n\lambda} \rightarrow C^{\lambda}_{s} \times (C^{\lambda}_{t})^{*}$$

determines a $Z_2^n - Z_2^n$ bi-module isomorphism from $Z_2^{n\lambda}/\overline{Z}_2^{\,n\lambda}$ to $\delta^\lambda \otimes_R (\Delta^\lambda)^*$. For all $s, t, u, v \in M(\lambda)$, there exist $R-$valued bi-linear form $\langle . \rangle$ such that

$$C^{\lambda}_{s,t}C^{\lambda}_{u,v} \equiv \langle C^{\lambda}_{t}.C^{\lambda}_{u}\rangle C^{\lambda}_{s,v} \mod \overline{Z}_2^{n\lambda}.$$

This bi-linear form plays an essential role in theory of cellular algebra

Lemma 3.3. Z_2^n be a cellular algebra with cell datum $(\Lambda, M, C, *)$. Let $\lambda \in \Lambda$ and $d \in \Delta^{\lambda}$ be non-zero. Then $a \to a \otimes d^*$ is a Z_2^n-module isomorphism of Δ^{λ} onto $\Delta^{\lambda} \otimes d^* \subseteq \Delta^{\lambda} \otimes_R (\Delta^{\lambda})^*$.

Proof. $(\Delta^{\lambda})^*$ is a free module and hence torsion free. Thus as Z_2^n-modules,

$$\Delta^{\lambda} \cong \Delta^{\lambda} \otimes_R Rd^* = \Delta^{\lambda} \otimes d^*.$$

hence $x \to x \otimes d^*$ is an isomorphism.

Definition 3.4.[8] A cellular algebra is said to be cyclic cellular if every cell module of Z_2^n is cyclic.

Lemma 3.5. If Z_2^n is a cellular algebra with cell datum $(\Lambda, M, C, *)$ then following are equivalent.

(i) Z_2^n is cellular.

(ii) For each $\lambda \in \Lambda$, there exists an element $a_{\lambda} \in Z_2^{n\lambda}$ such that

(a) $a_{\lambda} \equiv a_{\lambda}^* \mod \overline{Z}_2^{n\lambda}$

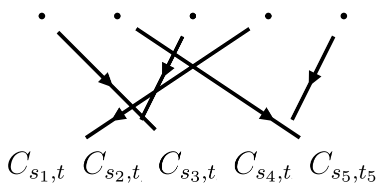
(b) $Z_2^{n\lambda} = Z_2^n a_{\lambda} Z_2^n + \overline{Z}_2$

(c) $(Z_2^n a_{\lambda} + \overline{Z}_2^{n\lambda})/\overline{Z}_2^{n\lambda} \cong \Delta^{\lambda}$, as Z_2-modules.

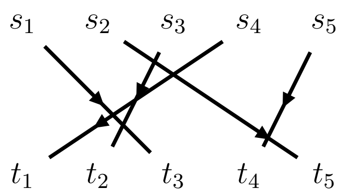
Proof. Suppose that Z_2^n is cyclic cellular. For each $\lambda \in \Lambda$, let δ^{λ} be the generator of the cell module Δ^{λ}. Let $a_{\lambda} \in Z_2^{n\lambda}$ be any lifting of $\alpha_{\lambda}^{-1}(\delta^{\lambda} \otimes (\delta^{\lambda})^*)$. Then $(\delta^{\lambda} \otimes (\delta^{\lambda})^*)^* = (\delta^{\lambda} \otimes (\delta^{\lambda})^*)$ implies 2(a) is true. $Z_2^n(\delta^{\lambda} \otimes (\delta^{\lambda})^*)Z_2^n = (\Delta^{\lambda} \otimes_R (\Delta^{\lambda})^*)$ implies 2(b) is true.
We obtain Z_2^n-module isomorphism $za_{\lambda} + \overline{Z}_2^{n\lambda} \to z\delta^{\lambda \otimes (\delta^{\lambda})^*}$, by restricting α_{λ}. By lemma 3.3., $x \otimes (\delta^{\lambda})^* \to x$ is an Z_2^n-module isomorphism from $\Delta^{\lambda} \otimes (\delta^{\lambda})^*$ onto Δ^{λ}. By composing these two isomorphisms we get $za_{\lambda} + \overline{Z}_2^{n\lambda} \to z\delta^{\lambda}$ such that $(Z_2^n a_{\lambda} + \overline{Z}_2^{n\lambda})/\overline{Z}_2^{n\lambda} \cong \Delta^{\lambda}$. This proves 2(c).
Conversely, if (2) holds, then in particular 2(c) implies that each cell module is cyclic. Hence Z_2^n is cellular.

4.The iterated inflation structure of $\overrightarrow{S_n} = Z_2 \wr S_n$

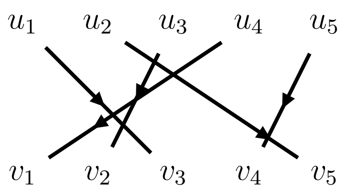
As stated by Konig and Xi in [5], prove that Brauer algebra is cellular by exhibiting a structure called iterated inflation. Special cases can be found in the papers of Graham and Lehrer[1]. R.Green[3] has applied iterated inflations to show that the wreath product $A \wr S_n$ is cellular for any cellular algebra A by showing it as an iterated inflation of tensor products of group algebras of symmetric group.
Let Z_2 be a cellular algebra with cell datum $(\Lambda, M, C, *)$ where * denotes the anti involution. Let $|\Lambda| = s$, and $\lambda_1, ..., \lambda_s$ be the elelments of Λ such that $\lambda_i > \lambda_j$ for all $i < j$. Thus the numbering is in compatible with partial ordering on Λ. Let Δ^{λ} be the right cell module for every $\lambda \in \Lambda$. We shall write the elements of cellular basis $C^{\lambda}_{s,t}$ as $C_{s,t}$. Then the basis of $\overrightarrow{S_n} = Z_2 \wr S_n$ consists the elements of the form $(\alpha; C_{s_1,t_1}, ..., C_{s_n,t_n})$ for $\alpha \in S_n$ and C_{s_k,t_k} is some element in the basis of Z_2. We shall denote this basis as P. The elements of P are represented by the diagrams



This can be represented as



Thus the elements P can be represented as



This diagram represents the element
$((1,3,2,5,4)(6,8,7,10,9); C_{u_4v_1}, C_{u_3v_2}, C_{u_1v_3}, C_{u_5v_4}, C_{u_2v_5}) \in \overrightarrow{S_5} = Z_2 \wr S_5$.
where $u_l, v_m \in M(\lambda)$ for some $\lambda \in \Lambda$. For each $l \in \{1, ..., s\}$, let μ_l be the number of elements u_l in $M(\lambda)$ such that there exists a composition $\mu = \mu_1, ..., \mu_s$ of n for any such diagram. We call it *layer index* of the diagram and also the element which it represents in P. Let kP be the $k-$span of all elementsof P with layer index μ, and $I(n,s)$ be the set of all $s-$part composition of n with non-negative integer entries. Thus $\overrightarrow{S_n} = Z_2 \wr S_n = \bigoplus_{\mu \in I(n,s)} kP_\mu$. For layer index μ, the tuple $(u_1, ..., u_n)$ of n elements of $\sqcup_{\lambda\in\Lambda} M(\lambda)$ denotes the half diagram such that it has exactly μ_k elements of $M(\lambda_k)$ for each k. Let U_μ be the set of all half diagram of type μ. Then there exists a unique element $\epsilon \in R_\mu$ such that $(u_{(1)\epsilon}, ..., u_{(n)\epsilon})$ lies in the set $M(\lambda_1)^{\mu_1} \times ... \times M(\lambda_s)^{\mu_s}$. We call this ϵ the *shape* of the half diagram $(u_1, ..., u_n)$.
Let $\alpha \in S_n$ be a permutation such that u_k is connected to $v_{(k)\alpha}$. Then there is an element

$$(\alpha; C_{u_{(1)\alpha^{-1}}, v_1}, ..., C_{u_{(n)\alpha^{-1}}, v_n})$$

in $\overrightarrow{S_n}$ which has a layer index μ. This element can be split into three parts namely the half diagrams $(u_1, ..., u_n)$ of top rows and $(v_1, ..., v_n)$ of bottom rows of type μ and the element $(\beta_1, ..., \beta_s)$ of the group $S_{\mu_1} \times ... \times S_{\mu_s}$ where $\mu_k \in S_{\mu_k}$. This β_k records how the elements of $M(\lambda_k)$ on the top row is connected to the elements of $M(\lambda_k)$ in the bottom row. If ϵ, δ are the shapes of $(u_1, ..., u_n)$ and $(v_1, ..., v_n)$ respctively and β is the image of $(\beta_1, ..., \beta_s)$ under the natural identification of $S_{\mu_1} \times ... \times S_{\mu_s}$ with Young subgroup S_μ of S_n. Then $\beta = \epsilon^{-1}\beta\delta$. We now take B_μ to be the $k-$vector space with basis b_μ. Then the above decomposition has a $k-$linear bijection

$$B_\mu \otimes kS_\mu \otimes B_\mu \to kZ_2^n - \mu$$

given by the mapping

$$(u_1, ..., u_n) \otimes \beta \otimes (v_1, ..., v_n),$$

to

$$(\epsilon^{-1}\beta\delta; C_{u_{(1)\epsilon^{-1}\beta\delta}, v_1}, ..., C_{u_{(n)\epsilon^{-1}\beta\delta}, v_n})$$

where ϵ is the shape of $(u_1, ..., u_n)$ and δ is the shape of $(v_1, ..., v_n)$. Thus we have a decomposition $\overrightarrow{S_n} = \bigoplus_{\mu \in I_{(n,s)}} B_\mu \otimes kS_\mu \otimes B_\mu$, and this decomposition will allow us to exhibit the desired iterated inflation structure.
Now, we need ordering of $I_{(n,s)}$. If $(\mu_1, ..., \mu_s)$ and $\gamma_1, ..., \gamma_s$ are elements of $I_{(n,s)}$, then define $\mu \trianglerighteq_\Lambda \gamma$ such that

$$\sum \mu_k \geq \sum \gamma_k$$

where $\lambda_k \geq \lambda_l$ for each k. This is called the partial $\Lambda-$dominance order.

Lemma.4.1. Suppose that we have $a_1, ..., a_s, b_1, ..., b_s \in \{1, ..., s\}$ such that $\lambda_{a_i} \geq \lambda_{b_i}$ for each i in the poset Λ and let $\mu = (\mu_1, ..., \mu_s)$ and $\gamma = (\gamma_1, ..., \gamma_s)$ so that $\mu, \gamma \in I_{(n,s)}$. Then $\mu \trianglerighteq_\Lambda \gamma$ and if one of the inequalities $\lambda_{a_i} \geq \lambda_{b_i}$ is strict, then $\mu \triangleright_\Lambda \gamma$.

Proof: For each k, $\lambda_k \geq \lambda_l$ implies

$$\sum \mu_k \geq \sum \gamma_k$$

where

$$\sum \mu_k = |\{i : \lambda_{a_i} \geq \lambda_l\}|,$$

$$\sum \gamma_k = |\{i : \lambda_{b_i} \geq \lambda_l\}|.$$

But

$$\{i : \lambda_{a_i} \geq \lambda_l\} \subset \{i : \lambda_{b_i} \geq \lambda_l\} \text{ for } \lambda_{a_i} \geq \lambda_{b_i}.$$

Thus if there is one strict inequality $\lambda_{a_i} > \lambda_{b_i}$, then we get $\mu \neq \gamma$ and hence $\mu \triangleright_\Lambda \gamma$.

Proposition 4.2. Let $\mu \in I_{(n,s)}$, and $u = (u_1, ...,)u_n, v = (v_1, ..., v_n) \in B_\mu$. Let $\beta_1, ..., \beta_s \in S_\mu$ be such that the element of basis of Z_2 corresponding to the pure tensor $u \otimes \beta \otimes v$ has a layer index μ. Also, let $f = (\alpha; f_1, ..., f_n)$ be a pure tensor in $\overrightarrow{S_n} = Z_2 \wr S_n$. Then $(u \otimes \beta \otimes v).f \cong u \otimes \beta\phi_\mu(v, f) \otimes \psi_\mu(v, f)$ modulo elements of basis of Z_2 whose layer index is strictly less than μ, where $\phi_\mu(v, f) \in S_\mu$ and $\psi_\mu(v, f) \in B_\mu$ are independent of u and β.

Proof. Let $\epsilon, \delta \in B_\mu$ be the shapes of u and v respectively, so that $u \otimes \beta \otimes v$ corresponds to the element

$$(\epsilon^{-1}\beta\delta; C_{u_{(1)}\epsilon^{-1}\beta\delta, v_1}, ..., C_{u_{(n)}\epsilon^{-1}\beta\delta, v_n}).$$

Then

$$\begin{aligned}&(u \otimes \beta \otimes v)(\alpha; f_1, ..., f_n)\\ =&(\epsilon^{-1}\beta\delta; C_{[u_{(1)}(\epsilon^{-1}\beta\delta)^{-1}, v_1]}, ..., C_{[u_{(n)}(\epsilon^{-1}\beta\delta^{-1}, v_n]})\\ =&(\epsilon^{-1}\beta\delta\alpha; C_{[u_{(1)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, v_1]f_1}, ..., C_{[u_{(n)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, v_n]f_n}).\end{aligned}$$

For $k = 1, ..., n$, let $s_k \in \{1, ...s\}$ be such that $u_{(k)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, v_{(k)\alpha^{-1}} \in M(\lambda_{s_k})$. Then from (1),

$$C_{[u_{(k)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, v_{(k)\alpha^{-1}}]f_k} \equiv \sum_{p \in M(\lambda)} R_{f_k}(v_{(k)\alpha^{-1}}, p_k) C_{[u_{(k)}(\epsilon^{-1}\beta\delta\alpha)^{-1}}, p_k]$$

modulo cellular basis elements of lower cell index. Using this we get
$(u \otimes \beta \otimes v)(\alpha; f_1, ..., f_n) \equiv$
$\sum_{p_1} \cdots \sum_{p_n} (\prod_{k=1}{}^n R_{f_k}(v_{(k)\alpha^{-1}}, p_k))(\epsilon^{-1}\beta\delta\alpha; C_{[u_{(1)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, p_1]}, ..., C_{[u_{(n)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, p_n]})$.....(*)
modulo elements of the basis of Z_2 of the form

$$(\epsilon^{-1}\beta\delta\alpha; C_{s_1, t_1}{}^{\lambda_{t_1}}, ..., C_{s_n, t_n}{}^{\lambda_{t_n}}) \text{ ......(**)}$$

where for each k $\lambda_{s_k} \geq \lambda_{t_k}$ and for atleast one k the the inequality is strict. Let $\gamma = (\gamma_1, ..., \gamma_s)$be the layer index of (**). By lemma 4.1. we have $\mu \rhd_\Lambda \gamma$, so that $(u \otimes \beta \otimes v)(\alpha; f_1, ..., f_n)$ is congruent (*) modulo elements of lower layer index.
Now, p_k lies in the same set $M(\lambda_{s_k})$ as $v_{(k)\alpha^{-1}}$, and from this we see that the shape of $(p_1, ..., p_n)$ is the unique element ψ of B_μ such that $\delta\alpha = \phi\psi$ for $\phi \in S_\mu$. Thus in (*) we have $(\epsilon^{-1}\beta\delta\alpha; C_{[u_{(1)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, p_1]}, ..., C_{[u_{(n)}(\epsilon^{-1}\beta\delta\alpha)^{-1}, p_n]})$
$= (\epsilon^{-1}\beta\phi\psi; C_{[u_{(1)}(\epsilon^{-1}\beta\phi\psi)^{-1}, p_1]}, ..., C_{[u_{(n)}(\epsilon^{-1}\beta\phi\psi)^{-1}, p_n]})$.
which corresponds to the pure tensor $u \otimes \beta\phi \otimes (p_1, ..., p_n)$ and hence (*) is equal to

$$u \otimes \beta\phi \otimes (\sum_{p_1} \cdots \sum_{p_n} (\prod_{k=1}{}^n R_{f_k}(v_{(k)\alpha^{-1}}, p_k))(p_1, ..., p_n)).$$

Thus, by setting $\phi_\mu(v, f)$ to be the unique element ϕ of S_μ so that $\delta\alpha = \phi\psi$ for $\psi \in B_\mu$ and $\psi_\mu(v, f)$ to be

$$(\prod_{k=1}{}^n R_{f_k}(v_{(k)\alpha^{-1}}, p_k))(p_1, ..., p_n)).$$

Further we observe that $(u \otimes \beta \otimes v)(\alpha; f_1, ..., f_n) \equiv u \otimes \beta\phi_\mu(v, f) \otimes \psi_\mu(v, f)$ modulo lower layers whose values depend only on v and f as required. □

Theorem 4.3. Let Z_2 be the cellular algebra with anti involution $*$ and poset Λ of cell indices. Let ${B_n}^s$ be the set of all multi partitions of n of length s with the partial order : if $(a_1, ..., a_s), (b_1, ..., b_s) \in {B_n}^s$, then $(a_1, ..., a_s) \geq (b_1, ..., b_s)$ implies $(|a_1|, ..., |a_s|) \trianglerighteq_\Lambda (|b_1|, ..., |b_s|)$ or that $|a_k| = |b_k|$ and $a_k \trianglelefteq b_k$ for each k. Then $\overrightarrow{S_n} = Z_2 \wr S_n$ is a cellular algebra for $\alpha \in S_n$ and $f_1, ..., f_n \in Z_2^n$ by

$$(\alpha; f_1, ..., f_n)^* = (\alpha^{-1}; {f^*}_{(1)\alpha}, ..., {f^*}_{(n)\alpha}).$$

5. The cell modules and simple modules of the wreath product $\overrightarrow{S_n}$.

We know that cell modules $\Delta\lambda_j$ are indexed by the cell indices $\lambda_1, ..., \lambda_s$. These are indexed by length s multipartitions of n. Let $\eta_1, ..., \eta_s$ be such a multipartition and μ the composition $(|\eta_1|, .., |\eta_s|)$, such that $\mu_j = \eta_j$.
$\Delta^{(\eta_1, ..., \eta_s)}$ as a $k-$ vector space may be identified with

$$S^{\eta_1} \otimes \cdots \otimes S^{\eta_s} \otimes V_\mu,$$

Let $(\theta_1, ..., \theta_n) \in \Lambda$ such that

$$(\theta_1, ..., \theta_n) = (\lambda_1, ..., \lambda_1(\mu_1 times), ..., \lambda_s, ..., \lambda_s(\mu_s times))$$

Let $(X_1, ..., X_n)$ be half diagram in B_μ. Then its shape is the unique element $\delta \in R_\mu$ such that it lies in $M(\theta_1\delta^{-1}) \times \cdots \otimes M(\theta_n\delta^{-1})$. Hence

$$B_\mu = \sqcup_{\delta \in B_\mu} M(\theta_1\delta^{-1}) \times \cdots \times M(\theta_n\delta^{-1}).$$

Therefore the half diagram $(X_1, ..., X_n)$ is identified with thepure tensor $C_{X_1 \otimes \cdots \otimes C_{X_n}}$. and obtain the natural identification of $k-$vector spaces

$$V_\mu = \bigoplus_{\delta \in B_\mu} \Delta^{(\theta_1\delta^{-1})} \otimes \cdots \otimes \Delta^{(\theta_n\delta^{-1})}.$$

Further for $x_j \in S^{\eta_j}$ and $u_1 \otimes \cdots \otimes u_n$ is a pure tensor in V_μ the pure tensor of $\Delta^{(\eta_1, ..., \eta_s)}$ is

$$a_1 \otimes \cdots \otimes a_s \otimes u_1 \cdots \otimes u_n,$$

Then for $\phi_\mu(v, f) \in S_\mu$ it may be verified that the map taking the pure tensor

$$v_1 \otimes \cdots \otimes v_n \otimes b_1 \cdots \otimes b_s \otimes \theta$$

in $\Theta^{\mu}((\Delta^{\lambda_1}, ..., \Delta^{\lambda_s}), (S^{\eta_1}, ..., S^{\eta_s}))$ where $\theta \in R_{\mu}$ to the pure tensor

$$b_1 \cdots \otimes b_s \otimes v_{(1)\theta^{-1}} \otimes \cdots \otimes v_{(n)\theta^{-1}}$$

in $\Delta^{(\eta_1,...,\eta_s)}$ is an isomorphism of $\overrightarrow{S_n}-$ modules. Thus by [3] we have the following remark,

Remark 5.1 The cell module $\Delta^{(\eta_1,...,\eta_s)}$ is isomorphic to the module $\Theta^{\mu}((\Delta^{\lambda_1}, ..., \Delta^{\lambda_s}), (S^{\eta_1}, ..., S^{\eta_s}))$

Proposition 5.2. Let $n_1, ..., n_s$ be non-negative integers. Let $B_{n_1} \times ... \times B_{n_s}$ be the poset of cell indices with the order $\lambda_i \geq \mu_i$ for all i. Then the group algebra $k(S_{n_1} \times ... \times S_{n_s})$ is a cellular algebra with respect to the mapping $(\alpha_1, ..., \alpha_s) \to (\alpha_1^{-1}, ..., \alpha_s^{-1})$ for all $\alpha_i \in S_{n_i}$ and cell module associated to $(\lambda_1, ..., \lambda_s)$ is $S^{\lambda_1} \otimes \cdots \otimes S^{\lambda_s}$ with the action

$$(x_1 \otimes \cdots \otimes x_s).(x_1\alpha_1) \otimes \cdots (x_s\alpha_s) \text{ for } x_i \in S^{\lambda} \text{ and } \alpha_i \in S_{n_i}.$$

The cell form is given on pure tensor by

$$\langle x_1 \otimes \cdots \otimes x_s, y_1 \otimes \cdots \otimes y_s \rangle = \langle x_1, y_1 \rangle \cdots \langle x_s, y_s \rangle$$

where each bilinear form on the right hand side is the appropriate cell form of S^{λ_i}.

Proposition 5.3. From [3] we have that if $Z_2$is a $k-$algebra with anti involution $*$, then

$$Z_2^n \cong \bigoplus_{\mu \in I} V_{\mu} \otimes B_{\mu} \otimes V_{\mu}$$

of ${Z^n}_2$ where I is the partially ordered set, each V_{μ} is a $k-$vector space and each B_{μ} is cellular algebra over k with respect to cell datum $(\Lambda_{\mu}, M_{\mu}, C, *)$. Hence Z_2^n can be identified with this direct sum of tensor products and $V_{\mu} \otimes B_{\mu} \otimes V_{\mu}$ as the $\mu-$th layer of Z_2^n. Also for each $\mu \in I$ there is unique $B_{\mu}-$valued $k-$bilinear form ϕ_{μ} on V_{μ} such that for any $u, v, x, y \in V_{\mu}$ and $b, c \in B_{\mu}$ we have

$$\phi_{\mu}(y, u) = \phi_{\mu}(u, y)^* \text{ and}$$
$$(x \otimes c \otimes y)(u \otimes b \otimes v) \equiv x \otimes c\phi_{\mu}(y, u)b \otimes v \text{ mod } H(< \mu).$$

where $H(< \mu) = \bigoplus_{\gamma < \mu} V_{\gamma} \otimes B_{\gamma} \otimes V_{\gamma}$

Further, for $(\mu, \lambda) \in \Lambda$, let $\Delta^{(\mu,\lambda)}$ denoted as Δ^{λ} be the right cell module of Z_2 so that for any $x, y \in V_{\mu}$ and $z, w \in \Delta^{\lambda}$, we have

$$\langle z \otimes x, w \otimes y \rangle = \langle z, w\phi_{\mu}(y, x) \rangle_{\lambda}$$

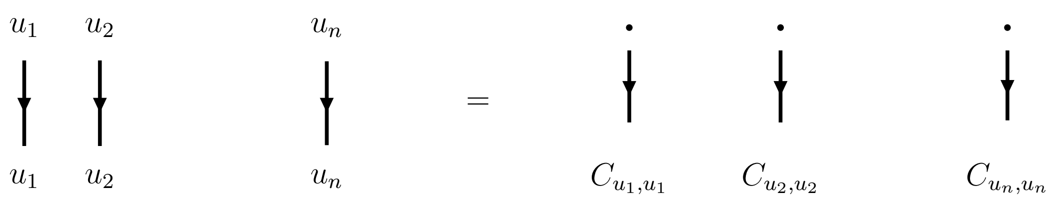
as the cell form.

Proposition 5.4. The wreath product $\overrightarrow{S_n} = Z_2 \wr S_n$ is cyclic cellular if Z_2^n is cyclic cellular.

Proof. By Proposition 5.2., we understand that the multiplication within each layer of $\overrightarrow{S_n} = Z_2 \wr S_n$ is determined by a bilinear formϕ_{μ}. Let $(u_1, ..., u_n), (v_1, ..., v_n)$ be the half diagram in V_{μ}, so that $u = C_{u_1} \otimes \cdots \otimes C_{u_n}$ and $v = C_{v_1} \otimes \cdots \otimes C_{v_n}$ are pure tensorsin V_{μ}. Then

$$(u \otimes e \otimes u)(v \otimes e \otimes v) \equiv u\phi_{\mu}(u, v) \otimes v ......(***)$$

modulo lower layers. The element $(u \otimes e \otimes u)$ of $\overrightarrow{S_n} = Z_2 \wr S_n$ is represented by the diagram



Exactly the similar way can represent the element $v \otimes e \otimes v$ by a diagram. The product $(u \otimes e \otimes u)(v \otimes e \otimes v)$ will now be represented as

$C_{u_1,u_1}C_{v_1,v_1}$ $C_{u_2,u_2}C_{v_2,v_2}$ $C_{u_n,u_n}C_{v_n,v_n}$

..(****)

For $i = 1, ..., n$, and $u_i \in M(\lambda_{s_i})$, we can expand the products $C_{u_i,u_i}C_{v_i,v_i}$ in terms of the linear combination of cellular basis elements $C^{\lambda_{t_i}}_{X,Y}$ where $\lambda_{t_i} \leq \lambda_{s_i}$ we get the diagramvof the form

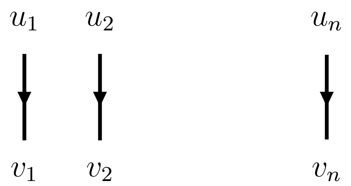
C_{X_1,Y_1} C_{X_2,Y_2} C_{X_n,Y_n}

By Lemma 4.1. it follows that all such diagrams have layer index utmost μ and the element v_i do not lie in $M(\lambda_{s_i})$ implies that all the diagrams in the expansion have layer index strictly less than μ. ALso in by (***), in such a case we see that $\phi_\mu(u, v) = 0$.
Suppose that for each i, v_i lies in $M(\lambda_{s_i})$, then as stated in [3], since

$$C_{u_i,u_i}C_{v_i,v_i} \equiv \langle C_{u_i}, C_{v_i}\rangle C_{u_i,v_i}$$

modulo cellular basis elements of lower index, where $\langle . \rangle$ is the suitable cell form. By lemma 4.1. (****) is congruent modulo lower layers to

$\langle C_{u_1}, C_{v_1}\rangle \cdots \langle C_{u_n}, C_{v_n}\rangle$



representing the element $\langle C_{u_1}, C_{v_1}\rangle \cdots \langle C_{u_n}, C_{v_n}\rangle(u \otimes e \otimes v)$. Therefore in this case

$$\phi(u, v) = \langle C_{u_1}, C_{v_1}\rangle \cdots \langle C_{u_n}, C_{v_n}\rangle.$$

From proposition 5.2., if for $z, w \in \Delta^\lambda$, $z_1 \otimes \cdots \otimes z_s \otimes u_1 \otimes \cdots \otimes u_n$ and $w_1 \otimes \cdots \otimes w_s \otimes v_1 \otimes \cdots \otimes v_n$ are pure tensors in the cell module $\Delta^{\eta_1,\ldots,\eta_s}$ then

$$\langle z_1 \otimes \cdots \otimes z_s \otimes u_1 \otimes \cdots \otimes u_n, w_1 \otimes \cdots \otimes w_s \otimes v_1 \otimes \cdots \otimes v_n\rangle = \langle z_1, u_1\rangle \cdots \langle z_s, u_s\rangle\langle w_1, v_1\rangle \cdots \langle w_1, v_1\rangle$$

If u_i and v_i lie in the same Δ^λ, for each $i = 1, ..., n$,

$$\langle z_1 \otimes \cdots \otimes z_s \otimes u_1 \otimes \cdots \otimes u_n, w_1 \otimes \cdots \otimes w_s \otimes v_1 \otimes \cdots \otimes v_n\rangle = 0$$

otherwise.
By remark 5.1.,

$$\Delta^{(\eta_1,...,\eta_s)} \cong S^{\eta_1} \otimes \cdots \otimes S^{\eta_s} \otimes V_\mu \cong \bigoplus_{\theta\in R_\mu} S^{\eta_1} \otimes \cdots \otimes S^{\eta_s} \otimes \Delta^{(\theta_1\delta_{-1})} \otimes \Delta^{(\theta_n\delta^{-1})}$$

For $\theta \in R_\mu$ let $\Gamma_\theta = S^{\eta_1} \otimes \cdots \otimes S^{\eta_s} \otimes \Delta^{(\theta_1\delta_{-1})} \otimes \Delta^{(\theta_n\delta^{-1})}$. If θ, β be distinct elements of R_μ and $u \in \Gamma_\theta$ and $v \in \Gamma_\beta$ then $(u, v) = 0$. This implies that if R_θ is the radical restriction of Γ_θ of $\langle . \rangle$, then the radical cell of $\Delta^{(\eta_1,...,\eta_s)}$ is $\bigoplus_{\theta\in R_\mu} R_\theta$.

Thus we have the following results on the simple and cell modules $P^{(\eta_1,...,\eta_s)}$ and semi-simplicity of $\overrightarrow{S_n} = Z_2 \wr S_n$.

Theorem 5.4. The set $(\hat{B}_n^s)_0$ consists of exactly those set of elements $(\eta_1, ..., \eta_s) \in \hat{B}_n^s$ such that $\eta_j = ()$ whenever $\lambda_j \in \Lambda\Lambda_0$ so that the cell radical of $\Delta^{(\eta_1,...,\eta_s)}$ is a proper submodule of $\Delta^{(\eta_1,...,\eta_s)}$ and $(\hat{B}_n^s)_0$ indexes the simple modules of $\overrightarrow{S_n} = Z_2 \wr S_n$.

Theorem 5.5. If $(\eta_1, ..., \eta_s) \in (\hat{B}_n^s)_0$ then from proposition 4.3., there exists an isomorphism of $k-$ vector spaces

$$P^{(\eta_1,...,\eta_s)} \cong Q^{\eta_1} \otimes \cdots \otimes Q^{\eta_s} \otimes P^{\theta_1\delta^{-1}} \otimes \cdots \otimes P^{\theta_n\delta^{-1}}.$$

Theorem 5.6. If $(\eta_1, ..., \eta_s) \in (\hat{B}_n^s)_0$ then $P^{(\eta_1,...,\eta_s)} \cong \Delta^{(\eta_1,...,\eta_s)}$ if and only if $Q^{\eta_j} = S^{\eta_j}$ for $j = 1, ..., s$ and whenever $\eta_j \neq ()$, $P^{\lambda_j} \cong \Delta^{\lambda_j}$.

Theorem 5.7. [3] If Z_2 is a cellular algebra, then $\overrightarrow{S_n} = Z_2 \wr S_n$ is semisimple if and only if both Z_2 and kS_n are semisimple.

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