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Applied Some Mixture 2 and 3 Distribution for Daily Exchange Rate American Dollar vs Indonesian Rupiah Probability Modelling

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ABSTRACT

As a world superpower, the United States has a very stable exchange rate and has a big impact on the currencies of other countries, like Indonesia. Probability modeling is therefore essential for analyzing the change in exchange rates between the Indonesian rupiah (IDR) and the US dollar (USD). In addition to comparing the distributions of two parameters, this study also discusses the use of several mixture 2 and 3 component distribution probability models, such as mixture 2 log-normal (ML2), mixture 2 Gamma (MG2), mixture 2 Weibull (MW2), mixture 3 Log-Normal (ML3), mixture 3 Gamma (MG3), and a mixture 3 Weibull (MW3). The maximum likelihood method is used for parameter estimation, and numerical methods like Akaike Information Cretarius (AIC) and Bayesian Information Cretarius (BIC) are used to select the best model, also known as the Goodness of Fit (GOF). Then, the GOF between the model distribution and the theoretical data is evaluated. The ML3 distribution-based daily USD/IDR exchange rate data can be best modeled using the MLE approach, as demonstrated by the results. We are able to reasonably forecast the risks associated with daily exchanges in the future on the basis of the identified models.

KEYWORDS: Exchange Rate, micture distribution, Log-Normal, Gamma, Weibull

I. INTRODUCTION

Investors are very interested in the foreign exchange (forex) market because of the rapid growth of digitalization. Investors buy currencies from one country and exchange them for other currencies to take advantage of price fluctuations. When looking at a country's currency rate in the context of forex trading, one of the currencies that serves as a reference is the American Dollar, or USD rate. Other nations use the USD rate as a foreign exchange reserve rate. In a similar vein, the Indonesian Rupiah, or IDR rate, can be compared to the rupiah rate against the USD rate to assess its strength (Dewi et al., 2022). Additionally, Indonesia automatically evaluates its trade activities in USD currency because it is a partner in US export and import activities. Due to Indonesia's use of the US dollar for international trade [1,2], the USD plays an increasingly significant role. Because its trading activities are evaluated using the US Dollar (USD), an unstable Rupiah (IDR) exchange rate will typically interfere with trading because it can result in economic losses.

An interesting topic to discuss is the significance of the exchange rate as an economic indicator. The probability of the exchange rate in the future is very important to discuss, and the exchange rate that changes actively results in the discussion of forecasting. Forecasting exchange rates is done by some researchers using statistical theory The Box-Jenkins approach to time series modeling and forecasting is utilized by many researchers. There are drawbacks to forecasting using the Box-Jenkins method. The existence of a linear relationship between the variables is assumed by this method. However, time series data are frequently nonlinear in the real world [3 - 6]. Second, utilizing the Box-Jenkins method for the model selection procedure is highly dependent on the researchers' expertise and experience [7]. The Box-Jenkins method's model selection procedure is highly dependent on the researcher's skill and experience. 2015. As a result, the accuracy of the Box-Jenkins method for modeling and forecasting is insufficient. All things considered, the paper analyzes the idea of probabilistic displaying, trailed by information. Because it is based on actual data, probabilistic modeling can be used to predict. In addition, they can instantly reflect data changes and do not necessitate lengthy historical time series for accurate estimation. Furthermore, probabilistic modeling excels at capturing the data's inherent uncertainty. The mathematical precedent of Chang and Melick has little effect on probabilistic modeling [8].

With regard to the frequency of the data, probabilistic models typically determine the best-fit probability density function. Theoretical probability distribution models that correspond to the empirical distributions of changes in spot exchange rates are still sought after by researchers. Improved test statistics and more accurate exchange rate pricing models should result from better theoretical models of these empirical distributions. For a long time, researchers believed that either a normal or a lognormal probability distribution was the most appropriate way to describe the empirical distribution of changes in exchange rates. These assumed probability distributions have been refuted by recent empirical studies; There is no conclusive agreement among these studies regarding any one alternative model. Most of the time, these studies back discrete mixture distributions. [9] compares the fit of the unbounded Johnson family to the mixtures of two normals and the skewed Student t when modeling six major trading currencies and two Latin American currencies. Other researchers have used mixture distributions to study probability modeling for exchange rates. Overall, the results favor the skewed Student t, while the unbounded Johnson family performs better in terms of VaR than other families.

The mixture distribution for the exchange rate and gold price in Malaysia based on the crisis time and normal situation was developed by Phoong and Phoong using the MLE and the Bayesian method [10]. The results show that the statistical methods were used to fit two-component mixture models, and the mixture distribution that was obtained is roughly similar. This research focuses on modeling daily USD to IDR exchange rate movements by employing a variety of two-parameter distributions and mixture probability models, including Log-Normal (L), Gamma (G), Weibull (W), Mixture 2 component Log-Normal (ML2), Mixture 2 Component Gamma (MG2), Mixture 2 component Weibull (MW2), Mixture 3 component Log-Normal (ML3), Mixture 3 Component Gamma (MG3), and Mix Maximum likelihood (MLE) parameter estimation, as well as numerical methods like Akaike Information Creation (AIC) and Bayesian Information Creation (BIC) and graphical methods like density plots and cumulative plots, are used to select the best model.

II. DATA SET

The American Dollar to Indonesian Rupiah (USD/IDR) daily exchange rate is included in the data set. The Central Bank of Indonesia (BI) website (www.bi.go.id) serves as the source for these data. From January 24, 2001 to December 31, 2021, the data set covered with 5145 observations all together. Figures 1 and 2 clearly show that the daily USD/IDR histogram and daily USD/IDR exchange rate data are related, respectively. Figure 2's histogram depicts multiple waves and describes data that most likely have more than one distribution or mixture distribution. This study will therefore employ some probability modeling with a mix of distributions with two and three components. Table 1 summarizes a few of the statistics. That table provides a description of the initial statistical information regarding the USD/IDR exchange rate.

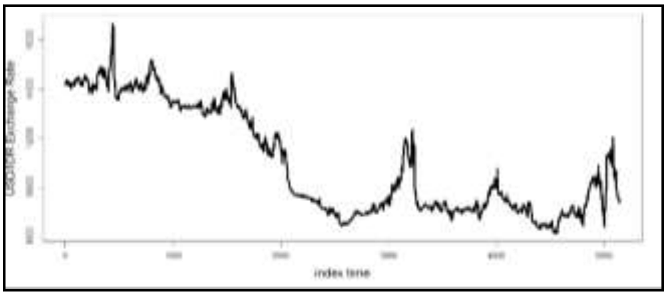

Fig. 1. Daily Exchange Rate USD/IDR Data

The values shown are descriptive statistics like the mean, variance, kurtosis, skewness, and maximum and minimum data, which typically describe the data.

Table 1: Statistics of daily Exchange Rate USD/IDR.

Statistics	*Velue*
Mean	11008.6
Varians	4840899
Minimum	8124
Maximum	16657.3
Skewness	5.096749e-16
Kurtosis	0.0078

The values of kurtosis and skewness that are less than or equal to 1 in table 1 indicate unequal probability models like Weibull and Gamma. In this study, exchange rate data can be modeled with Log Normal and Normal. This is also made clear by the data histogram in Figure 2, which shows that data that probably have more than one distribution or mixture distribution are described by more than one wave. The models used in this study were chosen for the right reasons.

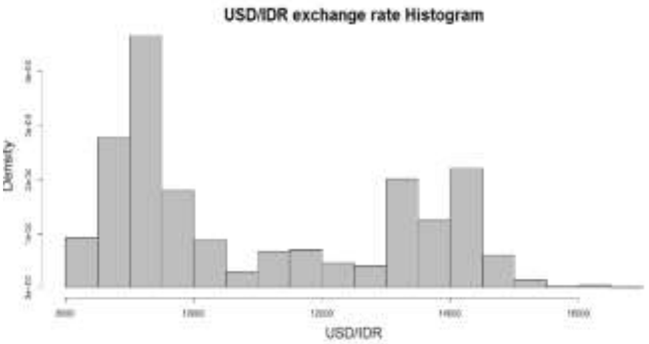

Fig. 2. The Histogram Daily Exchange Rate USD/IDR

III. METHODOLOGY

This research involves several stages in the process of obtaining probability modeling that is in accordance with exchange rate data. Further discussion is as follows:

A. Probability Density Function (PDFs) or Exchange Rate Models.

The analysis of daily exchange rate data over a number of years is necessary for exchange rate modeling. Utilizing statistical distribution functions to describe the variations in the exchange rate is desirable in order to cut down on the amount of money spent and time spent processing long-term daily exchange rate data. Probability density functions are the primary means of describing the characteristics of an exchange rate. The boundaries of likelihood conveyance works that depict day to day conversion scale recurrence circulation are assessed utilizing measurable information from a couple of years. In the current study, L, G, W, ML2, MG2, MW2, ML3, MG3, and MW3 are utilized to describe the characteristics of exchange rates. Although numerous probability density functions (PDFs) have been proposed recently, The maximum likelihood method is utilized to calculate the parameters that define each distribution function. Table 2 displays the PDFs and Likelihood function (LL). [11-25] are just a few of the early researchers who contributed to the development of the mixture distribution and provided the initial explanation for it.

Table 2. List of PDFs and LL Used in This Study

PDFs and LL	The Formula
L	$f(x;\mu,\sigma)=\frac{1}{x\sqrt{2\pi(\sigma)^2}}e^{-\frac{(\ln(x)-\mu)^2}{2(\sigma)^2}}$ $LL=\sum_{i=1}^{n}\ln\left(f(x_i;\mu,\sigma)\right)$
G	$g(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$ $LL=\sum_{i=1}^{n}\ln\langle g(x_i;\alpha,\beta)\rangle$
W	$h(x;k,c)=\frac{k}{c}\left(\frac{x}{c}\right)^{k-1}e^{-\left(\frac{x}{c}\right)^{(k)}}$ $LL=\sum_{i=1}^{n}\ln\left(h(x_i;k,c)\right)$
ML2	$w_1f(x;\mu_1,\sigma_1)+w_2f(x;\mu_2,\sigma_2),\quad w_1+w_2=1$ $LL=\sum_{i=1}^{n}\ln\left(w_1f(x_i;\mu_1,\sigma_1)+w_2f(x_i;\mu_2,\sigma_2)\right)$
MG2	$w_1g(x;\alpha_1,\beta_1)+w_2g(x;\alpha_2,\beta_2),\quad w_1+w_2=1$ $LL=\sum_{i=1}^{n}\ln\left(w_1g(x;\alpha_1,\beta_1)+w_2g(x;\alpha_2,\beta_2)\right)$
MW2	$w_1h(x;k_1,c_1)+w_2h(x;k_2,c_2),\quad w_1+w_2=1$ $LL=\sum_{i=1}^{n}\ln\left(w_1h(x;k_1,c_1)+w_2h(x;k_2,c_2)\right)$
ML3	$w_1f(x;\mu_1,\sigma_1)+w_2f(x;\mu_2,\sigma_2)+w_3f(x;\mu_3,\sigma_3),\quad w_1+w_2+w_3=1$ $LL=\sum_{i=1}^{n}\ln\left(w_1f(x_i;\mu_1,\sigma_1)+w_2f(x_i;\mu_2,\sigma_2)+w_3f(x_i;\mu_3,\sigma_3)\right)$
MG3	$w_1g(x;\alpha_1,\beta_1)+w_2g(x;\alpha_2,\beta_2)+w_3g(x;\alpha_3,\beta_3),\quad w_1+w_2+w_3=1$ $LL=\sum_{i=1}^{n}\ln\left(w_1g(x_i;\alpha_1,\beta_1)+w_2g(x_i;\alpha_2,\beta_2)+w_3g(x_i;\alpha_3,\beta_3)\right)$
MW3	$w_1h(x;k_1,c_1)+w_2h(x;k_2,c_2)+w_3h(x;k_3,c_3),\quad w_1+w_2+w_3=1$ $LL=\sum_{i=1}^{n}\ln\left(w_1h(x_i;k_1,c_1)+w_2h(x_i;k_2,c_2)+w_3h(x_i;k_3,c_3)\right)$

B. Model Selection

For selecting the best-fitted probability distribution, the following goodness-of-fit (GOF) measures are taken into consideration here. The numerical criteria Akaike Information criterion (AIC) and Bayesian information criterion (BIC) were used to determine the GOF criteria of the distributions, which are based on graphical inspection probability density function (pdf) and cumulative density function (cdf). Despite the fact that their AIC, BIC, and Log Likelihood (ln (L)) results differed, graphical inspection consistently produced the same outcome. The distribution with the lowest AIC and BIC values was chosen as the best-fit result.

$$AIC = 2k - 2\ LL$$

$$BIC = k \log n - 2\ LL$$

where LL is log-likelihood function evaluated at the MLEs and k refers to the number of parameters in the model. For each parameter θ_i, MLE involves maximizing the likelihood function by solving the following:

$$\frac{\partial LL}{\partial \theta_i}=0,\qquad i=1,2,\cdots$$

This method is used to get the likelihood functions for the parameters of the chosen models. In this case, numerical methods were used to get these parameter estimates, so we use this method. The statistical software R and packages that include some of the cited models can be used by interested readers. See, for instance, Delignette-Muller and Dutang [26].

IV. RESULT

The USD/IDR daily exchange rate histogram in Figure 2 depicts the likely modeling with multiple distributions in this study. This study will therefore employ some probability modeling with a mix of distributions with two and three components. The maximum likelihood estimation technique is utilized to estimate the parameters of the probability distributions. All probability models' parameter estimates are shown in Table 4. The findings regarding the goodness of fit measures are presented in Table 3. Figure 4 presents a pdf plot with the outcomes got from the L, G, W, ML2, MG2, and MW2 dissemination. As a result, the results show that comparing the probabilities of using the competitor distributions and the L, G, W, ML2, MG2, and MW2 distributions for the analysis of the daily exchange rate USD/IDR is not more accurate. A pdf plot of the results from the ML3, MG3, and MW3 distributions is shown in Figure 4. We can easily identify the distributions that performed better than the others, as shown in the figures. As a result, the results show that the probability of analyzing the USD/IDR daily exchange rate using these distributions is more accurate than using the other distributions in this study. In addition, when selecting the optimal distribution for this study, the plot of the cumulative distribution function will be used to draw more convincing conclusions. For this reason, Figures 5, 6, and 7 are also presented. It is abundantly clear from figure 5 that two-parameter distributions, such as L, G, and W, are unable to approximate the observed distribution function.

Table 3. The Goodness and fit test result of the daily exchange rate USD/IDR

Model	*AIC*	*BIC*	*Ln(L)*
L	93337.2	93350.3	-46666.6
G	93461.1	93474.2	-46728.5
W	94058.4	94071.5	-47027.2
ML2	89163.6	89196.4	-44576.8
MG2	89149.3	89182.1	-44569.6
MW2	89322.9	89355.6	-44656.4
ML3	**88410.5**	**88462.8**	**-44197.2**
MG3	88453.8	88506.1	-44218.9
MW3	88888.3	88940.7	-44436.1

On the other hand, a mixture of two and three components, such as ML2, MG2, MW2, ML3, MG3, and MW3, is able to accurately approximate the observed distribution function. The conclusion that the Mixture 2 and 3 component distributions are the best models produced in this study is also clarified in Figures 6 and 7. The ML3, MG3, and MW3 are clearly very good at capturing the observation distribution function, as shown in figure 7. The conclusion that the distributions presented in this paper are the best models produced by this study is also clarified in Figure 7. The pdf and CDF plots clearly show that the distributions ML3, MG3, and MW3 are the best model for analyzing the frequency of exchange rate data, as demonstrated by the goodness of fit test performed on the model using this graphical approach. The L, G, and W distributions appear to have maximum AIC and BIC for the daily USD/IDR exchange rate. Consequently, we conclude that none of the selected distributions can be used to describe the data's distributions. We discovered that the distributions ML2, MG2, MW2, ML3, MG3, and MW3 provided a better fit than the other ones with smaller AIC and BIC. This study also used numerical goodness-of-fit test methods like AIC, BIC, and LL values. Table 3 will display these three values for each distribution. The ML3 distribution, or mixture of three components, is the best model because it has the lowest AIC and BIC values, as shown in the table. The log-likelihood (LL) model and other tests of model goodness are included in Table 3. Based on the values shown, it can also be concluded that the ML3 distribution is the best in this study.

Table 4. Computed parameters of different distribution

	L	*G*	*W*	*ML2*	*MG2*	*MW2*	*ML3*	*MG3*	*MW3*
μ	9.28	–	–	–	–	–	–	–	–
σ	0.19	–	–	–	–	–	–	–	–
α	–	25.94	–	–	–	–	–	–	–
β	–	0.0024	–	–	–	–	–	–	–
k	–	–	5.41	–	–	–	–	–	–
c	–	–	11934.1	–	–	–	–	–	–
μ_1	–	–	–	9.126	–	–	9.12	–	–
σ_1	–	–	–	0.054	–	–	0.046	–	–
μ_2	–	–	–	9.492	–	–	9.35	–	–

σ_2	–	–	–	0.088	–	–	0.13	–	–
μ_3	–	–	–	–	–	–	9.54	–	–
σ_3	–	–	–	–	–	–	0.039	–	–
α_1	–	–	–	–	320.000	–	–	480.00	–
β_1	–	–	–	–	0.0347	–	–	0.052	–
α_2	–	–	–	–	140.000	–	–	480.00	–
β_2	–	–	–	–	0.0105	–	–	0.034	–
α_3	–	–	–	–	–	–	–	120.00	–
k_1	–	–	–	–	–	23.44	–	–	27.11
c_1	–	–	–	–	–	9311.3	–	–	9255.8
k_2	–	–	–	–	–	11.06	–	–	31.09
c_2	–	–	–	–	–	13569.4	–	–	14124.3
k_3	–	–	–	–	–	–	–	–	7.30
c_3	–	–	–	–	–	–	–	–	12589.8
w_1	–	–	–	0.561	0.562	0.505	0.501	0.511	0.461
w_2	–	–	–	0.439	0.438	0.495	0.217	0.322	0.218
w_3	–	–	–	–	–	–	0.282	0.167	0.321

V. DISCUSSION

The goal of this study is to identify the appropriate nine models or distributions for describing the daily exchange rate distribution by analyzing the USD/IDR rate on a daily basis. In conclusion, the results obtained by the ML3 distribution were superior to those obtained by other well-known distributions. The AIC, BIC, and LL goodness of fit test models are the foundation for this conclusion. When comparing the empirical distributions to the adjusted ML3 distribution, the graphical method known as the pdf plot was also observed. The fact that the majority of the analyses performed with the USD/SGD exchange rate utilized a mixture distribution is an interesting aspect of our findings. Additionally, we can use the quantile function distribution to simulate the number of daily USD/IDR exchange rates in the future with the ML3 distribution's adjusted parameters.

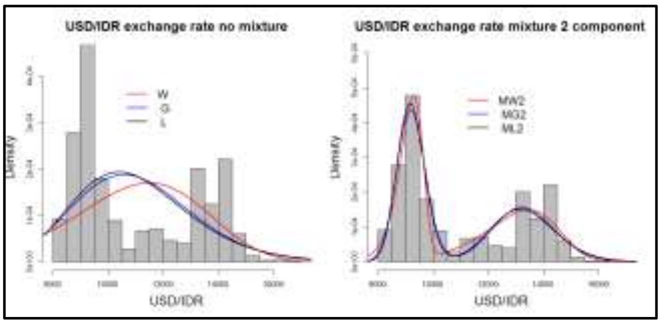

Fig. 3. Pdf plot for comparisons Predicted and observed daily USD/IDR Exchange rate for no mixture (W,L,G) and two mixture (MW2, MG2 and ML2)

VI. CONCLUSION

The probability of the USD/IDR daily exchange rate occurring was looked at in this paper. To fit the data, the nine probability distributions (L, G, W, ML2, MG2, MW2, ML3, MG3, and MW3) were chosen. MLE was used to specifically estimate the parameters of the nine probability distributions by analyzing the various types of data in this work. It could be demonstrated in this paper that the MLE was effective in this study.

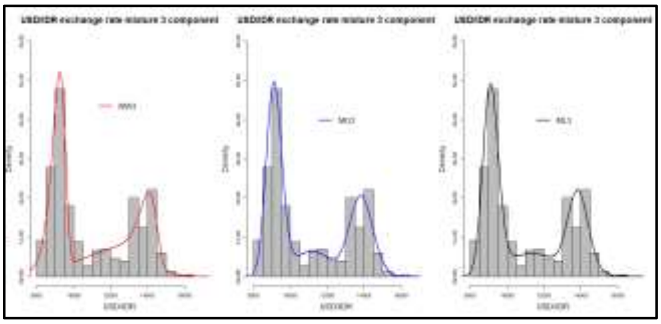

Figure 5. Empirical and theoretical cumulative distributions functions (cdf) W, G and L for the daily exchange rate USD/IDR

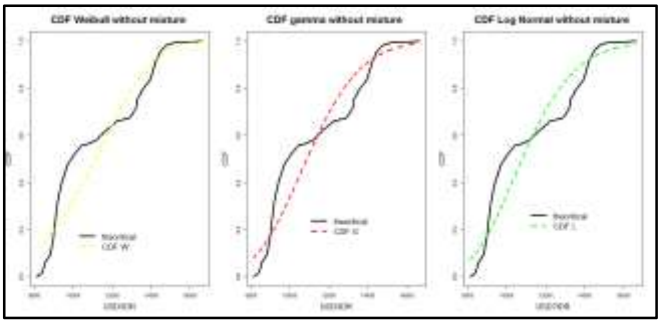

Fig. 5. Empirical and theoretical cumulative distributions functions (cdf) W, G and L for the daily exchange rate USD/IDR

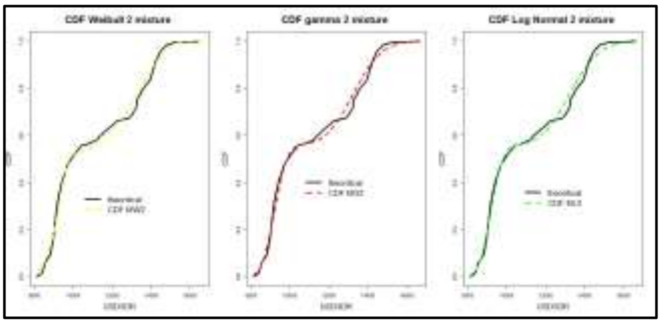

Fig. 6. Empirical and theoretical cumulative distributions functions (cdf) MW2, MG2 and ML2 for the daily exchange rate USD/IDR

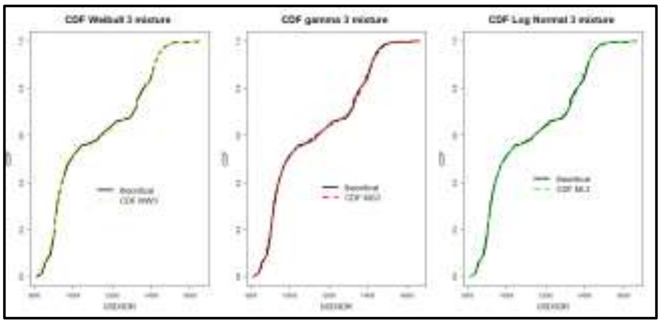

Fig. 7. Empirical and theoretical cumulative distributions functions (cdf) MW3, MG3, and ML3 for the daily exchange rate USD/IDR

The primary objective of this study is to identify the appropriate models or distributions that can be used to describe the distribution of the daily USD/IDR exchange rate data. In conclusion, the results obtained by the ML3 distribution were superior to those obtained by other well-known distributions. The AIC, BIC, and LL goodness of fit test models are the foundation for this conclusion. When comparing the empirical distributions to those that had been adjusted by the ML3 distribution, the graphical technique of pdf and CDF plots was also observed.

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