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Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions 11*n* + *k*

Neeraj Anant Pande[1]

[1]Associate Professor, Department of Mathematics & Statistics,
Yeshwant Mahavidyalaya (College), Nanded – 431602, Maharashtra, INDIA
napande@gmail.com

Abstract: ***All 10 infinite prime containing arithmetical progressions 11n + k are considered for spacings between primes of identical form in blocks of 10^n for $1 \le n \le 12$ till one trillion. The minimum spacings between primes of same forms in these blocks; for very first and last pairs with such minimum spacings, the first prime candidate in the pairs are determined along with the number of times such minimum spacings between them occur in these blocks. Similar work for maximum spacings is also undertaken. Finally the comparison of number of primes with different digits in units place and tens & Units places is done.***

Keywords: Arithmetical progressions, block-wise spacings, prime, prime digits**.**
Mathematics Subject Classification 2010: 11A41, 11N05, 11N25**.**

1. Introduction

Prime numbers are topic of investigation in this work. Their properties that make them prone to intense analysis are, amongst others, their infinitude [1], our hitherto inability of fitting them in a straightforward and simple formula after efforts to the extent that now some have started believing that such formula just doesn't exist, their irregular distribution [2] amongst the set of integers and many more.

The simplest expected formula for any list of infinite numbers will be of first degree. What can be more elementary than an arithmetical progression? This is the reason the occurrence of primes in arithmetical progression is being keenly dealt with. But almost 180 year ago [3], the possibilities of any arithmetical progression containing finite, infinite and all primes have been sorted out. Dirichlet came up with clear answer for any arithmetical progression $an + b$: If a and b have gcd greater than 1, then it contains at most finite number of primes, if this gcd is exactly 1, it contains infinitely many primes and any arithmetical progression cannot contain all primes; not even $2n + 1$, as it fails to cover 2! Of course, the tricky candidates like $1n + 1$ or $1n + 0$ are prohibited as they mean nothing but the set of all integers!!

In depth study of prime containing arithmetical progression of all types $3n + k$, $4n + k$, $5n + k$, $6n + k$, $7n + k$, $8n + k$, $9n + k$, $10n + k$ has been recently done [4]–[15]. Here we undertake analysis of primes in $11n + k$. Work on density distribution of primes in them is already done [16]. We take on analysis of spacings between primes in them.

2. Minimum Spacings between Primes in 11*n* + *k* in Blocks of 10 Powers

For the reasons stated in very beginning, all primes cannot be analyzed in one go. If one is very much onto doing so, it comes at the cost of approximations. Here we are not interested in such asymptotic outcomes. So we have to confine ourselves to finite range in which we will do our study. We prefer earlier adopted block-wise approach. Considering all blocks of all powers of 10, like 10, 10^2, 10^3, . . . , 10^{12}, till 10^{12}, we do our investigations.

The first requirement for this deeper analysis was prime number database; that too all primes till as high a limit as 1,000,000,000,000. Formula non-fitting nature of primes compels to first generate all these primes and then to go ahead for additional work like analysis. This cumbersome task must be accomplished by choice of the best prime generating algorithm. It must be deterministic and covering complete desired range, in addition to, of course, being as efficient as possible on many fronts like number of steps required by it, time of execution, number of resources like memory of electronic computer, its CPU cycles etc. For our huge work, we could do a wise choice due to exhaustive comparisons of many prime generating algorithms in [17] – [23]. On the top of this for actual implementation purpose a good programming language was needed to take care of heavy resource requirements. The choice of Java [24] proved correct due to many managed features in it.

Amongst all 11 arithmetical progressions $11n + k$ for $k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, except the first one $11n + 0 = 11n$, due to the Dirichlet's property other 10 contain infinitely many primes.

First it is the turn of minimum spacings in primes of same form.

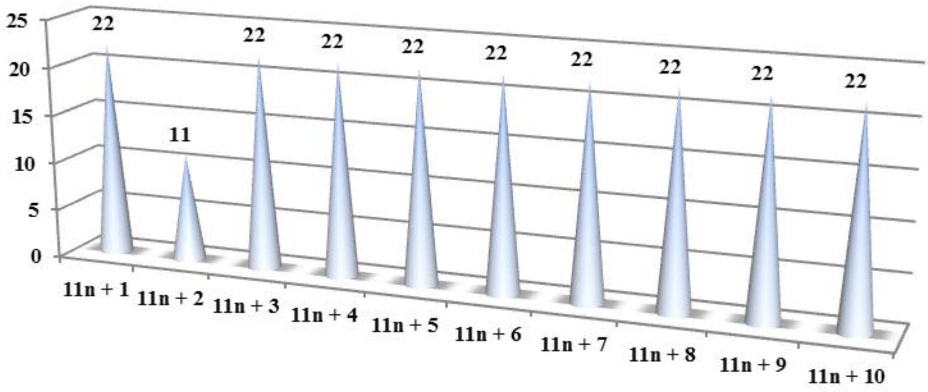


Figure 1: Minimum Block Spacing between Primes of form 11*n* + *k*

As seen, except for the form 11*n* + 2, the minimum in-block spacing between primes of all other forms is 22. For 11*n* + 2, as an exception, it is 11. This is attributed to the presence of only even prime 2 in this progression. And there is no question of spacing between primes of form 11*n* + 0 as contains a solo prime 11.

First & last primes in 10^n sized blocks with these minimum block spacings with next of same form are determined to be as follows :

Table 1: First Starters of Minimum Block Spacings between Primes of form 11*n* + *k* in Blocks of 10^n.

Sr. No.	*Blocks of Size (of 10 Power)*	*First Primes of different forms with Respective Minimum Block Spacing*									
		11n + 1	*11n + 2*	*11n + 3*	*11n + 4*	*11n + 5*	*11n + 6*	*11n + 7*	*11n + 8*	*11n + 9*	*11n + 10*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100 till 10^{12}	67	2	157	37	577	61	7	19	31	109

Table 2: Last Starters of Minimum Block Spacings between Primes of form 11*n* + *k* in Blocks of 10^n.

Sr. No.	*Blocks of Size (of 10 Power)*	*Last Primes of different forms with Respective Minimum Block Spacing*				
		11n + 1	*11n + 2*	*11n + 3*	*11n + 4*	*11n + 5*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	999,999,996,601	2	999,999,984,547	999,999,999,937	999,999,997,771
3.	1,000 till 10^{12}	999,999,996,601	2	999,999,993,589	999,999,999,937	999,999,997,771

Sr. No.	*Blocks of Size (of 10 Power)*	*Last Primes of different forms with Respective Minimum Block Spacing*				
		11n + 6	*11n + 7*	*11n + 8*	*11n + 9*	*11n + 10*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	999,999,998,509	999,999,999,577	999,999,991,141	999,999,978,217	999,999,999,877
3.	1,000 till 10^{12}	999,999,998,509	999,999,999,577	999,999,999,589	999,999,978,217	999,999,999,877

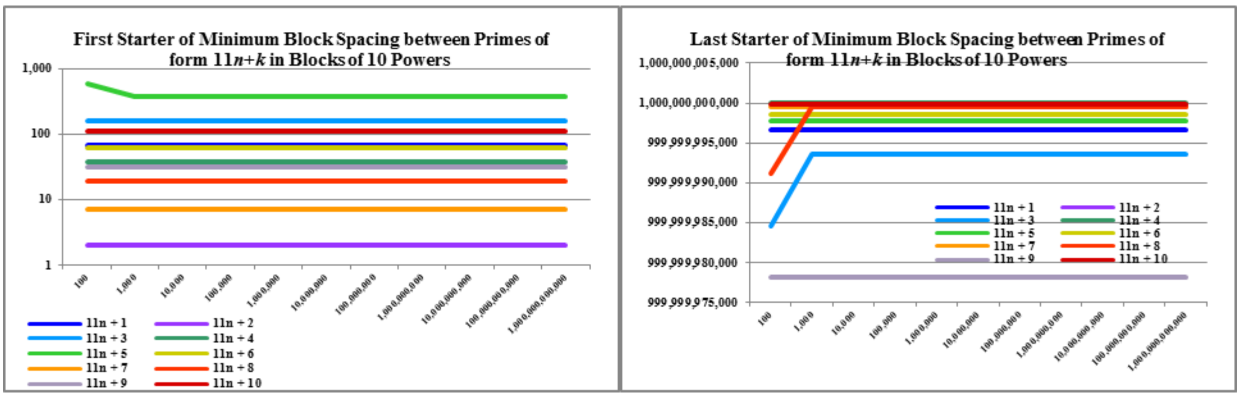


Figure 2: First & Last Starters of Minimum Spacings between Primes of form 11*n* + *k* in 10^n Blocks.

Interestingly, although the first as well last primes of different forms with respective minimum block spacing remain same after block size 1,000 for most of the forms, their count does increase significantly.

Table 3: Frequency of Minimum Block Spacings between Primes of form 11*n* + *k*.

Sr. No.	*Blocks of Size (of 10 Power)*	*Number of Times Minimum Block Spacing Occurring for Primes of form 11n + k for k =*									
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1.	10	Not Found	NF	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	159,346,679	1	159,338,012	159,360,480	159,339,473	159,353,405	159,342,992	159,351,494	159,345,490	159,350,064
3.	1,000	203,001,170	1	202,987,479	203,008,536	202,975,924	203,008,945	202,992,798	202,989,289	202,993,932	203,002,872
4.	10,000	207,363,911	1	207,353,331	207,371,436	207,337,585	207,374,472	207,355,275	207,355,663	207,359,703	207,367,176
5.	100,000	207,799,882	1	207,790,392	207,808,045	207,774,351	207,811,525	207,792,549	207,793,055	207,795,660	207,803,325
6.	1,000,000	207,843,430	1	207,834,225	207,851,892	207,817,875	207,855,424	207,835,858	207,836,784	207,839,424	207,847,164
7.	10,000,000	207,847,839	1	207,838,617	207,856,270	207,822,182	207,859,798	207,840,117	207,841,200	207,843,826	207,851,579
8.	100,000,000	207,848,271	1	207,839,041	207,856,697	207,822,609	207,860,269	207,840,552	207,841,602	207,844,257	207,852,039
9.	1,000,000,000	207,848,307	1	207,839,090	207,856,726	207,822,650	207,860,315	207,840,599	207,841,647	207,844,293	207,852,091
10.	10,000,000,000	207,848,313	1	207,839,094	207,856,729	207,822,658	207,860,321	207,840,603	207,841,649	207,844,303	207,852,100
11.	100,000,000,000	207,848,313	1	207,839,094	207,856,729	207,822,658	207,860,321	207,840,605	207,841,649	207,844,303	207,852,100
12.	1,000,000,000,000	207,848,313	1	207,839,094	207,856,729	207,822,658	207,860,321	207,840,605	207,841,649	207,844,303	207,852,100

Dropping the special case of 11*n* + 2 of unique occurrence of minimum spacing, like we have already dropped that of 11n + 0 of no occurrence of any spacing, these values are compared graphically.

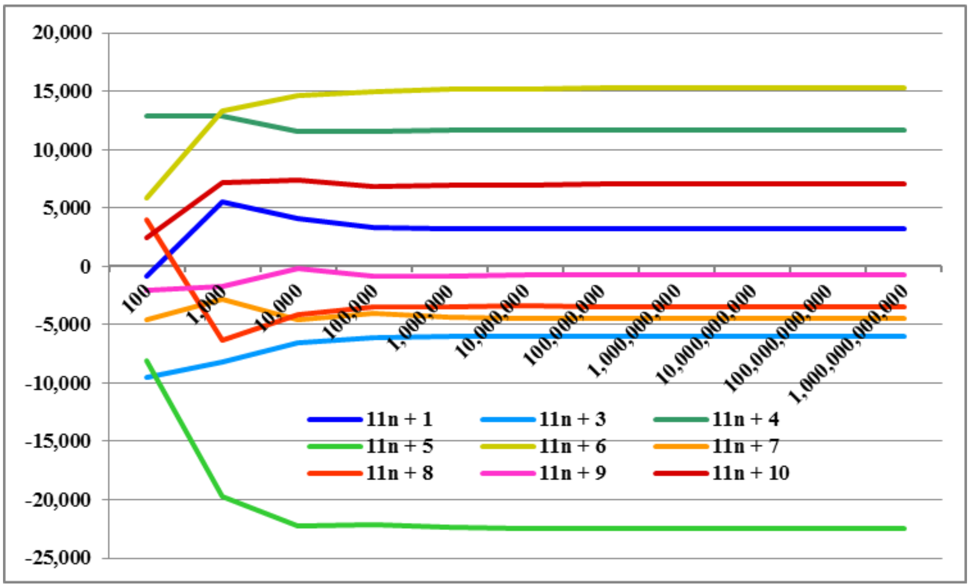


Figure 3: Average Deviation in Occurrences of Minimum Block Spacing between Primes of form 11*n* + *k* in Blocks of 10 Powers.

As seen clearly, for forms 11*n* + 3, 11*n* + 5, 11*n* + 7, 11*n* + 8, 11*n* + 9 the number of occurrences of minimum block spacings is less compared to average of all while those of other 3 forms are majoritily above it.

3. Maximum Spacings between Primes in 11*n* + *k* in Blocks of 10 Powers

As rarity of primes increases in general, the maximum spacing between primes is expected to rise with rise in the block size.

Table 4: Maximum Block Spacing between Primes of form 11*n* + *k* in Blocks of 10^n.

Sr. No.	*Blocks of Size (of 10 Power)*	*Maximum Spacing Between Primes of form*									
		11n + 1	*11n + 2*	*11n + 3*	*11n + 4*	*11n + 5*	*11n + 6*	*11n + 7*	*11n + 8*	*11n + 9*	*11n + 10*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	88	88	88	88	88	88	88	88	88	88
3.	1,000	990	990	990	990	990	990	990	990	990	990
4.	10,000	5,104	4,774	5,280	5,456	4,840	4,884	4,950	4,686	4,752	5,610
5.	100,000	5,236	4,884	5,280	5,456	4,906	5,016	5,170	4,752	4,862	5,610
6.	1,000,000 till 10^{12}	5,236	4,884	5,280	5,478	4,906	5,016	5,170	4,752	4,862	5,610

After rising at most till block size of 10^6, they remain stable till 10^{12} with respect to inspection limit of one trillion. Their deviations of common average value are depicted in following graph.

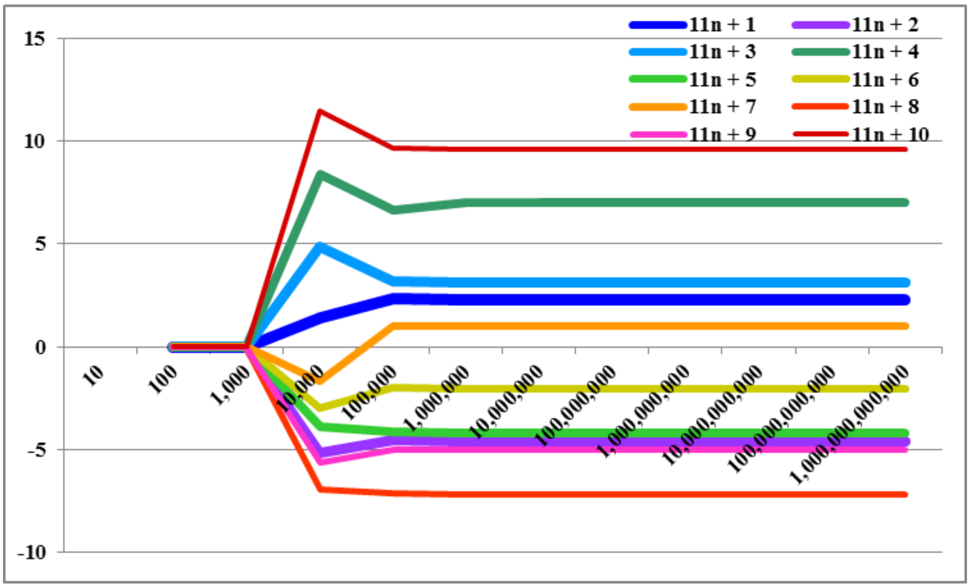


Figure 4: % Deviation of Maximum Block Spacing in Blocks of 10^n between Primes forms $11n + k$ from Average.

First primes in 10^n sized blocks with these maximum block spacings with immediate next prime of same form are as follows :

Table 5: First Starters of Maximum Block Spacings between Primes of form $11n + k$ in Blocks of 10^n.

Sr. No.	*Blocks of Size (of 10 Power)*	*First Prime with Respective Maximum Block Spacing*				
		Form 11n + 1	*Form 11n + 2*	*Form 11n + 3*	*Form 11n + 4*	*Form 11n + 5*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	12,409	3,709	1,609	103	709
3.	1,000	37,259,003	17,783,009	38,159,003	47,418,001	39,706,001
4.	10,000	151,110,404,677	661,170,501,763	804,139,840,837	795,170,314,283	925,223,102,743
5.	100,000	487,042,335,121	324,777,916,553	804,139,840,837	795,170,314,283	67,531,317,067
6.	1,000,000 till 10^{12}	487,042,335,121	324,777,916,553	804,139,840,837	939,019,099,121	67,531,317,067

Sr. No.	*Blocks of Size (of 10 Power)*	*First Prime with Respective Maximum Block Spacing*				
		Form 11n + 6	*Form 11n + 7*	*Form 11n + 8*	*Form 11n + 9*	*Form 11n + 10*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	2,503	19,609	11,701	2,011	4,003
3.	1,000	60,325,007	23,234,009	18,629,003	51,951,007	31,638,001
4.	10,000	738,483,581,149	159,237,184,751	828,945,994,841	193,171,340,129	773,296,091,611
5.	100,000 till 10^{12}	208,943,188,427	387,801,208,681	759,097,606,231	601,873,566,677	773,296,091,611

Similarly the last primes in 10^n sized blocks with these maximum block spacings with immediate next prime of same form are also determined.

Table 6: Last Starters of Maximum Block Spacings between Primes of form $11n + k$ in Blocks of 10^n.

Sr. No.	*Blocks of Size (of 10 Power)*	*Last Prime with Respective Maximum Block Spacing*				
		Form 11n + 1	*Form 11n + 2*	*Form 11n + 3*	*Form 11n + 4*	*Form 11n + 5*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	999,999,947,101	999,999,977,209	999,999,922,903	999,999,986,803	999,999,932,101
3.	1,000	999,989,212,003	999,996,711,001	999,988,911,001	999,994,715,009	999,997,066,007
4.	10,000	151,110,404,677	661,170,501,763	804,139,840,837	795,170,314,283	925,223,102,743
5.	100,000	487,042,335,121	324,777,916,553	804,139,840,837	795,170,314,283	67,531,317,067
6.	1,000,000 till 10^{12}	487,042,335,121	324,777,916,553	804,139,840,837	939,019,099,121	67,531,317,067

Sr. No.	*Blocks of Size (of 10 Power)*	*Last Prime with Respective Maximum Block Spacing*				
		Form 11n + 6	*Form 11n + 7*	*Form 11n + 8*	*Form 11n + 9*	*Form 11n + 10*
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	999,999,998,311	999,999,973,309	999,999,899,203	999,999,919,609	999,999,921,601
3.	1,000	999,987,616,007	999,996,090,001	999,994,824,001	999,996,622,007	999,982,678,001
4.	10,000	738,483,581,149	159,237,184,751	828,945,994,841	193,171,340,129	773,296,091,611
5.	100,000 till 10^{12}	208,943,188,427	387,801,208,681	759,097,606,231	601,873,566,677	773,296,091,611

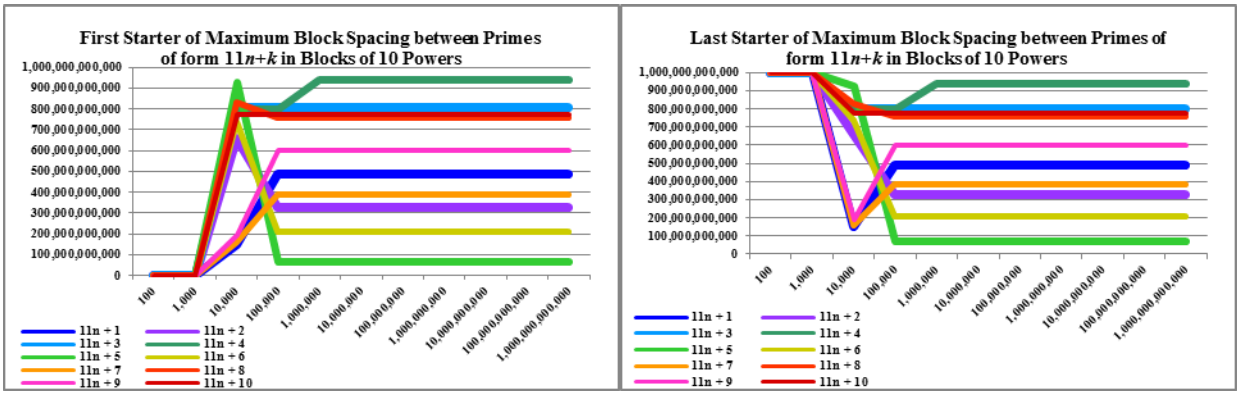


Figure 5: First & Last Starters of Maximum Spacings between Primes of form $11n + k$ in 10^n Blocks.

The number of maximum spacings between primes of same form in higher blocks of 10^n till 1 trillion gets soon settled to 1.

Table 7: Frequency of Maximum Block Spacings between Primes of form $11n + k$.

Sr. No.	*Blocks of Size (of 10 Power)*	*Number of Times Maximum Block Spacing Occurring for Primes of form 11n + k for k =*									
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1.	10	Not Found	NF	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	22,212,527	22,222,460	22,225,170	22,219,859	22,221,303	22,215,624	22,218,716	22,227,917	22,213,089	22,222,985
3.	1,000	106,432	106,743	106,281	105,885	106,410	106,118	106,329	106,110	106,115	106,521
4.	10,000 till 10^{12}	1	1	1	1	1	1	1	1	1	1

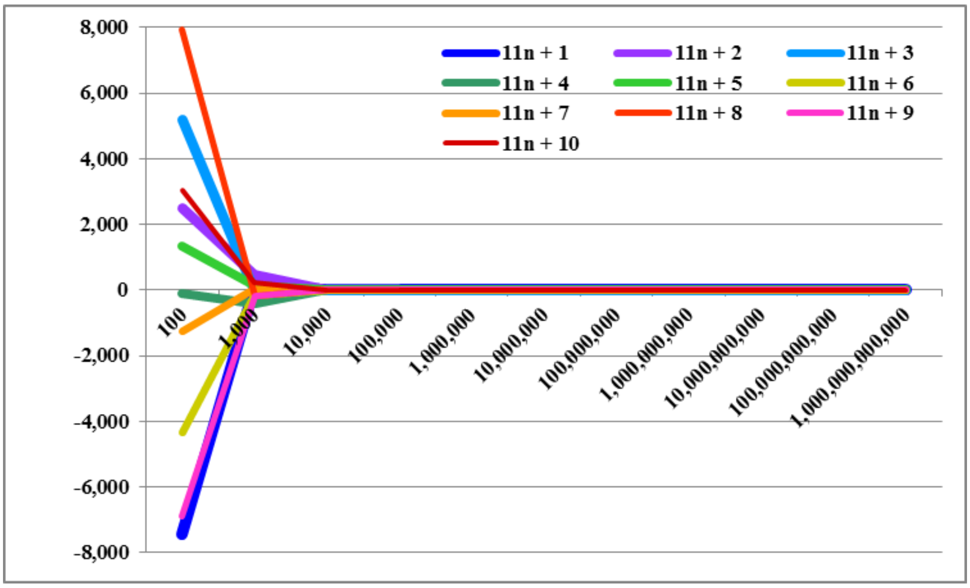


Figure 6: Average Deviation in Occurrences of Maximum Block Spacing between Primes of form $11n + k$ in Blocks of size 10^n.

4. Units Place & Tens Place Digits in Twin Prime Pair Starters of form $11n + k$

Like for primes in arithmetical progression of earlier forms, units place digits in arithmetical progression of current forms are also analyzed.

Table 8: Number of Primes of form $11n + k$ with Different Units Place Digits till One Trillion.

Sr. No.	*Units Place Digit*	*Number of Primes of form*										
		11n	*11n + 1*	*11n + 2*	*11n + 3*	*11n + 4*	*11n + 5*	*11n + 6*	*11n + 7*	*11n + 8*	*11n + 9*	*11n + 10*
1.	1	1	940,189,409	940,189,338	940,195,847	940,203,882	940,198,057	940,196,263	940,203,095	940,194,617	940,184,750	940,205,721
2.	2	0	0	1	0	0	0	0	0	0	0	0
3.	3	0	940,211,556	940,193,744	940,195,621	940,190,141	940,205,185	940,202,169	940,201,087	940,200,551	940,191,086	940,188,764
4.	5	0	0	0	0	0	1	0	0	0	0	0
5.	7	0	940,202,778	940,198,964	940,207,883	940,189,980	940,196,906	940,199,013	940,194,189	940,205,643	940,207,313	940,194,331
6.	9	0	940,190,886	940,210,665	940,192,537	940,197,583	940,194,671	940,195,524	940,194,327	940,189,776	940,198,068	940,210,095

Except digits 2 and 5 at units place in primes which are unique cases and except the primes in form $11n + 0 = 11n$, which contains only one primes, number of primes in other progressions $11n + k$ with different digits in units places compare with each other as in figure.

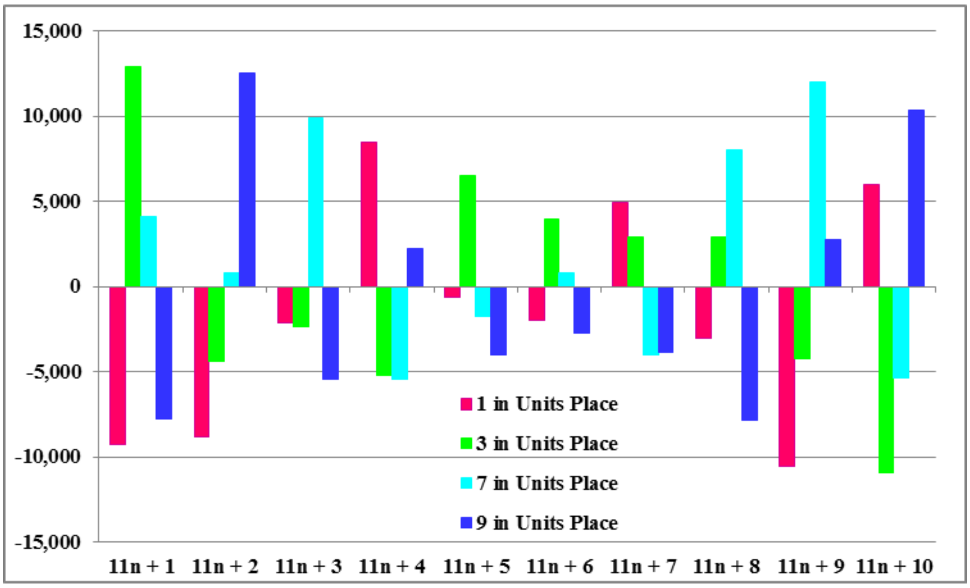


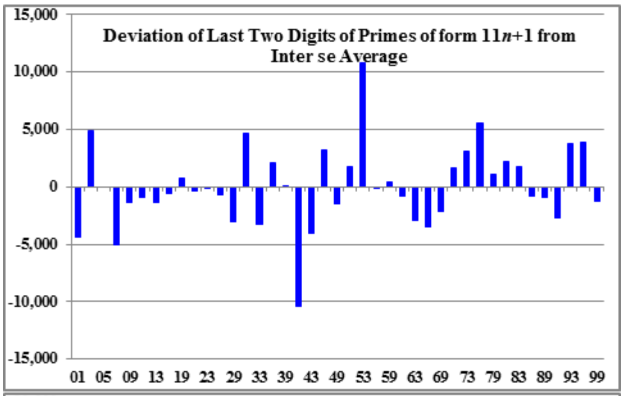
Figure 7: Deviation of Units Place Digits in Primes of form $11n + k$ from Average.

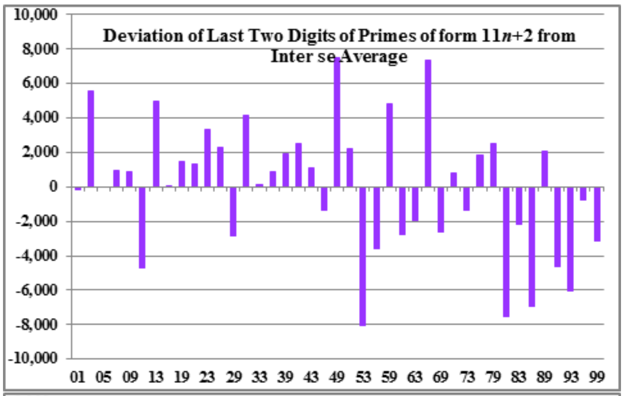
Consideration of tens and units place digits together in primes of forms $11n + k$ till one trillion give following values.

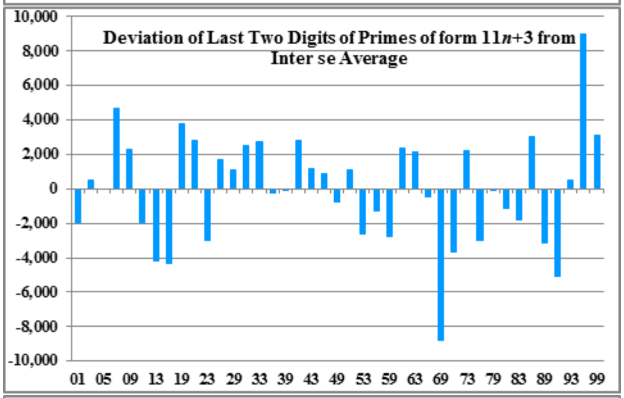
Table 9: Number of Primes of form $11n + k$ with Different Tens and Units Place Digits till One Trillion.

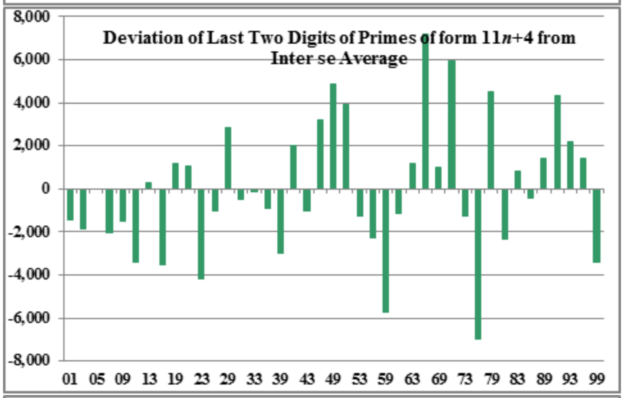
Sr. No.	*Tens & Units Place Digits*	*Number of Primes of form*										
		11n	*11n + 1*	*11n + 2*	*11n + 3*	*11n + 4*	*11n + 5*	*11n + 6*	*11n + 7*	*11n + 8*	*11n + 9*	*11n + 10*
1.	01	0	94,015,455	94,019,667	94,017,840	94,018,125	94,019,719	94,025,271	94,019,697	94,018,717	94,019,640	94,027,093
2.	02	0	0	1	0	0	0	0	0	0	0	0
3.	03	0	94,024,755	94,025,373	94,020,332	94,017,699	94,018,788	94,019,664	94,015,039	94,018,075	94,020,077	94,019,240
4.	05	0	0	0	0	0	1	0	0	0	0	0
5.	07	0	94,014,778	94,020,781	94,024,440	94,017,489	94,021,276	94,026,286	94,019,554	94,019,734	94,018,702	94,018,484
6.	09	0	94,018,481	94,020,658	94,022,072	94,018,049	94,016,222	94,021,136	94,017,341	94,021,085	94,019,807	94,023,186
7.	11	1	94,018,963	94,015,078	94,017,815	94,016,131	94,021,789	94,014,111	94,019,688	94,019,060	94,019,357	94,029,638
8.	13	0	94,018,540	94,024,815	94,015,625	94,019,839	94,022,759	94,024,261	94,016,423	94,019,959	94,017,671	94,020,812
9.	17	0	94,019,334	94,019,846	94,015,442	94,015,994	94,014,498	94,017,960	94,024,449	94,019,921	94,020,414	94,021,447
10.	19	0	94,020,603	94,021,282	94,023,605	94,020,739	94,017,811	94,025,145	94,022,241	94,026,883	94,025,777	94,020,481
11.	21	0	94,019,537	94,021,133	94,022,631	94,020,603	94,024,939	94,026,901	94,018,238	94,019,108	94,015,180	94,019,181
12.	23	0	94,019,850	94,023,137	94,016,774	94,015,356	94,023,642	94,025,243	94,020,732	94,020,383	94,019,084	94,020,912
13.	27	0	94,019,215	94,022,121	94,021,485	94,018,513	94,022,283	94,020,484	94,025,833	94,017,493	94,021,954	94,017,991
14.	29	0	94,016,857	94,016,956	94,020,897	94,022,364	94,022,653	94,019,268	94,021,688	94,020,442	94,015,197	94,021,107
15.	31	0	94,024,542	94,023,963	94,022,310	94,019,078	94,014,682	94,018,550	94,018,745	94,021,536	94,021,179	94,016,711
16.	33	0	94,016,657	94,019,979	94,022,539	94,019,397	94,015,357	94,019,845	94,019,358	94,023,170	94,018,280	94,023,052
17.	37	0	94,022,007	94,020,718	94,019,590	94,018,638	94,019,660	94,019,126	94,015,027	94,015,526	94,021,548	94,026,996
18.	39	0	94,019,957	94,021,709	94,019,754	94,016,560	94,021,438	94,014,250	94,017,556	94,016,353	94,023,644	94,024,142
19.	41	0	94,009,439	94,022,316	94,022,624	94,021,578	94,021,427	94,009,732	94,021,242	94,016,943	94,019,212	94,025,493
20.	43	0	94,015,781	94,020,936	94,020,980	94,018,522	94,023,747	94,013,631	94,025,467	94,018,921	94,017,962	94,021,646
21.	47	0	94,023,103	94,018,457	94,020,649	94,022,725	94,016,208	94,020,153	94,016,479	94,023,123	94,020,654	94,016,181
22.	49	0	94,018,336	94,027,330	94,019,004	94,024,399	94,025,592	94,015,248	94,015,374	94,015,029	94,018,048	94,022,416
23.	51	0	94,021,676	94,022,031	94,020,898	94,023,434	94,024,626	94,019,936	94,014,786	94,023,689	94,020,800	94,013,004
24.	53	0	94,030,718	94,011,739	94,017,145	94,018,250	94,025,967	94,017,950	94,020,791	94,020,044	94,014,901	94,018,082
25.	57	0	94,019,749	94,016,194	94,018,521	94,017,241	94,023,306	94,023,350	94,021,968	94,019,204	94,015,986	94,017,476
26.	59	0	94,020,340	94,024,659	94,017,003	94,013,816	94,020,488	94,023,245	94,024,534	94,018,877	94,018,202	94,018,358
27.	61	0	94,019,050	94,017,034	94,022,180	94,018,373	94,014,067	94,021,302	94,023,427	94,023,743	94,019,613	94,017,321
28.	63	0	94,016,969	94,017,878	94,021,955	94,020,711	94,020,239	94,018,056	94,022,274	94,022,054	94,022,569	94,012,661
29.	67	0	94,016,394	94,027,202	94,019,353	94,026,738	94,024,980	94,016,493	94,011,780	94,021,447	94,021,577	94,017,393
30.	69	0	94,017,754	94,017,163	94,010,994	94,020,517	94,019,246	94,015,743	94,016,550	94,016,636	94,015,357	94,022,484
31.	71	0	94,021,467	94,020,631	94,016,146	94,025,453	94,016,798	94,017,958	94,022,568	94,016,288	94,022,031	94,017,149
32.	73	0	94,022,971	94,018,436	94,021,996	94,018,299	94,018,924	94,017,537	94,021,294	94,019,593	94,020,914	94,016,983
33.	77	0	94,025,376	94,021,680	94,016,821	94,012,575	94,014,104	94,016,866	94,022,358	94,025,379	94,019,167	94,022,317

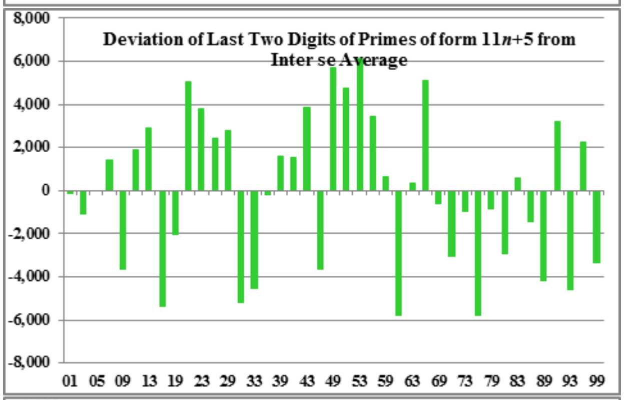
Sr. No.	Tens & Units Place Digits	Number of Primes of form										
		11n	11n + 1	11n + 2	11n + 3	11n + 4	11n + 5	11n + 6	11n + 7	11n + 8	11n + 9	11n + 10
34.	79	0	94,020,995	94,022,335	94,019,673	94,024,057	94,019,031	94,020,331	94,011,612	94,017,065	94,015,466	94,018,261
35.	81	0	94,022,128	94,012,312	94,018,697	94,017,227	94,016,963	94,017,130	94,019,978	94,020,414	94,014,329	94,020,825
36.	83	0	94,021,653	94,017,644	94,017,972	94,020,354	94,020,466	94,024,878	94,015,211	94,016,451	94,017,400	94,019,871
37.	87	0	94,019,060	94,012,900	94,022,814	94,019,098	94,018,450	94,019,123	94,020,942	94,023,617	94,022,798	94,020,252
38.	89	0	94,018,938	94,021,891	94,016,630	94,020,980	94,015,662	94,018,120	94,025,857	94,020,781	94,024,447	94,018,702
39.	91	0	94,017,152	94,015,173	94,014,706	94,023,880	94,023,047	94,025,372	94,024,726	94,015,119	94,013,409	94,019,306
40.	93	0	94,023,662	94,013,807	94,020,303	94,021,714	94,015,296	94,021,104	94,024,498	94,021,901	94,022,228	94,015,505
41.	97	0	94,023,762	94,019,065	94,028,768	94,020,969	94,022,141	94,019,172	94,015,799	94,020,199	94,024,513	94,015,794
42.	99	0	94,018,625	94,016,682	94,022,905	94,016,102	94,016,528	94,023,038	94,021,574	94,016,625	94,022,123	94,020,958

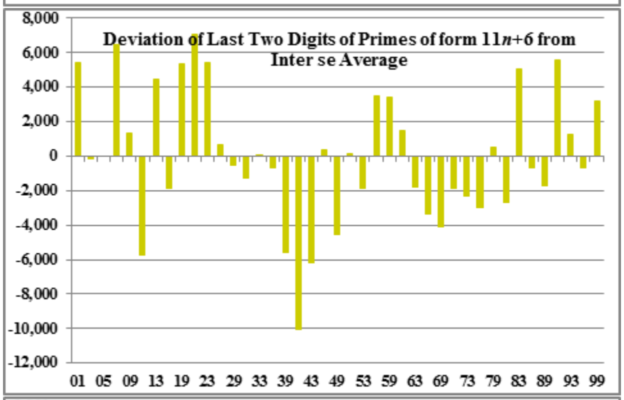


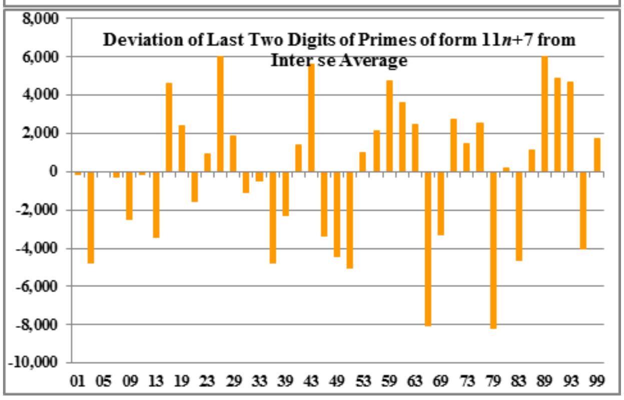


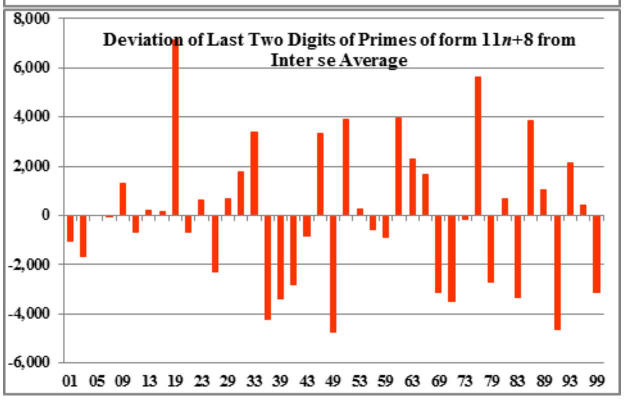












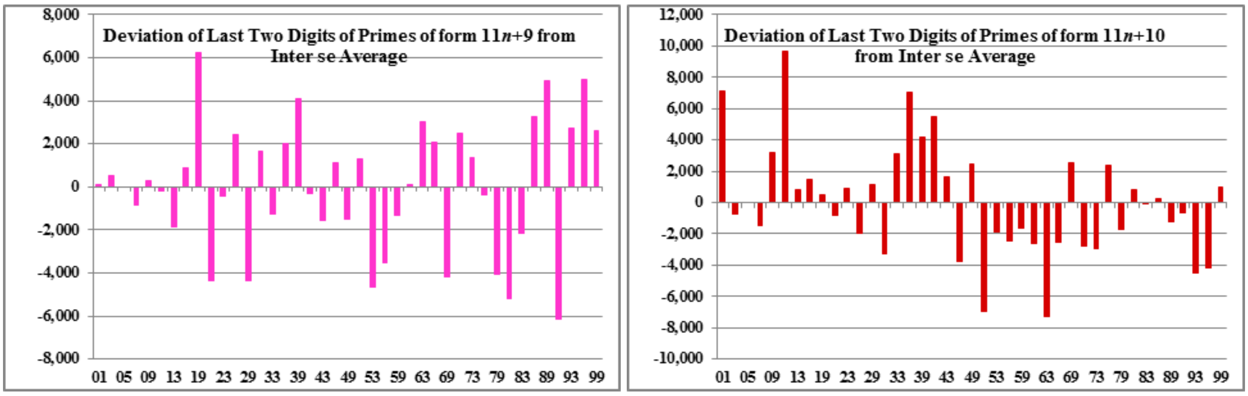


Figure 8: Deviation of Last Two Digits in Primes of form $11n + k$ from Average.

These graphs are for 10 significant arithmetical progression forms by neglecting the cases of digit combinations where there are unique primes.

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