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Domination Nirmala Indices of Graphs

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ARTICLE INFO	ABSTRACT
Published Online: **29 June 2023** Corresponding Author: **V. R. Kulli**	In this study, we introduce the domination Nirmala index, modified domination Nirmala index and their corresponding exponentials of a graph.Furthermore, we compute these domination Nirmala indices for some standard graphs, windmill graphs, book graphs.

KEYWORDS: domination degree, domination Nirmala index, modified domination Nirmala index, graph.

I. INTRODUCTION

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. For undefined terms and notations, we refer the books [1, 2].

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [3]. Recently, some new graph indices were studied in [4, 5, 6].

The domination degree $d_d(u)$ [7] of a vertex u in a graph G is defined as the number of minimal dominating sets of G which contains u.

Recently, the so-called Nirmala index was put forward, defined as [8]

$$N(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)+d_G(v)}.$$

Ref. [8] was soon followed by a series of publications [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

Inspired by work on Nirmala indices, we introduce the domination Nirmala index of a graph G as follows:

The domination Nirmala index of a graph G is defined as

$$DN(G)=\sum_{uv\in E(G)}\sqrt{d_d(u)+d_d(v)}$$

where $d_d(u)$ is the domination degree of a vertex u in G.

Considering the domination Nirmala index, we introduce the domination Nirmala exponential of a graph G and defined it as

$$DN(G,x)=\sum_{uv\in E(G)}x^{\sqrt{d_d(u)+d_d(v)}}.$$

We introduce the modified domination Nirmala index of a graph G as follows:

The modified domination Nirmala index of a graph G is defined as

$$^{m}DN(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)+d_d(v)}}.$$

Considering the modified domination Nirmala index, we introduce the modified domination Nirmala exponential of a graph G and defined it as

$$^{m}DN(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_d(u)+d_d(v)}}}.$$

We define the product domination Nirmala index of a graph G as

$$pDN(G)=\left(\sum_{uv\in E(G)}\sqrt{d_d(u)+d_d(v)}\right)\left(\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)+d_d(v)}}\right).$$

In this paper, the domination Nirmala index, modified domination Nirmala index and their corresponding exponentials of some standard graphs, windmill graphs, book graphs are computed.

II. THE DOMINATION NIRMALA INDEX AND ITS EXPONENTIAL OF GRAPHS

1. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1. If K_n is a complete graph with n vertices, then

$$DN(K_n)=\frac{n(n-1)}{\sqrt{2}}.$$

Proof: If K_n is a complete graph, then $d_d(u) = 1$.

From definition, we have

$$DN(K_n) = \sum_{uv \in E(K_n)} \sqrt{d_d(u) + d_d(v)}$$

$$= \frac{n(n-1)}{2}\sqrt{1+1} = \frac{n(n-1)}{\sqrt{2}}.$$

Proposition 2. If S_{n+1} is a star graph with $d_d(u) = 1$, then

$$DN(S_{n+1}) = \sqrt{2}n.$$

Proposition 3. If $S_{p+1,q+1}$, is a double star graph with $d_d(u) = 2$, then

$$DN(S_{p+1,q+1}) = 2(p+q+1).$$

Proposition 4. Let $K_{m,n}$ be a complete bipartite graph with $2 \leq m \leq n$. Then

$$DN(K_{m,n}) = mn\sqrt{m+n+2}.$$

Proof: Let $G=K_{m,n}$, m, $n \geq 2$ with $d_d(u) = m+1$

$= n+1$, for all $u \in V(G)$.

From definition, we have

$$DN(K_{m,n}) = \sum_{uv \in E(K_{m,n})} \sqrt{d_d(u) + d_d(v)}$$

$$= mn\sqrt{(m+1)+(n+1)} = mn\sqrt{m+n+2}.$$

In the following proposition, by using definition, we obtain the domination Nirmala exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$.

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Proposition 5. The domination Nirmala exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $$DN(K_n, x) = \sum_{uv \in E(G)} x^{\sqrt{d_d(u)+d_d(v)}}$$

$$= \frac{n(n-1)}{2} x^{\sqrt{1+1}} = \frac{n(n-1)}{2} x^{\sqrt{2}}.$$

(ii) $$DN(S_{n+1}, x) = nx^{\sqrt{2}}.$$

(iii) $$DN(S_{p+1,q+1}, x) = (p+q+1)x^2.$$

(iv) $$DN(K_{m,n}, x) = mnx^{\sqrt{m+n+2}}.$$

2. RESULTS FOR FRENCH WINDMILL GRAPHS

The French windmill graph F_n^m is the graph obtained by taking m □ 3 copies of K_n, n □ 3 with a vertex in common. The graph F_n^m is presented in Figure 1. The French windmill graph F_3^m is called a friendship graph.

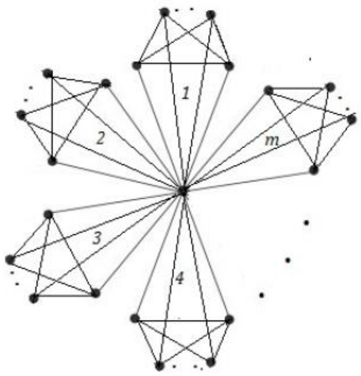

Figure 1. French windmill graph F_n^m

Let F be a French windmill graph F_n^m. Then

$d_d(u) = 1$, if u is the center vertex,

$= (n-1)^{m-1}$, otherwise.

Theorem 1. Let F be a French windmill graph F_n^m. Then

$${}^{m}DN(F) = m(n-1)\sqrt{1+(n-1)^{(m-1)}}$$

$$+[(mn(n-1)/2) - m(n-1)]\sqrt{2(n-1)^{(m-1)}}.$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$DN(F) = \sum_{uv \in E(G)} \sqrt{d_d(u) + d_d(v)}$$

$$= \sum_{uv \in E_1(G)} \sqrt{d_d(u) + d_d(v)} + \sum_{uv \in E_2(G)} \sqrt{d_d(u) + d_d(v)}$$

$$= m(n-1)\sqrt{1+(n-1)^{(m-1)}}$$

$$+[(mn(n-1)/2) - m(n-1)]\sqrt{(n-1)^{(m-1)} + (n-1)^{(m-1)}}$$

$$= m(n-1)\sqrt{1+(n-1)^{(m-1)}}$$

$$+[(mn(n-1)/2) - m(n-1)]\sqrt{2(n-1)^{(m-1)}}.$$

Corollary 1.1. Let F_3^m be a friendship graph. Then

$$DN(F_3^m) = 2m\sqrt{1+2^{(m-1)}} + m\sqrt{2^m}.$$

In the following theorem, by using definition, we obtain the domination Nirmala exponential of F_n^m and F_3^m.

Theorem 2. The domination Nirmala exponential of F_n^m and F_3^m are given by

(i) $DN\left(F_n^m, x\right) = m(n-1)x^{\sqrt{1+(n-1)^{(m-1)}}}$

$$+[(mn(n-1)/2) - m(n-1)]x^{\sqrt{2(n-1)^{(m-1)}}}.$$

(ii) $DN\left(F_3^m, x\right) = 2mx^{\sqrt{1+2^{(m-1)}}} + mx^{\sqrt{2^m}}.$

3. RESULTS FOR GoK_p

Theorem 3. Let $H=GoK_p$, where G is a connected graph with n vertices and m edges; and K_p is a complete graph. Then

$$DN(H) = \frac{1}{\sqrt{2}}(2m + np^2 + np)\sqrt{(p+1)^{n-1}}.$$

Proof: If $H=GoK_p$, then $d_d(u) = (p+1)^{n-1}$. In F, there are $\frac{p(p-1)}{2}$. edges. Thus H has $\frac{1}{2}(2m + np^2 + np)$ edges. Thus

$$DN(H) = \sum_{uv\in E(H)} \sqrt{d_d(u) + d_d(v)}$$

$$= \frac{1}{2}(2m + np^2 + np)\sqrt{(p+1)^{n-1} + (p+1)^{n-1}}$$

$$= \frac{1}{\sqrt{2}}(2m + np^2 + np)\sqrt{(p+1)^{n-1}}.$$

In the following theorem, by using definition, we obtain the domination Nirmala exponential of GoK_p.

Theorem 4. The domination Nirmala exponential of GoK_p is given by

$$DN\left(GoK_p, x\right) = \frac{1}{2}(2m + np^2 + np)x^{\sqrt{2(p+1)^{n-1}}}.$$

4. RESULTS FOR B_n

The book graph B_n, $n\geq 3$, is a cartesian product of star S_{n+1} and path P_2.

For B_n, $n\geq 3$, we have $d_d(u) = 3$, if u is the center vertex,

$= 2^{n-1} + 1$, otherwise.

Theorem 5. If B_n, $n\geq 3$, is a book graph, then

$$DN(B_n) = \sqrt{6} + 2n\sqrt{4 + 2^{n-1}} + n\sqrt{2\left(2^{n-1} + 1\right)}.$$

Proof: In B_n, there are three types of edges as follow:

$E_1 = \{uv \square E(B_n) \mid d_d(u)=d_d(v)=3\}$, $|E_1| = 1$ $E_2 = \{uv \square E(B_n) \mid d_d(u) = 3, d_d(v)= 2^{n-1}+1\}$, $|E_2| = 2r$.

$E_3 = \{uv \square E(B_n) \mid d_d(u) = d_d(v)= 2^{n-1}+1\}$, $|E_3| = r$.

By definition, we have

$$DN(B_n) = \sum_{uv\in E(H)} \sqrt{d_d(u) + d_d(v)}$$

$$= 1\sqrt{3+3} + 2n\sqrt{3 + \left(2^{n-1}+1\right)}$$

$$+ n\sqrt{\left(2^{n-1}+1\right) + \left(2^{n-1}+1\right)}$$

$$= \sqrt{6} + 2n\sqrt{4 + 2^{n-1}} + n\sqrt{2\left(2^{n-1}+1\right)}.$$

In the following theorem, by using definition, we obtain the domination Nirmala exponential of B_n.

Theorem 6. The domination Nirmala exponential of GoK_p is given by

$$DN(B_n, x) = x^{\sqrt{6}} + 2nx^{\sqrt{4+2^{n-1}}} + nx^{\sqrt{2\left(2^{n-1}+1\right)}}.$$

II. THE MODIFIED DOMINATION NIRMALA INDEX AND ITS EXPONENTIAL OF GRAPHS

5. RESULTS FOR SOME STANDARD GRAPHS

Proposition 6. If K_n is a complete graph with n vertices, then

$$^{m}DN(K_n) = \frac{n(n-1)}{2\sqrt{2}}.$$

Proof: If K_n is a complete graph, then $d_d(u) = 1$.

From definition, we have

$$^{m}DN(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_d(u) + d_d(v)}}$$

$$= \frac{n(n-1)}{2}\frac{1}{\sqrt{1+1}} = \frac{n(n-1)}{2\sqrt{2}}.$$

Proposition 7. If S_{n+1} is a star graph with $d_d(u) = 1$, then

$$^{m}DN(S_{n+1}) = \frac{n}{\sqrt{2}}.$$

Proposition 8. If $S_{p+1,q+1}$ is a double star graph with $d_d(u) = 2$, then

$$^{m}DN\left(S_{p+1,q+1}\right) = \frac{p+q+1}{2}.$$

Proposition 9. Let $K_{m,n}$ be a complete bipartite graph with $2 \leq m \leq n$. Then

$$^{m}DN\left(K_{m,n}\right) = \frac{mn}{\sqrt{m+n+2}}.$$

In the following proposition, by using definition, we obtain the modified domination Nirmala exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$.

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Proposition 10. The modified domination Nirmala exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $^{m}DN(K_n,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_d(u)+d_d(v)}}}$

$$=\frac{n(n-1)}{2}x^{\frac{1}{\sqrt{1+1}}}=\frac{n(n-1)}{2}x^{\frac{1}{\sqrt{2}}}.$$

(ii) $^{m}DN(S_{n+1},x)=nx^{\frac{1}{\sqrt{2}}}.$

(iii) $^{m}DN(S_{p+1,q+1},x)=(p+q+1)x^{\frac{1}{2}}.$

(iv) $^{m}DN(K_{m,n},x)=mnx^{\frac{1}{\sqrt{m+n+2}}}.$

6. RESULTS FOR FRENCH WINDMILL GRAPHS

Theorem 7. Let F be a French windmill graph F_n^m. Then

$$^{m}DN(F)=\frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{\sqrt{2(n-1)^{(m-1)}}}$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$^{m}DN(F)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)+d_d(v)}}$$

$$=\sum_{uv\in E_1(G)}\frac{1}{\sqrt{d_d(u)+d_d(v)}}+\sum_{uv\in E_2(G)}\frac{1}{\sqrt{d_d(u)+d_d(v)}}$$

$$=\frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{\sqrt{(n-1)^{(m-1)}+(n-1)^{(m-1)}}}$$

$$=\frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)}}}+\frac{[(mn(n-1)/2)-m(n-1)]}{\sqrt{2(n-1)^{(m-1)}}}.$$

Corollary 7.1. Let F_3^m be a friendship graph. Then

$$^{m}DSN(F_3^m)=\frac{2m}{\sqrt{1+2^{(m-1)}}}+\frac{m}{\sqrt{2^m}}.$$

In the following theorem, by using definition, we obtain the modified domination Nirmala exponential of F_n^m and F_3^m.

Theorem 8. The modified domination Nirmala exponential of F_n^m and F_3^m are given by

(i) $^{m}DN(F_n^m,x)=m(n-1)x^{\frac{1}{\sqrt{1+(n-1)^{(m-1)}}}}$

$$+[(mn(n-1)/2)-m(n-1)]x^{\frac{1}{\sqrt{2(n-1)^{(m-1)}}}}.$$

(ii) $^{m}DN(F_3^m,x)=2mx^{\frac{1}{\sqrt{1+2^{(m-1)}}}}+mx^{\frac{1}{\sqrt{2^m}}}.$

7. RESULTS FOR GoK_p

Theorem 9. Let $H=GoK_p$, where G is a connected graph with n vertices and m edges; and K_p is a complete graph. Then

$$^{m}DN(H)=\frac{2m+np^2+np}{2\sqrt{2(p+1)^{n-1}}}.$$

Proof: If $H=GoK_p$, then $d_d(u)=(p+1)^{n-1}$. In F, there are $\frac{p(p-1)}{2}$. edges. Thus H has $\frac{1}{2}(2m+np^2+np)$ edges. Thus

$$^{m}DN(H)=\sum_{uv\in E(H)}\frac{1}{\sqrt{d_d(u)+d_d(v)}}$$

$$=\frac{(2m+np^2+np)}{2}\frac{1}{\sqrt{(p+1)^{n-1}+(p+1)^{n-1}}}$$

$$=\frac{2m+np^2+np}{2\sqrt{2(p+1)^{n-1}}}.$$

In the following theorem, by using definition, we obtain the domination Nirmala exponential of GoK_p.

Theorem 10. The domination Nirmala exponential of GoK_p is given by

$$^{m}DN(GoK_p,x)=\frac{1}{2}(2m+np^2+np)x^{\sqrt{2(p+1)^{n-1}}}.$$

8. RESULTS FOR B_n

The book graph B_n, $n\geq3$, is a cartesian product of star S_{n+1} and path P_2.

For B_n, $n\geq3$, we have $d_d(u)=3$, if u is center vertex,

$= 2^{n-1}+1$, otherwise.

Theorem 11. If B_n, $n\geq3$, is a book graph, then

$$DN(B_n)=\sqrt{6}+2n\sqrt{4+2^{n-1}}+n\sqrt{2(2^{n-1}+1)}.$$

Proof: In B_n, there are three types of edges as follow:

$E_1 = \{uv \in E(B_n) \mid d_d(u)=d_d(v)=3\}$, $|E_1| = 1$.

$E_2 = \{uv \in E(B_n) \mid d_d(u) = 3, d_d(v)= 2^{n-1}+1\}$, $|E_2| = 2r$.

$E_3 = \{uv \in E(B_n) \mid d_d(u) = d_d(v)= 2^{n-1}+1\}$, $|E_3| = r$.

By definition, we have

$$^{m}DN(B_n)=\sum_{uv\in E(B_n)}\frac{1}{\sqrt{d_d(u)+d_d(v)}}$$

$$=\frac{1}{\sqrt{3+3}}+\frac{2n}{\sqrt{3+(2^{n-1}+1)}}+\frac{n}{\sqrt{(2^{n-1}+1)+(2^{n-1}+1)}}$$

$$=\frac{1}{\sqrt{6}}+\frac{2n}{\sqrt{4+2^{n-1}}}+\frac{n}{\sqrt{2(2^{n-1}+1)}}.$$

In the following theorem, by using definition, we obtain the modified domination Nirmala exponential of B_n.

Theorem 12. The domination Nirmala exponential of GoK_p is given by

$$^{m}DN(B_n,x)=x^{\frac{1}{\sqrt{6}}}+2nx^{\frac{1}{\sqrt{4+2^{n-1}}}}+nx^{\frac{1}{\sqrt{2(2^{n-1}+1)}}}.$$

Problems:

(i) Determine the properties of *DN*.

(ii) Establish the lower and upper bounds for *DN*.

(iii) Determine the properties of *pDN*.

(iv) Establish the lower and upper bounds for *pDN*.

III. CONCLUSION

In this paper, the domination Nirmala index, modified domination Nirmala index and their corresponding exponentials of some standard graphs, windmill graphs, book graphs are computed.

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