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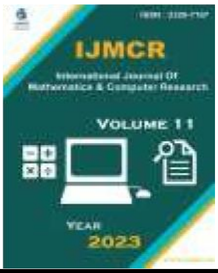

Efficient Area Calculation of Jackfruit Leaves: Leveraging Green's Theorem and Least Squares Function

Tita Nuril Istiqomah[1], Jovian Dian Pratama[2*], Widuri Wahyu Saputri[3], Nahdia Ayu Sabrina[4], Fidini Nur Sa'adah[5], Luluk Ainul Latifah[6], Pandu Reksa Wiguna[7], Wahyu Aditya Bagus Mahardika[8]
[1,2,3,4,5,6,7,8]Department of Mathematics, Faculty Science and Mathematics, Diponegoro University, Semarang, Indonesia.

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Corresponding Author:
Jovian Dian Pratama

ABSTRACT
Leaves are an important part of a plant. As a place for photosynthetic organs, leaves can be used as indicators to assess a plant's growth and physiological performance. Leaf measurement is carried out as an important effort to collect leaf area data which is useful in analyzing a leaf's growth. This study aims to apply the green theorem to calculate leaf area and compare leaf area with the least square approximation, Simpson 1/3, the elliptic approach, and the millimeter block method, with the research object used being Jackfruit leaves. From this research, we obtain the results as follows leaf area by applying the green theorem by least square approximation is 5946.75 mm^2, the leaf area with Simpson 1/3 method is 5938.25 mm^2, the leaf area with ellipse approximation is 6850.23 mm^2, and leaf area with millimeter block method is 5387.5 mm^2. From the several approximation methods used in this study, it was found that by using the Least Square Quadratic approximation and the Simpson 1/3 method, effective results were obtained for the observed object because the functions of the Least Square Quadratic approximation and the Simpson 1/3 method are almost close to the actual function so that the results obtained are close to accurate with an area the original of the observed object is jackfruit leaves.

KEYWORDS: Efficient area calculation, Jackfruit leaves, Green's theorem, Least square function, Leaf area measurement

I. INTRODUCTION

Each plant typically has numerous leaves, and leaves play a crucial function in a plant's life [1]. The primary photosynthetic organ, the leaf, has various shapes. However, the petiole, which joins the leaf to the stem node, and the flat blade (also known as the blade) make up most of the leaf structure [2]. Jackfruit leaves, also known as stalked leaves, are an example of an incomplete composition because they only have stalks and strands [1]. Since plants typically have this type of leaf arrangement, the leaf arrangement is comparable to that of jackfruit leaves, which can be found in various locations [1].

The jackfruit plant produces a single leaf with an elongated shape and grows alternately on the plant's branches. The upper leaf surface is bright green and smooth, while the lower leaf surface is dark green and rough, and the leaves have a blunt tip, a sharp leaf base, a flat leaf edge, leaf flesh that resembles paper, and green leaf veins [3]. If the length-to-width ratio is 1.5 to 2: 2, the shape and structure of jackfruit leaves resemble a shield (ovalis or ellipticus)[3].

Figure 1. Jackfruit Leaf Sample

Measuring the leaf area is crucial for assessing a plant's growth and physiological function. However, due to the variety of plant leaf shapes, measuring leaf area is a difficult task that takes some time and the right measuring tools [4].

The area of jackfruit leaves will be determined using a variety of techniques. The first technique is a theory in mathematics called Green's Theorem with the least squares function, which can be used to determine the area by matching the curve and is then anticipated to build specific mathematical equations. The second technique is Simpson's 1/3 method, used as an approximation to determine the area of jackfruit leaves. The third technique is known as the elliptic approximation approach, and it works by drawing an ellipse that approximates the shape of the leaf as closely as possible and then utilizing that information to determine the area of the leaf. The last technique is called the millimeter block method, and it entails manually measuring a leaf's area on a piece of paper in millimeters and tallying the number of blocks it has underneath it.

This study aims to compute leaf area using Green's theorem and compare it to leaf area calculated using Simpson's 1/3, the elliptical technique, Green's least square approximation theorem, and the millimeter block method. This study also intends to assess the efficacy and precision of various techniques in estimating jackfruit leaf area. Farmers, biologists, and other researchers interested in plant growth and the factors affecting it may find this research to be of service. Farmers or agricultural professionals can forecast greater harvests or alter their farming operations to promote plant health by understanding the most precise and effective method of measuring the leaf area of crops.

II. THEORETICAL REVIEW

Calculations will be made on the leaves' width and the veins' length in this study. Therefore, we will employ the Simpson 1/3, elliptic method, quadratic least square approximation, and block millimeters approaches to generate the function that will be utilized to determine the width of the leaves.

A. Quadratic Least Square

The least squares method is an approach (approximation) method for formulating equations obtained from data points. This technique matches curves that will later be anticipated to create certain accurate mathematical equations [5]. The following are examples of frequent mathematical equations:

- Linear Equation
  $$y = px + q \tag{1}$$
- Quadratic Equation
  $$y = ax^2 + bx + c \tag{2}$$

The least squares method will be applied in this study to approximate functions from known data to achieve the closest function. The quadratic least square is the least square approximation that the researcher will employ. Equation (2) serves as the approximation function for this method and is the distribution equation that expresses the quadratic equation's error distribution.

$$S = \sum (y - ax^2 - bx - c)^2 \tag{3}$$

A system of linear equations (SLE) of order three will be created by determining the function's first derivative (3) for variables a, b, and c. An augmented matrix of SST can then be organized as follows.

$$\begin{bmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & N \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum x^2 y \\ \sum xy \\ \sum y \end{bmatrix} \tag{4}$$

B. Simpson's Approach 1/3

A problem that cannot be resolved using analytical techniques may be solved using the Simpson 1/3 method, which is an approximation method. The Simpson 1/3 and the Simpson 1/8 methods are the two categories into which this method is classified. The Simpson 1/3 method's operation is based on the volume of a rotating object integration. However, it has been demonstrated that the Simpson 1/3 approach can determine the area of cargo ships below sea level [6]. The area of Jackfruit leaves will be approximated in this study using the Simpson 1/3 method, shown in the following picture.

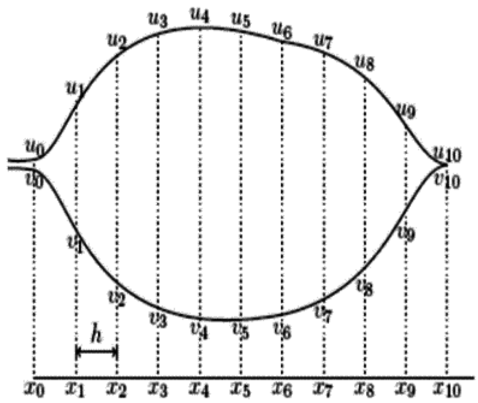

Figure 2. Measurements with the Simpson 1/3 method

Given the horizontally arranged leaves in Figure 2, if p is taken to be the leaf's length, h is taken to be the width of each partition, and n is taken to be the segment that must be satisfied, we get

$$h = \frac{p}{2n} \tag{5}$$

The point u_i can then be obtained by drawing a line from the division points x_0, x_1, x_2,..., x_{2n} to the top end of the leaf. Then, point v_i can be attained if a straight line is drawn from x_1 to the bottom end of the leaf. The next step is determining f_i from each partition using the formula below.

$$f_i = u_i - v_i \tag{6}$$

After obtainin f_0, f_1, f_2,..., f_{2n}, the Simpson 1/3 method, represented by [4], will be used to calculate the leaf area.

$$\text{Leaf Area} \approx \frac{h}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 \dots + 2f_{2n-2} + 4f_{2n-1} + f_{2n}) \tag{7}$$

C. Elliptical Approach

The focus of all points in one plane with the same total distance from two preset fixed points is an example of a cone section, and it has the condition $|PF| = e|PL|$ where $0 < e < 1$ [7].

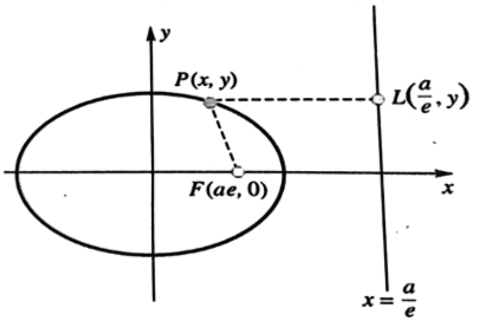

Figure 3. Ellipse

Assume $P(x, y)$, as seen in Figure 3, is a point on the ellipse. The projection on the directrix is then $L\left(\frac{a}{e}, y\right)$. Terms $|PF| = e|PL|$.

$$\sqrt{(x - ae)^2 + y^2} = e\sqrt{\left(x - \frac{a}{e}\right)^2} \quad (8)$$

We obtain the equivalent equation by squaring both sides and adding the terms.

$$x^2 - 2aex + a^2e^2 + y^2 = e^2\left(x^2 - \frac{2a}{e}x + \frac{a^2}{e^2}\right) \quad (9)$$

or

$$(1 - e^2)x^2 + y^2 = a^2(1 - e^2) \quad (10)$$

or

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1 \quad (11)$$

This final equation relates to a symmetric curve around the x- and y-axes and the original because it only contains x and y to even powers. Additionally, this symmetry requires a second focus at $(-ae, 0)$ and a second directrix at $x = -\frac{a}{e}$. The main axis, which runs through the center of the object and includes the two vertices and two foci, is also known as the minor axis.

Since $0 < e < 1$ for an ellipse, $(1 - e^2)$ is positive. If, for instance, $b = a\sqrt{1 - e\text{^}2}$, the equation above will take the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (12)$$

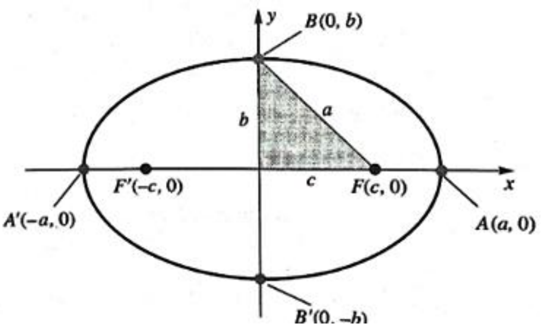

Figure 4. Ellipses and their Equations

The basic ellipse equation, equation (12), can be used to get an area in the shape of an ellipse. The area of the ellipse is then obtained by integrating the point y with the area of the ellipse, giving you the area of the ellipse as follows.

$$\text{Area of Ellipse} = \pi \times a \times b \quad (13)$$

Where the ellipse's diagonals are a and b.

D. Calculating Leaf Area with Millimeter Blocks

One technique for calculating leaf area is the Millimeter Block approach. This method of measuring leaf area involves plotting the leaf onto millimeter block paper, calculating the area the leaf covers, and obtaining the leaf area in units of mm^2 [8].

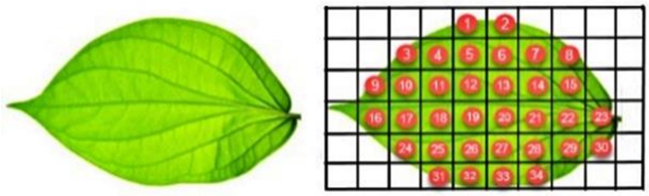

Figure 5. Calculating Leaf Area with Millimeter Blocks

This technique is used to quantify leaf area manually. With the caveat that only a few samples are used, this method is simple and practical and can yield satisfactory results [9]. The method is deemed less efficient when calculating leaf area with many leaf samples because it takes more time the more samples are used.

When calculating leaf area using this method, squares within millimeter blocks are counted as one if filled with leaves; otherwise, they are counted as half and combined with other boxes whose calculation results are also half.

E. Calculating Area with Integral

The following definition[10] can be used to define the area in Cartesian coordinates enclosed by the curves $f(x)$ and $g(x)$. Assume that D is an area enclosed by the lines $x = a, x = b$, and the x-axis, as well as the continuous graph [a,b] with $f(x) \geq 0$ on $[a, b]$. $D = \{(x, y) | a \leq x \leq b; 0 \leq y \leq f(x)\}$ can be used to represent region D, where f is continuous on $[a, b]$. Area D is indicated by

$$L = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i \quad (14)$$

$$L = \int_a^b f(x)dx \quad (15)$$

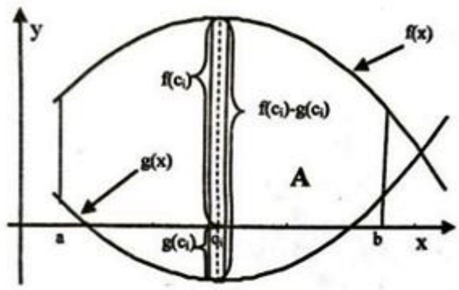

Figure 6. Area between 2 Curves

The limit of the sum for $n \to \infty$ of the areas of the approaching rectangles, or the area between the two curves in Figure 6, is similarly located there.

$$\lim_{n\to\infty} \sum_{i=1}^{n} [f(c_i) - g(c_i)]\Delta x_i \quad (16)$$

The area of region A, which is enclosed by the lines $x = a$, $x = b$ with continuous f and g and $f(x) \geq g(x) \forall x \in [a, b]$ and the curve $y = f(x)$,$y = g(x)$, is

$$A = \int_x^b \big(f(x) - g(x)\big)dx \qquad (17)$$

F. Green's Theorem

According to Green's Theorem[11], there is a correlation between the area of a double integral over a region C and the line integral over a closed curve C. For example, if $M(x, y)$ and $N(x, y)$ are continuous functions and have a continuous first partial derivative at R, and R is a closed region in the X and Y planes circumscribed by a closed curve C, then

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy \qquad (18)$$

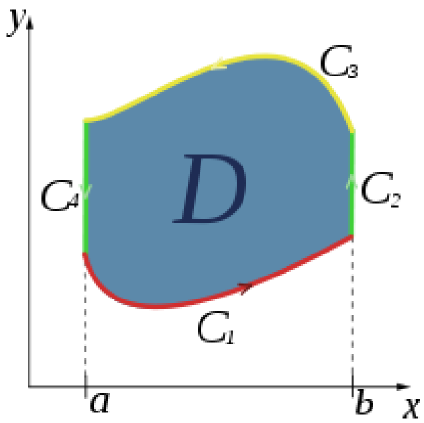

Figure 7. Region *D* bounded by Curve *C*

The idea of a line integral can be used to compute the area of an area because of Green's Theorem's impact on the plane. If a plane's area D has a boundary, say, curve C and curve C is simple, closed, and smooth in sections, then area D is given by:

$$A(D) = \frac{1}{2}\oint_C (x\,dy - y\,dx) \qquad (19)$$

III. Research Techniques

To determine leaf area, this study will use four practical methods: Green's theorem with quadratic least square functions, Simpson's 1/3 method, elliptic approach, and millimeter blocks. The scale is 1:5; thus, once these computations have produced their results, the output will be multiplied by 25 mm^2. This implies that 1 mm on the box equals 5mm. Due to the box's square shape, the area scale is 1:25, meaning that one box's one mm^2 area corresponds to 25 mm^2 of the actual area.

A. Green's Theorem with Quadratic Least Square Functions

The following are the processes for calculating leaf area using the least squares method:

a. On millimeter block paper, doodle a pattern of leaves.
b. Using Geogebra software to project the leaf pattern after copying it to cartesian coordinates in quadrant 1 on millimeter block paper.
c. Dividing the leaf pattern into curves f(x) and g(x), respectively.
d. Observe and record the points through the curve $f(x)$ or $g(x)$.
e. It uses the points found to obtain the functions $f(x)$ and $g(x)$ with the quadratic least square method described in the theoretical review.
f. Calculate the area of the leaf with the usual integral in the interval a to b.
g. The results of leaf area calculations will be obtained using the quadratic least square approximation and the usual integral.
h. Multiply your result by 25 mm^2.

B. 1/3 Simpson's approach

The following procedures [4] are used to find the leaf area using the Simpson 1/3 approximation.

a. Prepare the leaves so the area may be determined.
b. Using Geogebra software to project the leaf pattern after copying it to cartesian coordinates in quadrant one on millimeter block paper.
c. By measuring the length of the leaf, one may determine the value of the variable p.
d. Choose n's (the segment's) value such that $2n$ is an even segment.
e. Using the formula (5), ascertain the value of h.
f. In search of the value $x_0, x_1, x_2, \ldots, x_{2n}$ based on the h discovered.
g. To obtain the u_1 point, move the x_i point to the leaf's upper end.
h. To obtain point v_1, draw point x_i to the leaf's bottom end.
i. Applying formula (6) to each partition's f_i calculation.
j. Then, apply formula (7) to determine the leaf area.
k. Integrating 25 mm^2 to the result obtained.

C. Elliptical Method

The elliptic method was also applied in this study to determine leaf area. The steps are listed below.

a. We use GeoGebra software to project the leaf pattern after copying it to cartesian coordinates in quadrant one on millimeter block paper.
b. Over the picture of the leaf, draw an ellipse shape.
c. Find the ellipse's semi-major axis, the distance from the center to the outermost point, and the semi-minor axis, the distance from the innermost point to the center.
d. Using the area of an ellipse formula to determine an ellipse's area.
e. It brought 25 mm^2 to the result obtained.

D. Block Method in Millimeters

The millimeter block method will be compared along with the three earlier techniques. The steps are listed below.

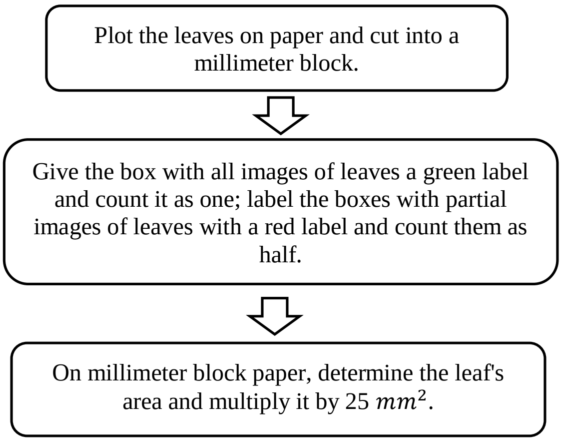

Figure 8. illustrates how to compute leaf area using the millimeter block technique.

IV. RESULTS AND DISCUSSION

The jackfruit leaf must first be sketched on millimeter block paper to determine its length and width before determining its area, as illustrated below.

Figure 9. To Be Drawn Jackfruit Leaves

The outcomes that followed drawing on millimeter block paper were as follows.

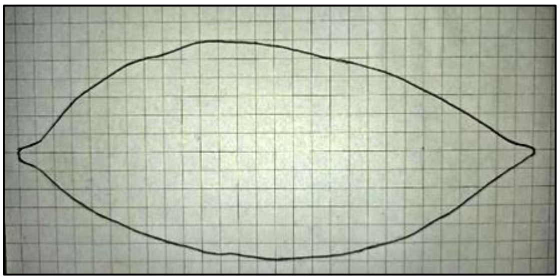

Figure 10. Sketch of Jackfruit Leaves Drawn on Millimeter Block Paper

The area of jackfruit leaves can then be determined using one of the four ways.

A. Calculating Leaf Area Using Green's Theorem

Green's theorem can be applied after finding the function using the quadratic least square approximation. Using Geogebra software, the leaf picture drawn on millimeter block paper is copied to Cartesian coordinates before the approximation.

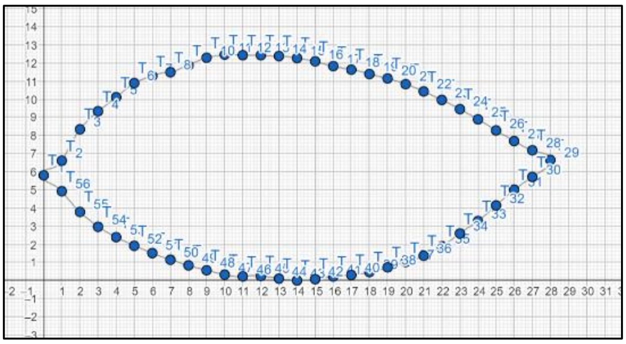

Figure 11. Geometrical Image of Jackfruit Leaves in Cartesian Coordinates (Using Geogebra)

Then, using quadratic least squares, we will search for the function $f(x)$ as the top curve and $g(x)$ as the bottom curve. The following 56 points are generated from Figure 11 and pass through the curves $f(x)$ and $g(x)$.

Table 1. Points Through the $f(x)$ Curve

Points	x	y	Points	x	y
T_1	0	5.8	T_{29}	28	6.65
T_2	1	6.61	T_{30}	27	5.71
T_3	2	8.34	T_{31}	26	5.01
T_4	3	9.34	T_{32}	25	4.14
T_5	4	10.11	T_{33}	24	3.28
T_6	5	10.89	T_{34}	23	2.59
T_7	6	11.29	T_{35}	22	1.89
T_8	7	11.49	T_{36}	21	1.36
T_9	8	11.87	T_{37}	20	1.02
T_{10}	9	12.29	T_{38}	19	0.72
T_{11}	10	12.46	T_{39}	18	0.46
T_{12}	11	12.43	T_{40}	17	0.3
T_{13}	12	12.43	T_{41}	16	0.22
T_{14}	13	12.38	T_{42}	15	0.07
T_{15}	14	12.27	T_{43}	14	0
T_{16}	15	12.09	T_{44}	13	0.1
T_{17}	16	11.83	T_{45}	12	0.24
T_{18}	17	11.63	T_{46}	11	0.22
T_{19}	18	11.39	T_{47}	10	0.32
T_{20}	19	11.15	T_{48}	9	0.56
T_{21}	20	10.83	T_{49}	8	0.83
T_{22}	21	10.43	T_{50}	7	1.14
T_{23}	22	9.96	T_{51}	6	1.51
T_{24}	23	9.45	T_{52}	5	1.91
T_{25}	24	8.89	T_{53}	4	2.39
T_{26}	25	8.28	T_{54}	3	2.96
T_{27}	26	7.69	T_{55}	2	3.78
T_{28}	27	7.18	T_{56}	1	4.92
T_{29}	28	6.65	T_1	0	5.8

The matrix for calculating the function f(x) is obtained by applying the quadratic least squares approach, which looks like this.

$$\begin{bmatrix} 3756718 & 164836 & 7714 \\ 164836 & 7714 & 406 \\ 7714 & 406 & 29 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 21221.54 \\ 913.58 \\ 60.1 \end{bmatrix} \quad (20)$$

The answer derived from the SPLTV above using the Gauss Jordan method, which is solved using Maple software, is:

$a = -0.03095132157$

$b = 0.8353660681$

$c = 6.794823137$

Thus, the function

$$f(x) = -0.03095132157x^2 + 0.8353660681x + 6.794823137 \quad (21)$$

Similarly, the following matrix is produced to determine the values of $a, b,$ and c in $g(x)$.

$$\begin{bmatrix} 3756718 & 164836 & 7714 \\ 164836 & 7714 & 406 \\ 7714 & 406 & 29 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 21551.54 \\ 912.58 \\ 60.1 \end{bmatrix} \quad (22)$$

Solutions a, b, and c for the g(x) function are discovered using the Maple software, with values of

$a = 0.03128608507$

$b = -0.8404537317$

$c = 5.516667408$

A function is created with the variables a, b, and c above

$$g(x) = 0.03128608507x^2 - 0.8404537317x + 5.516667408 \quad (23)$$

GeoGebra is used to project the functions f(x) and g(x), and the result is a closed curve, as illustrated below.

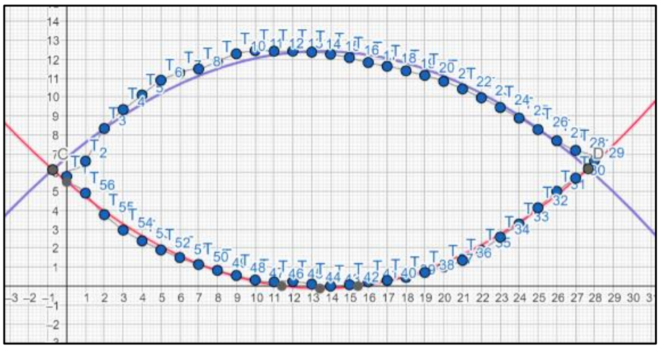

Figure 12. Using Geogebra, a closed curve projection of $f(x)$ and $g(x)$

The S limit for calculating area is derived from Figure 12 using the outcome of Green's Theorem, namely.

$$S = \{(x, y) | -0.742244182301 \leq x \leq 27.6684914389299,\ 0.03128608507x^2 - 0.8404537317x + 5.516667408 \leq y \leq -0.03095132157x^2 + 0.8353660681x + 6.794823137\} \quad (24)$$

It is produced by applying Green's Theorem's conclusion.

$$A(S) = \frac{1}{2} \oint_C xdy - ydx \quad (25)$$

$$= \int_{-0.742244182301}^{27.6684914389299} (0.03128608507x^2 - 0.8404537317x + 5.516667408 - (-0.03095132157x^2 + 0.8353660681x + 6.794823137)) \quad (26)$$

$$= 237.8743825 \quad (27)$$

So, using the application of Green's theorem, the area of a jackfruit leaf is 237.87 Cartesian units or 5946.75 mm^2.

B. Leaf Area with Simpson's Method 1/3

When the length of the leaf is known, it can be divided into even segments ($2n$) to calculate the leaf area using Simpson's 1/3 approach. Given that the leaf's length (p) equals 28 Cartesian units, any n will divide p evenly when multiplied by $2n$. In this investigation, $n = 14$, hence using formula (5),

$$h = \frac{28}{2(14)} \quad (28)$$

$$h = 1 \quad (29)$$

This indicates that each partition has a width of one Cartesian unit.

Therefore, the partition point on the x-axis will be defined as x_i with the values of $i = 0,1,2, \ldots, 2n$. We can write the partition points in the following table based on the outcomes of determining the width of each partition (h).

Table 2. Leaf Partition Points

Point	(x, y)	Point	(x, y)
x_0	$(0, 0)$	x_{15}	$(15, 0)$
x_1	$(1, 0)$	x_{16}	$(16, 0)$
x_2	$(2, 0)$	x_{17}	$(17, 0)$
x_3	$(3, 0)$	x_{18}	$(18, 0)$
x_4	$(4, 0)$	x_{19}	$(19, 0)$
x_5	$(5, 0)$	x_{20}	$(20, 0)$
x_6	$(6, 0)$	x_{21}	$(21, 0)$
x_7	$(7, 0)$	x_{22}	$(22, 0)$
x_8	$(8, 0)$	x_{23}	$(23, 0)$
x_9	$(9, 0)$	x_{24}	$(24, 0)$
x_{10}	$(10, 0)$	x_{25}	$(25, 0)$
x_{11}	$(11, 0)$	x_{26}	$(26, 0)$
x_{12}	$(12, 0)$	x_{27}	$(27, 0)$
x_{13}	$(13, 0)$	x_{28}	$(28, 0)$
x_{14}	$(14, 0)$		

The partition point can also be seen in the image below.

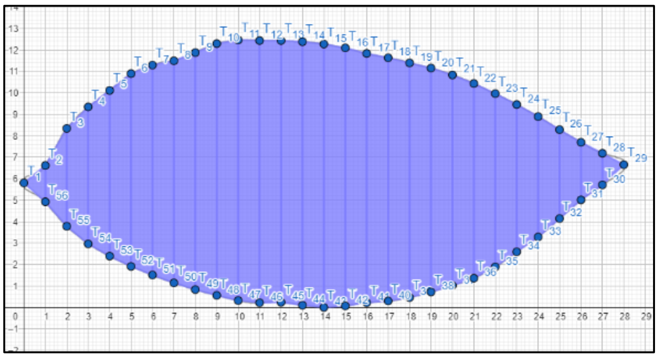

Figure 13. Projection of Partition Points and Partition Area Trapezoid on Cartesian Coordinates Using Geogebra

T_1 to T_{56}, the coordinate positions of the leaf edges and the starting point for estimating leaf area in this study, are acquired from the figure (see also Table 1).

Following the acquisition of leaf partitions at positions x_i, x_i will be drawn to the top and bottom of the leaf, respectively. A point of intersection between the leaf and x_i, which is drawn up to the top of the leaf, is designated u_i in the following. The point of intersection between the leaf and x_i, which is drawn to the bottom of the leaf, is known as v_i from then on. Formula (6) will yield the value of f_i from each partition after determining the values of each u_I and v_i.

Table 3. Acquisition of u_i, v_i, and f_i from each x_i

i	u_i	v_i	f_i	i	u_i	v_i	f_i
0	5.8	5.8	0	15	12.09	0.07	12.02
1	6.61	4.92	1.69	16	11.83	0.22	11.61
2	8.34	3.78	4.56	17	11.63	0.3	11.33
3	9.34	2.96	6.38	18	11.39	0.46	10.93
4	10.11	2.39	7.72	19	11.15	0.72	10.43
5	10.89	1.91	8.98	20	10.83	1.02	9.81
6	11.29	1.51	9.78	21	10.43	1.36	9.07
7	11.49	1.14	10.35	22	9.96	1.89	8.07
8	11.87	0.83	11.04	23	9.45	2.59	6.86
9	12.29	0.56	11.73	24	8.89	3.28	5.61
10	12.46	0.32	12.14	25	8.28	4.14	4.14
11	12.43	0.22	12.21	26	7.69	5.01	2.68
12	12.43	0.24	12.19	27	7.18	5.71	1.47
13	12.38	0.1	12.28	28	10	10	0
14	12.27	0	12.27				

Furthermore, based on formula (7), the area of Jackfruit leaves is obtained as calculated below.

$$Leaf\ Area \approx \frac{h}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \cdots + 2f_{2n-2} + 4f_{2n-1} + f_{2n}) \tag{30}$$

$$\begin{aligned} &\approx \frac{1}{3}(0 + 4(1.69) + 2(4.56) \\ &\quad + 4(6.38) + 2(7.72) \\ &\quad + 4(8.98) + 2(9.78) \\ &\quad + 4(10.35) + 2(11.04) \\ &\quad + 4(11.73) + 2(12.14) \\ &\quad + 4(12.21) + 2(12.19) \\ &\quad + 4(12.28) + 2(12.27) \\ &\quad + 4(12.02) + 2(11.61) \\ &\quad + 4(11.33) + 2(10.93) \\ &\quad + 4(10.43) + 2(9.81) \\ &\quad + 4(9.07) + 2(8.07) \\ &\quad + 4(6.86) + 2(5.61) \\ &\quad + 4(4.14) + 2(2.68) \\ &\quad + 4(1.47) + 0) \end{aligned} \tag{31}$$

$$\approx 237.53 \tag{32}$$

The Simpson method 1/3 was used to calculate the area of jackfruit leaves, which is equal to 237.53 Cartesian units or 5938.25 mm^2.

C. Leaf Area with an Elliptical Approach

Draw an ellipse shape on the leaf image using software in order to compute the leaf area using the elliptical method. by using the software to enter the elliptical equation. The formula $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$ can be used to determine the equation of an ellipse with center (p, q). We'll start by looking for the values (p, q) and (a, b).

$$(p, q) = \left(\frac{x_{max}-x_{min}}{2}, \frac{y_{max}-y_{min}}{2}\right) = (14, 6.23) \tag{33}$$

$$(a, b) = \left(\frac{x_{max}-x_{min}}{2}, \frac{y_{max}-y_{min}}{2}\right) = (14, 6.23) \tag{34}$$

The elliptical equation will be changed to become by substituting the values of a, b, p, and q.

$$\frac{(x-14)^2}{14^2} + \frac{(y-6.23)^2}{6.23^2} = 1 \tag{35}$$

$$\frac{x^2-28x+198}{196} + \frac{y^2-12.46y+38.81}{38.81} = 1 \tag{36}$$

$$38.81x^2 + 196y^2 - 1086.68x - 2442.16y + 15213.52 = 7606.76 \tag{37}$$

After that, enter the equation into the program to obtain an image of an ellipse. The ellipse shown below is obtained.

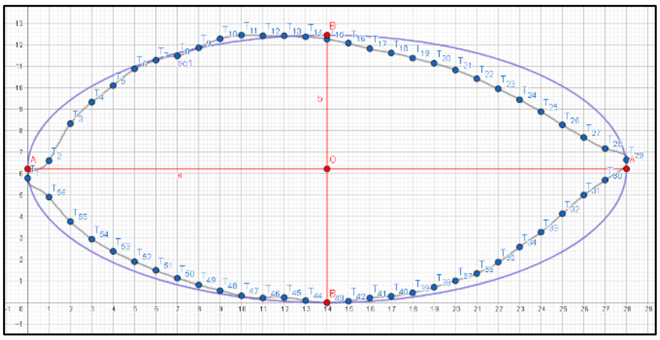

Figure 14. Elliptical curves and a Leaf Curve Approach Projection

Then, calculate the semi-major axis (distance of the farthest point from the ellipse's centre), which is equal to

$$a = \left(\frac{x_{max} - x_{min}}{2}\right) = \left(\frac{28 - 0}{2}\right) = 14$$

and the semi-minor axis (distance of the closest point to the ellipse's centre), which is equal to

$$b = \left(\frac{y_{max} - y_{min}}{2}\right) = \left(\frac{12.46 - 0}{2}\right) = 6.23$$

After that, use the formula Area of Ellipse to determine the area of the ellipse.

$$\text{Area of Ellipse} = \pi \times a \times b \quad (38)$$

$$\text{Area of Ellipse} = \pi \times 14 \times 6.23 = 274.009 \quad (39)$$

As a result, the elliptic technique yielded a leaf area measurement of 274,009 Cartesian units or 6850.225 mm^2.

D. Leaf Area with Millimeter Blocks

The image drawn on millimeter block paper will be labeled below to compute the area in millimeter blocks.

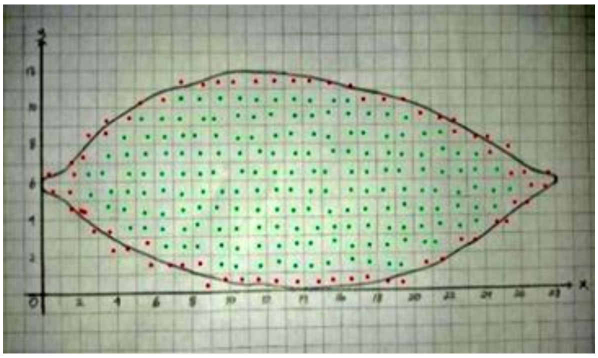

Figure 15. Illustration of jackfruit leaves with the colors red and green identified.

The square in the green image counts as 1, but the square in the red image only counts as half. There are 69 red boxes and 181 green boxes in the image above. There are 215.5 boxes in all. As a result, we may calculate the leaf area in millimeters, equal to 215.5 Cartesian units, or 5387.500 mm^2.

Table 4 shows the jackfruit leaf area determined using the quadratic least squares, Simpson 1/3, elliptic method, and block millimeters.

Table 4. Data of Jackfruit Leaf Area

No	Method	Jackfruit Leaf Area
1.	Quadratic Least Square	5946.75 mm^2
2.	Simpson's Method 1/3	5938.25 mm^2
3.	Elliptical Approach	6850.225 mm^2
4.	Milimeter Block	5387.5 mm^2

The differences from each approach based on the jackfruit leaf area shown in Table 4 are shown in Table 5 below.

Table 5. Difference between Jackfruit Leaf Areas Between Methods (in square millimeters)

Method	QLS	SM 1/3	EA	MB
QLS	0	8.5	903.475	559.25
SM 1/3	8.5	0	911.97	550.75
EA	903.475	911.97	0	1471.725
MB	559.25	550.75	1471.725	0

According to the above table, the difference between the Quadratic Least Squares (QLS) and Simpson's Method 1/3 (SM 1/3) approaches is the smallest, at 8.5 mm^2. The difference between the two procedures is slight because the least square method produces a curve similar to the original curve and Simpson's 1/3. At the same time, the Elliptical Approach (EA) and Millimeter Block (MB) approach approaches have a difference of 1471,725 mm^2 that is the most. The error in Figure 14 is significant due to the distance between the ellipses created by the locations that the curves f(x) and g(x) pass through.

V. CONCLUSIONS AND RECOMMENDATIONS

Based on the research findings, it is possible to determine the area of an object, mainly what is observed, such as jackfruit leaves, by utilizing Green's Theorem and helped by several approximations.

Green's theorem was applied to get the leaf area after determining the function of the quadratic least square approximation in this study. The leaf area was 237.87 Cartesian units or 5946.75 mm^2. From the various approximation techniques employed in this study, it was discovered that using the Quadratic Least Square approximation and the Simpson method 1/3 obtained effective results for the observed object because the functions of the Quadratic Least Square approximation and the Simpson method 1/3 are close to the actual function so that the results obtained are nearly accurate with the original area of the observed object, namely jackfruit leaves, so the results of the approx.

Future researchers are advised to pay close attention to the shape of the leaves to compute the area when conducting the same research with different things (other than jackfruit leaves). It is advised to look for the equation of a leaf's mathematical function using a different strategy by modifying the leaf's shape if the shape of the leaf being used as the study object does not resemble the shape of the jackfruit leaf. To reduce computation errors, it is also necessary to consider several elements that influence the magnitude of inaccuracy.

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