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Trace of the Adjacency Matrix $n \times n$ of the Cycle Graph to the Power of Six to Ten

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ARTICLE INFO	ABSTRACT
Published online: 14 July 2023 **Corresponding Author:** Fitri Aryani	The main aim of this research is to find the formula of the trace of adjacency matrix $n \times n$ from a cycle graph to the power of six to ten. The first step to obtain the general form is finding the general formula of adjacency matrix from a cycle graph to the power of six to ten. Furthermore, the formula of that trace of adjacency matrix obtained and proven by direct proof. We also present an implementation of the formula by an example.

KEYWORDS: cycle graph, adjacency matrix, direct proof, trace matrix

I. INTRODUCTION

Trace matrix is the sum of the entries on the main diagonal. How about trace of a matrix to the power? First, we determine matrix to the power by multiplying the matrix or multiplying the matrix n times. After that, we determine thetrace of that matrix to the power. This means that to calculate a trace of matrix to the power is quite complicated, if the matrix is raised to a large power. It is quite interesting to examine how to find the right general form to calculate the power of trace matrices without calculating powers or matrix multiplication. By simply substituting the matrix entries into the general form, the trace of the matrix to the power is obtained, without having to go through a long process of exponents or matrix multiplication.

According to [1] in 2012, traces of matrices to the power are often discussed in several areas of mathematics, such as Network Analysis, Number Theory, Dynamic Systems, Matrix Theory and Differential Equations. The calculation of trace of matrices to the power has been discussed by [2] in 2015 with a matrix of order 2×2 with a positive integer power. In this article, two general forms of trace of matrices to the power of positive integers are obtained. First, the general form of the trace matrix with a positive integer power for n is even, namely:

$$tr(A^n) = \sum_{r=0}^{n/2} \frac{(-1)^r}{r!} n[n-(r+1)][n-(r+2)] \cdots [up\ to\ r\ terms].\left(det(A)\right)^r \left(tr(A)\right)^{n-2r}$$

Second, the general form of the trace matrix to a positive integer power for n is odd, namely:

$$tr(A^n) = \sum_{r=0}^{n-1/2} \frac{(-1)^r}{r!} n[n-(r+1)][n-(r+2)] \cdots [up\ to\ r\ terms].\left(det(A)\right)^r \left(tr(A)\right)^{n-2r}$$

In 2017, [3] discussed a 2×2 order trace matrix to negative integer power. In this article, there are two general forms of exponential trace matrices, provided that the determinant of the matrix is not zero. First, the general form of the trace matrix is a negative integer for n even and n odd. Aryani and Yulianis [4] discussed about trace of matrices to the power, which discussed the general form of special form trace matrices to negative integer power for n odd and n even. The matrix used is $A = \begin{bmatrix} 0 & a \\ d & 0 \end{bmatrix} \forall a, d \in R$ to have an inverse. Then the general form of the special-shaped trace matrix with the power of negative integers is obtained, namely:

$$tr(A^{-n}) = \begin{cases} 0 & ,n \text{ is odd} \\ \dfrac{2}{(-1)^{\frac{n}{2}}(\det(A))^{\frac{n}{2}}} & ,n \text{ is even} \end{cases}$$

Still regarding rank trace matrices, there are several studies related to the trace matrix to the power with a matrix that varies from a 3 x 3 matrix size which can be seen in articles [5], and [6]. Furthermore, the trace matrix to the power on the complex matrix is carried out by [7] and [8], the special shape matrix in [9] and [10], and the special shape symmetrical toeplitz matrix in [11]. Trace matrices to power can also be performed on the adjacency matrices of an arbitrary graph, which has been done by researchers [12] and [13].

Article [12] conducts research on trace matrices to the power of adjacency matrices of complete graphs. The results obtained in this study are the general form of the trace of $n \times n$ adjacency matrix of complete graphs to the power of even number and odd number, as follows:

$$tr(A^k) = \sum_{r=1}^{n/2} S(k,r)n(n-1)^r(n-2)^{k-2r},$$

k is an even positive number

$$tr(A^k) = \sum_{r=1}^{(n-1)/2} S(k,r)n(n-1)^r(n-2)^{k-2r},$$

k is an odd positive number

$S(k,r)$ is a number that depends on k and r defined by:

$S(k,r) = 1, S\left(k,\frac{k}{2}\right) = 1, S\left(k, k-\frac{1}{2}\right) = \frac{k-1}{2}, S(k,r) = S(k-1,r) + S(k-2,r-1)$

In 2019, research [12] was developed by [13] who examined powers of negative two, negative three, and negative four in the same matrix. The results obtained from this study obtain the general form of the $n \times n$ adjacency matrix of complete graphs to the power of negative two, negative three, and negative four, namely:

$$tr(A^{-2}) = \frac{n\left((n-1)+(n-2)^2\right)}{(n-1)^2} \quad , n \geq 2.$$

The general form of the trace of $n \times n$ adjacency matrix of a complete graph to the negative power of three is:

$$tr(A^{-3}) = \frac{n-2(n-1)(n-2)-(n-2)^3}{(n-1)^3} \quad , n \geq 2.$$

and the general form of the trace of $n \times n$ adjacency matrix of a complete graph to the negative power of four is:

$$tr(A^{-4}) = \frac{n\left((n-1)^2+3(n-1)(n-2)^2+(n-2)^4\right)}{(n-1)^4} \quad , n \geq 2.$$

In addition to complete graphs, there are cycle graphs that can also be represented in a adjacency matrix. A cycle graph is a graph in which each vertex has degree two and a path that starts and ends at the same vertex [14]. So for each cycle graph C_n has n nodes and can be represented as a adjacency matrix C of size $n \times n$. The general form of the $n \times n$ adjacency matrix of the C_n cycle graph is shown in Equation (1) as follows:

$$C_n = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \quad (1)$$

(1)

Equation (1) above has been carried out on trace matrices to power two to five by [15] and the following results are obtained:

$tr(C_n^2) = 2n \quad , n \geq 6$

$tr(C_n^3) = 0 \quad , n \geq 8$

$tr(C_n^4) = 6n \quad , n \geq 10$

$tr(C_n^5) = 0 \quad , n \geq 12$

Based on the description of the results of previous studies regarding the trace matrices to the power, this article will continue research [15] regarding the trace of $n \times n$ adjacency matrices of cycle graphs with positive integer powers six to ten.

II. RESEACH METHOD

This research method uses literature study or literature review. The following describes the steps in this research, namely: if given the adjacency matrix of the cycle graph C_n in Equation (1), then to get the trace matrix to the power six to ten, we determine the matrix to the power six to ten first. After that, the form of the trace matrix is obtained. In more detail, the research steps are given as follows:

1. Given the adjacency matrix of the cycle graph C_n.
2. Prove C_n^6 in a way $C_n^5 \times C_n$.
3. Prove C_n^7 in a way $C_n^6 \times C_n$
4. Prove C_n^8 in a way $C_n^7 \times C_n$
5. Prove C_n^9 in a way $C_n^8 \times C_n$.
6. Prove C_n^{10} in a way $C_n^9 \times C_n$
7. Prove $tr(C_n^6)$, $tr(C_n^7)$, $tr(C_n^8)$, $tr(C_n^9)$, and $tr(C_n^{10})$ using direct proof.

8. Implements the general forms of $tr(C_n^6)$, $tr(C_n^7)$, $tr(C_n^8)$, $tr(C_n^9)$, and $tr(C_n^{10})$ with some related examples.

The proof of the general form of exponents of matrices uses the rules of mathematical induction, which are explained in [15] and [16]. Whereas for proof of the trace matrix using direct evidence that uses the definition of the trace matrix in [17], [18], and [19]. Elaboration of the definitions regarding matrix multiplication and exponents as well as theorems related to matrix exponents and trace matrices are in [20], [21], [22], and [23]

III. RESULTS AND DISCUSSION

The results of this study were obtained after following the steps described in the research method above. There are two general forms obtained, first, the general form for the $n \times n$ neighborhood matrix of circle graphs in Equation (1) to the power of six to ten. Second, the general form of the $n \times n$ trace of adjacency matrix of circle graphs in Equation (1) is raised to the power of six to ten.

A. The General Form of the $n \times n$ Adjacency Matrix From Cycle Graph to the Power of Six

Theorem 1 Given the $n \times n$ adjacency matrix from the cycle graph in Equation (1) then:

$$C_n^6 = \begin{bmatrix} 20 & 0 & 15 & 0 & 6 & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & 6 & 0 & 15 & 0 \\ 0 & 20 & 0 & 15 & 0 & 6 & 0 & 1 & \cdots & 0 & 0 & 1 & 0 & 6 & 0 & 15 \\ 15 & 0 & 20 & 0 & 15 & 0 & 6 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 6 & 0 \\ 0 & 15 & 0 & 20 & 0 & 15 & 0 & 6 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 6 \\ 6 & 0 & 15 & 0 & 20 & 0 & 15 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 6 & 0 & 15 & 0 & 20 & 0 & 15 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 6 & 0 & 15 & 0 & 20 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 & 0 & 15 & 0 & 20 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 20 & 0 & 15 & 0 & 6 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 20 & 0 & 15 & 0 & 6 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 15 & 0 & 20 & 0 & 15 & 0 & 6 \\ 6 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 15 & 0 & 20 & 0 & 15 & 0 \\ 0 & 6 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 6 & 0 & 15 & 0 & 20 & 0 & 15 \\ 15 & 0 & 6 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 6 & 0 & 15 & 0 & 20 & 0 \\ 0 & 15 & 0 & 6 & 0 & 1 & 0 & 0 & \cdots & 1 & 0 & 6 & 0 & 15 & 0 & 20 \end{bmatrix}$$

Proof: Theorem 1 will be proved by direct proof. C_n^5 has been obtained in article [16], so it can be used to prove C_n^6, which is described as follows.

$$C_n^6 = C_n^5 \cdot C_n = \begin{bmatrix} 0 & 10 & 0 & 5 & 0 & \cdots & 1 & 0 & 5 & 0 & 10 \\ 10 & 0 & 10 & 0 & 5 & \cdots & 0 & 1 & 0 & 5 & 0 \\ 0 & 10 & 0 & 10 & 0 & \cdots & 0 & 0 & 1 & 0 & 5 \\ 5 & 0 & 10 & 0 & 10 & \cdots & 0 & 0 & 0 & 1 & 0 \\ 0 & 5 & 0 & 10 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 10 & 0 & 5 & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 10 & 0 & 10 & 0 & 5 \\ 5 & 0 & 1 & 0 & 0 & \cdots & 0 & 10 & 0 & 10 & 0 \\ 0 & 5 & 0 & 1 & 0 & \cdots & 5 & 0 & 10 & 0 & 10 \\ 10 & 0 & 5 & 0 & 1 & \cdots & 0 & 5 & 0 & 10 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The result of multiplying the matrix entries from $C_n^5 \cdot C_n$ can be analyzed as follows:

1. For entries with a value of 20.
   Entries in $a_{i,j}$ with $i = j = 1,2,\cdots,n$
   If you pay attention to the multiplication of the same rows and columns in the matrix above, it can be concluded that there are two values that are worth 10 and the others are zero. So the multiplication of these entries is worth 20.
2. For entries with a value of 15.
   a. Entries in $i = j - 2 \ \ with\, j = 3,4,5,\dots,n$
   b. Entries in $i = j + 2 \ \ with\, j = 1,2,3,\dots,(n-2)$
   c. Entries in $i = j + n - 2 \ with\, j = 1,2$
   d. Entries in $i = j - n + 2 \ with\, j = n-1, n$

   Multiplying the rows and columns in the entries mentioned here, there are two places that are worth, namely 10.1+5.1=15, the rest are multiplication by zero. So the value of these entries is 15.
3. For entries with a value of 6.
   a. Entries in $i = j - 4 \ \ with\, j = 5,6,7,\dots,n$
   b. Entries in $i = j + 4 \ \ with\, j = 1,2,3,\dots,(n-4)$
   c. Entries in $i = j - n + 4 \ with\, j = n-3, n-2, n-1, n$
   d. Entries in $i = j + n - 4 \ with\, j = 1,2,3,4$

   Multiplication of rows and columns in the entries mentioned here, there are two places that have value, namely at 5.1+1.1=6, the rest are multiplication by zero. So the value of these entries is 6.
4. For entries with a value of 1.
   a. Entries in $i = j - 6 \ \ with\, j = 7,8,9,\dots,n$
   b. Entries in $i = j + 6 \ \ with\, j = 1,2,3,\dots,(n-6)$
   c. Entries in $i = j - n + 6 \ with\, j = n-5, n-4, n-3,\dots,n$
   d. Entries in $i = j + n - 6 \ with\, j = 1,2,3,4,5,6$

   Multiplication of rows and columns in the entries mentioned here, there is one place that has a value of 1 .1=1 , the rest are multiplication by zero. So the value of these entries is 1.
5. For the other entries it is 0 namely:
   The multiplication of the rows and columns in the entries mentioned here all have a multiplication of zero. So the value of these entries is 0.

So, it can be concluded that the values in the multiplication entries of the $C_n^5 \cdot C_n$ matrix are 0, 1, 6 , 15 and 20. Based on the proof above, Theorem 1 is proven. ■

B. The General Form of the $n \times n$ Adjacency Matrix From Cycle Graph to the Power of Seven

Theorem 2 Given the $n \times n$ adjacency matrix from the cycle graph in Equation (1) then:

$$C_n^7 = \begin{bmatrix} 0 & 35 & 0 & 21 & 0 & 7 & 0 & 1 & \cdots & 1 & 0 & 7 & 0 & 21 & 0 & 35 \\ 35 & 0 & 35 & 0 & 21 & 0 & 7 & 0 & \cdots & 0 & 1 & 0 & 7 & 0 & 21 & 0 \\ 0 & 35 & 0 & 35 & 0 & 21 & 0 & 7 & \cdots & 0 & 0 & 1 & 0 & 7 & 0 & 21 \\ 21 & 0 & 35 & 0 & 35 & 0 & 21 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 7 & 0 \\ 0 & 21 & 0 & 35 & 0 & 35 & 0 & 21 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 7 \\ 7 & 0 & 21 & 0 & 35 & 0 & 35 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 7 & 0 & 21 & 0 & 35 & 0 & 35 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 7 & 0 & 21 & 0 & 35 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 35 & 0 & 21 & 0 & 7 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 35 & 0 & 35 & 0 & 21 & 0 & 7 \\ 7 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 35 & 0 & 35 & 0 & 21 & 0 \\ 0 & 7 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 21 & 0 & 35 & 0 & 35 & 0 & 21 \\ 21 & 0 & 7 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 21 & 0 & 35 & 0 & 35 & 0 \\ 0 & 21 & 0 & 7 & 0 & 1 & 0 & 0 & \cdots & 7 & 0 & 21 & 0 & 35 & 0 & 35 \\ 35 & 0 & 21 & 0 & 7 & 0 & 1 & 0 & \cdots & 0 & 7 & 0 & 21 & 0 & 35 & 0 \end{bmatrix}$$

Proof : The result of multiplying the matrix entries of $C_n^6 \cdot C_n$ can be analyzed as follows:

1. For entries with a value of 35.
   a. Entries in $i = j - 1$ with $j = 2,3,4,5,\dots,n$

   b. Entries in $i = j + 1$ with $j = 1,2,3,\dots,(n-1)$

   c. Entries in $i = 1$ with $j = n$

   d. Entries in $i = n$ with $j = 1$

   Multiplying the rows and columns in the entries mentioned here, there are two places that are worth, namely 20.1+15.1=35, the rest are multiplication by zero. So the value of these entries is 35.

2. For entries with a value of 21.
   a. Entries in $i = j - 3$ with $j = 4,5,6,7,\dots,n$

   b. Entries in $i = j + 3$ with $j = 1,2,3,\dots,(n-3)$

   c. Entries in $i = j - n + 3$ with $j = n-2, n-1, n$

   d. Entries in $i = j + n - 3$ with $j = 1,2,3$

   Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 15.1+6.1=6, the rest are multiplication by zero. So the value of these entries is 21.

3. For entries with a value of 7.
   a. Entries in $i = j - 5$ with $j = 6,7,8,9,\dots,n$

   b. Entries in $i = j + 5$ with $j = 1,2,3,\dots,(n-5)$

   c. Entries in $i = j - n + 5$ with $j = n-4, n-3,\dots,n$

   d. Entries in $i = j + n - 5$ with $j = 1,2,3,4,5$

   Multiplication of rows and columns in the entries mentioned here, there are two places that are worth, namely 6.1+1.1=7, the rest are multiplication by zero. So the value of these entries is 7.

4. For entries with a value of 1.
   a. Entries in $i = j - 7$ *dengan* $j = 8,9,\dots,n$

   b. Entries in $i = j + 7$ *dengan* $j = 1,2,3,\dots,(n-7)$

   c. Entries in $i = j - n + 7$ *dengan* $j = n-6, n-5,\dots,n$

   d. Entries in $i = j + n - 7$ *dengan* $j = 1,2,3,\dots,7$

   Multiplication of rows and columns in the entries mentioned here, there is one place that has a value of 1 .1=1 , the rest are multiplication by zero. So the value of these entries is 1.

5. For the other entries it is 0 namely:
   The multiplication of the rows and columns in the entries mentioned here all have a multiplication of zero. So the value of these entries is 0.

So, it can be concluded that the values in the multiplication entries of the matrix $C_n^6 \cdot C_n$ are 0, 1, 7 , 21 and 35. Based on the above proof, Theorem 2 is proven. ∎

C. The General Form of the $n \times n$ Adjacency Matrix From Cycle Graph to the Power of Eight

Teorema 3 Given the n×n adjacency matrix from the circle graph in Equation (1) then:

$$C_n^8 = \begin{bmatrix} 70 & 0 & 56 & 0 & 28 & 0 & 8 & 0 & \cdots & 0 & 8 & 0 & 28 & 0 & 56 & 0 \\ 0 & 70 & 0 & 56 & 0 & 28 & 0 & 8 & \cdots & 1 & 0 & 8 & 0 & 28 & 0 & 56 \\ 56 & 0 & 70 & 0 & 56 & 0 & 28 & 0 & \cdots & 0 & 1 & 0 & 8 & 0 & 28 & 0 \\ 0 & 56 & 0 & 70 & 0 & 56 & 0 & 28 & \cdots & 0 & 0 & 1 & 0 & 8 & 0 & 28 \\ 28 & 0 & 56 & 0 & 70 & 0 & 56 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 8 & 0 \\ 0 & 28 & 0 & 56 & 0 & 70 & 0 & 56 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 8 \\ 8 & 0 & 28 & 0 & 56 & 0 & 70 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 8 & 0 & 28 & 0 & 56 & 0 & 70 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 70 & 0 & 56 & 0 & 28 & 0 & 8 \\ 8 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 70 & 0 & 56 & 0 & 28 & 0 \\ 0 & 8 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 56 & 0 & 70 & 0 & 56 & 0 & 28 \\ 28 & 0 & 8 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 56 & 0 & 70 & 0 & 56 & 0 \\ 0 & 28 & 0 & 8 & 0 & 1 & 0 & 0 & \cdots & 28 & 0 & 56 & 0 & 70 & 0 & 56 \\ 56 & 0 & 28 & 0 & 8 & 0 & 1 & 0 & \cdots & 0 & 28 & 0 & 56 & 0 & 70 & 0 \\ 0 & 56 & 0 & 28 & 0 & 8 & 0 & 1 & \cdots & 8 & 0 & 28 & 0 & 56 & 0 & 70 \end{bmatrix}$$

Proof : The result of multiplying the matrix entries from $C_n^7 \cdot C_n$ can be analyzed as follows:

1. For entries with a value of 70.
   Entries in $a_{i,j}$ with $i = j = 1,2,\cdots,n$

   If you pay attention to the multiplication of the same rows and columns in the matrix above, it can be concluded that there are two values worth 35, namely 35.1 + 35.1 and the rest are zero. So the multiplication of these entries is worth 70.

2. For entries with a value of 56.
   a. Entries in $i = j - 2$ with $j = 3,4,5, \dots, n$

   b. Entries in $i = j + 2$ with $j = 1,2,3, \dots, (n-2)$

   c. Entries in $i = j + n - 2$ with $j = 1,2$

   d. Entries in $i = j - n + 2$ with $j = n - 1, n$

   Multiplication of rows and columns in the entries mentioned here, there are two places that are worth, namely 35.1+21.1=56, the rest are multiplication by zero. So the value of these entries is 56.
3. For entries with a value of 28.
   a. Entries in $i = j - 4$ with $j = 5,6,7, \dots, n$

   b. Entries in $i = j + 4$ with $j = 1,2,3, \dots, (n-4)$

   c. Entries in $i = j - n + 4$ with $j = n - 3, n - 2, n - 1, n$

   d. Entries in $i = j + n - 4$ with $j = 1,2,3,4$

   Multiplying the rows and columns in the entries mentioned here, there are two places that are worth, namely 21.1+7.1=28, the rest are multiplication by zero. So the value of these entries is 28.
4. For entries with a value of 8.
   a. Entries in $i = j - 6$ with $j = 7,8,9, \dots, n$

   b. Entries in $i = j + 6$ with $j = 1,2,3, \dots, (n-6)$

   c. Entries in $i = j - n + 6$ with $j = n - 5, n - 4, n - 3, \dots, n$

   d. Entries in $j + n - 6$ with $j = 1,2,3,4,5,6$

   Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 7.1+1.1=8, the rest are multiplication by zero. So the value of these entries is 8.
5. For entries with a value of 8.
   a. Entries in $i = j - 8$ with $j = 9, \dots, n$

   b. Entries in $i = j + 8$ with $j = 1,2,3, \dots, (n-8)$

   c. Entries in $i = j - n + 8$ with $j = n - 7, n - 6, n - 5, \dots, n$

   d. Entries in $i = j + n - 8$ with $j = 1,2,3, \dots, 8$

   Multiplication of rows and columns in the entries mentioned here, there is one place that has a value of 1.1 = 1, the rest are multiplication by zero. So the value of these entries is 1.
6. For the other entries it is 0 namely:
   The multiplication of the rows and columns in the entries mentioned here all have a multiplication of zero. So the value of these entries is 0.

So, it can be concluded that the values in the multiplication entries of the matrix $C_n^7 \cdot C_n$ are 0, 1, 8 , 28, 56, and 70. Based on the above proof, Theorem 3 is proven. ■

D. The General Form of the $n \times n$ Adjacency Matrix From Cycle Graph to the Power of Nine

Theorem 4 Given adjacency matrix $n \times n$ of cycle graph in Equation (1) then:

$$C_n^9 = \begin{bmatrix} 0 & 126 & 0 & 84 & 0 & 36 & 0 & 9 & \cdots & 9 & 0 & 36 & 0 & 84 & 0 & 126 \\ 126 & 0 & 126 & 0 & 84 & 0 & 36 & 0 & \cdots & 0 & 9 & 0 & 36 & 0 & 84 & 0 \\ 0 & 126 & 0 & 126 & 0 & 84 & 0 & 36 & \cdots & 1 & 0 & 9 & 0 & 36 & 0 & 84 \\ 84 & 0 & 126 & 0 & 126 & 0 & 84 & 0 & \cdots & 0 & 1 & 0 & 9 & 0 & 36 & 0 \\ 0 & 84 & 0 & 126 & 0 & 126 & 0 & 84 & \cdots & 0 & 0 & 1 & 0 & 9 & 0 & 36 \\ 36 & 0 & 84 & 0 & 126 & 0 & 126 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 9 & 0 \\ 0 & 36 & 0 & 84 & 0 & 126 & 0 & 126 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 9 \\ 9 & 0 & 36 & 0 & 84 & 0 & 126 & 0 & & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 9 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 126 & 0 & 84 & 0 & 36 & 0 \\ 0 & 9 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 126 & 0 & 126 & 0 & 84 & 0 & 36 \\ 36 & 0 & 9 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 126 & 0 & 126 & 0 & 84 & 0 \\ 0 & 36 & 0 & 9 & 0 & 1 & 0 & 0 & \cdots & 84 & 0 & 126 & 0 & 126 & 0 & 84 \\ 84 & 0 & 36 & 0 & 9 & 0 & 1 & 0 & \cdots & 0 & 84 & 0 & 126 & 0 & 126 & 0 \\ 0 & 84 & 0 & 36 & 0 & 9 & 0 & 1 & \cdots & 36 & 0 & 84 & 0 & 126 & 0 & 126 \\ 126 & 0 & 84 & 0 & 36 & 0 & 9 & 0 & \cdots & 0 & 36 & 0 & 84 & 0 & 126 & 0 \end{bmatrix}$$

Proof: The result of multiplying the matrix entries from $C_n^8 \cdot C_n$ can be analyzed as follows:

1. For entries with a value of 126.
   a. Entries in $i = j - 1$ with $j = 2,3,4,5, \dots, n$

   b. Entries in $i = j + 1$ with $j = 1,2,3, \dots, (n-1)$

   c. Entries in $i = 1$ with $j = n$

   d. Entries in $i = n$ with $j = 1$

   Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 70 .1+56 .1=126, the rest are multiplication by zero. So the value of these entries is 126.
2. For entries with a value of 84.
   a. Entries in $i = j - 3$ with $j = 4,5,6,7, \dots, n$

   b. Entries in $i = j + 3$ with $j = 1,2,3, \dots, (n-3)$

   c. Entries in $i = j - n + 3$ with $j = n - 2, n - 1, n$

   d. Entries in $i = j + n - 3$ with $j = 1,2,3$

   Multiplying the rows and columns in the entries mentioned here, there are two places that are worth, namely 56 .1+28 .1=84, the rest are multiplication by zero. So the value of these entries is 84.
3. For entries with a value of 36.
   a. Entries in $i = j - 5$ with $j = 6,7,8,9, \dots, n$

   b. Entries in $i = j + 5$ with $j = 1,2,3, \dots, (n-5)$

   c. Entries in $i = j - n + 5$ with $j = n - 4, n - 3, \dots, n$

   d. Entries in $i = j + n - 5$ with $j = 1,2,3,4,5$

Multiplying the rows and columns in the entries mentioned here, there are two places that are worth, namely 28 .1+8 .1=36, the rest are multiplication by zero. So the value of these entries is 36.

4. For entries with a value of 9.
   a. Entries in $i = j - 7$ with $j = 8,9,\ldots,n$

   b. Entries in $i = j + 7$ with $j = 1,2,3,\ldots,(n-7)$

   c. Entries in $i = j - n + 7$ with $j = n-6, n-5,\ldots,n$

   d. Entries in $i = j + n - 7$ with $j = 1,2,3,\ldots,7$

   Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 8.1+1.1=9, the rest are multiplication by zero. So the value of these entries is 9.
5. For entries with a value of 1.
   a. Entries in $i = j - 9$ with $j = 10,11,,\ldots,n$

   b. Entries in $i = j + 9$ with $j = 1,2,3,\ldots,(n-9)$

   c. Entries in $i = j - n + 9$ with $j = n-8, n-7,\ldots,n$

   d. Entries in $i = j + n - 9$ with $j = 1,2,3,\ldots,9$

   Multiplication of rows and columns in the entries mentioned here, there is one place that has a value of 1 .1 = 1, the rest are multiplication by zero. So the value of these entries is 1.
6. For the other entries it is 0 namely:
   The multiplication of the rows and columns in the entries mentioned here all have a multiplication of zero. So the value of these entries is 0.

So, it can be concluded that the values in the multiplication entries of the matrix $C_n^9 \cdot C_n$ are 0, 1, 9, 36, 9, and 126. Based on the above proof, Theorem 4 is proven. ■

E. The General Form of the $n \times n$ Adjacency Matrix From Cycle Graph to the Power of Ten

Teorema 5 Given adjacency matrix $n \times n$ of cycle graph in Equation (1) then:

$$C_n^{10} = \begin{bmatrix}
252 & 0 & 210 & 0 & 120 & 0 & 45 & 0 & \cdots & 0 & 45 & 0 & 120 & 0 & 210 & 0 \\
0 & 252 & 0 & 210 & 0 & 120 & 0 & 45 & \cdots & 10 & 0 & 45 & 0 & 120 & 0 & 210 \\
210 & 0 & 252 & 0 & 210 & 0 & 120 & 0 & \cdots & 0 & 10 & 0 & 8 & 0 & 28 & 0 \\
0 & 210 & 0 & 252 & 0 & 210 & 0 & 120 & \cdots & 1 & 0 & 10 & 0 & 8 & 0 & 28 \\
120 & 0 & 210 & 0 & 252 & 0 & 210 & 0 & \cdots & 0 & 1 & 0 & 10 & 0 & 8 & 0 \\
0 & 120 & 0 & 210 & 0 & 252 & 0 & 210 & \cdots & 0 & 0 & 1 & 0 & 10 & 0 & 8 \\
45 & 0 & 120 & 0 & 210 & 0 & 252 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 10 & 0 \\
0 & 45 & 0 & 120 & 0 & 210 & 0 & 252 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 10 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 10 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 252 & 0 & 210 & 0 & 120 & 0 & 45 \\
45 & 0 & 10 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 252 & 0 & 210 & 0 & 120 & 0 \\
0 & 45 & 0 & 10 & 0 & 1 & 0 & 0 & \cdots & 210 & 0 & 252 & 0 & 210 & 0 & 120 \\
120 & 0 & 45 & 0 & 10 & 0 & 1 & 0 & \cdots & 0 & 210 & 0 & 252 & 0 & 210 & 0 \\
0 & 120 & 0 & 45 & 0 & 10 & 0 & 1 & \cdots & 45 & 0 & 210 & 0 & 252 & 0 & 210 \\
210 & 0 & 120 & 0 & 45 & 0 & 10 & 0 & \cdots & 0 & 45 & 0 & 210 & 0 & 252 & 0 \\
0 & 210 & 0 & 120 & 0 & 45 & 0 & 10 & \cdots & 9 & 0 & 45 & 0 & 210 & 0 & 252
\end{bmatrix}$$

Proof: The result of multiplying the matrix entries from $C_n^9 \cdot C_n$ can be analyzed as follows:

1. For entries with a value of 252.
   Entries in $a_{i,j}$ with $i = j = 1,2,\cdots,n$

   If you pay attention to the multiplication of the same rows and columns in the matrix above, it can be concluded that there are two values worth 126, namely 126 .1.+126 .1=252 and the rest are zero. So the multiplication of these entries is 252.
2. For entries with a value of 210.
   a. Entries in $i = j - 2$ with $j = 3,4,5,\ldots,n$

   b. Entries in $i = j + 2$ with $j = 1,2,3,\ldots,(n-2)$

   c. Entries in $i = j + n - 2$ with $j = 1,2$

   d. Entries in $i = j - n + 2$ with $j = n-1, n$

   Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 126 .1.+84 .1=210, the rest are multiplication by zero. So the value of these entries is 210.
3. For entries with a value of 120.
   a. Entries in $i = j - 4$ with $j = 5,6,7,\ldots,n$

   b. Entries in $i = j + 4$ with $j = 1,2,3,\ldots,(n-4)$

   c. Entries in $i = j - n + 4$ with $j = n-3, n-2, n-1, n$

   d. Entries in $i = j + n - 4$ with $j = 1,2,3,4$

   Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 84 .1.+36 .1=120, the rest are multiplication by zero. So the value of these entries is 120.
4. For entries with a value of 45.
   a. Entries in $i = j - 6$ with $j = 7,8,9,\ldots,n$

   b. Entries in $i = j + 6$ with $j = 1,2,3,\ldots,(n-6)$

   c. Entries in $i = j - n + 6$ with $j = n-5, n-4, n-3,\ldots,n$

   d. Entries in $i = j + n - 6$ with $j = 1,2,3,4,5,6$

   Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 36 .1.+9.1=45, the rest are multiplication by zero. So the value of these entries is 45.
5. For entries with a value of 10.
   a. Entries in $i = j - 8$ with $j = 9,\ldots,n$

b. Entries in $i = j + 8$ with $j = 1,2,3,\dots,(n-8)$

c. Entries in $= j - n + 8$ with $j = n-7, n-6, n-5, \dots, n$

d. Entries in $i = j + n - 8$ with $j = 1,2,3,\dots,8$

Multiplying the rows and columns in the entries mentioned here, there are two places that have a value of 9 .1.+1 .1=10, the rest are multiplication by zero. So the value of these entries is 10.

6. For entries with a value of 1.

a. Entries in $i = j - 10$ with $j = 11,12,\dots,n$

b. Entries in $i = j + 10$ with $j = 1,2,3,\dots,(n-10)$

c. Entries in $i = j - n + 10$ with $j = n-9, n-8, n-7, \dots, n$

d.

e. Entries in $i = j + n - 10$ with $j = 1,2,3,\dots,10$

Multiplication of rows and columns in the entries mentioned here, there is one place that has a value of 1 .1 = 1, the rest are multiplication by zero. So the value of these entries is 1.

7. For the other entries it is 0 namely:
The multiplication of the rows and columns in the entries mentioned here all have a multiplication of zero. So the value of these entries is 0.

So, it can be concluded that the values in the multiplication entries of the $C_n^9 \cdot C_n$ matrix are 0, 1, 10 , 45, 120, 210, and 252. Based on the proof above, Theorem 5 is proven. ■

F. *General Form of Trace of Adjacency Matrix $n \times n$ of Cycle Graph*

The result of the next research is by using the results from Theorem 1 to Theorem 5, then trace of adjacency matrix $n \times n$ of the cycle graph can be obtained and presented in Corollary 1 to Corollary 5.

Corollary 1 Given adjacency matrix of cycle graph expressed in Equation (1) then:

$tr(A_n^6) = 20\,n, \qquad n \geq 6$

Proof: By using Theorem 1 and definition *trace* of matrix, then obtained:

$$tr(C_n^6) = tr\begin{bmatrix}
20 & 0 & 15 & 0 & 6 & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & 6 & 0 & 15 & 0 \\
0 & 20 & 0 & 15 & 0 & 6 & 0 & 1 & \cdots & 0 & 0 & 1 & 0 & 6 & 0 & 15 \\
15 & 0 & 20 & 0 & 15 & 0 & 6 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 6 & 0 \\
0 & 15 & 0 & 20 & 0 & 15 & 0 & 6 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 6 \\
6 & 0 & 15 & 0 & 20 & 0 & 15 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 6 & 0 & 15 & 0 & 20 & 0 & 15 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 6 & 0 & 15 & 0 & 20 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 6 & 0 & 15 & 0 & 20 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 20 & 0 & 15 & 0 & 6 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 20 & 0 & 15 & 0 & 6 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 15 & 0 & 20 & 0 & 15 & 0 & 6 \\
6 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 15 & 0 & 20 & 0 & 15 & 0 \\
0 & 6 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 6 & 0 & 15 & 0 & 20 & 0 & 15 \\
15 & 0 & 6 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 6 & 0 & 15 & 0 & 20 & 0 \\
0 & 15 & 0 & 6 & 0 & 1 & 0 & 0 & \cdots & 1 & 0 & 6 & 0 & 15 & 0 & 20
\end{bmatrix}$$

$$= \underbrace{20 + 20 + \cdots + 20}_{n}$$

$$= 20\,n$$

Corollary 2 Given adjacency matrix of cycle graph which expressed in Equation (1) then:

$tr(A_n^7) = 0, \qquad n \geq 6$

Proof: By Using Theorem 2 and definition of trace matrix, then obtained:

$$tr(C_n^7) = tr\begin{bmatrix}
0 & 35 & 0 & 21 & 0 & 7 & 0 & 1 & \cdots & 1 & 0 & 7 & 0 & 21 & 0 & 35 \\
35 & 0 & 35 & 0 & 21 & 0 & 7 & 0 & \cdots & 0 & 1 & 0 & 7 & 0 & 21 & 0 \\
0 & 35 & 0 & 35 & 0 & 21 & 0 & 7 & \cdots & 0 & 0 & 1 & 0 & 7 & 0 & 21 \\
21 & 0 & 35 & 0 & 35 & 0 & 21 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 7 & 0 \\
0 & 21 & 0 & 35 & 0 & 35 & 0 & 21 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 7 \\
7 & 0 & 21 & 0 & 35 & 0 & 35 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 7 & 0 & 21 & 0 & 35 & 0 & 35 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 7 & 0 & 21 & 0 & 35 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 35 & 0 & 21 & 0 & 7 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 35 & 0 & 35 & 0 & 21 & 0 & 7 \\
7 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 35 & 0 & 35 & 0 & 21 & 0 \\
0 & 7 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 21 & 0 & 35 & 0 & 35 & 0 & 21 \\
21 & 0 & 7 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 21 & 0 & 35 & 0 & 35 & 0 \\
0 & 21 & 0 & 7 & 0 & 1 & 0 & 0 & \cdots & 7 & 0 & 21 & 0 & 35 & 0 & 35 \\
35 & 0 & 21 & 0 & 7 & 0 & 1 & 0 & \cdots & 0 & 7 & 0 & 21 & 0 & 35 & 0
\end{bmatrix}$$

$$= \underbrace{0 + 0 + \cdots + 0}_{n}$$

$$= 0$$

Corollary 3 Given the adjacency matrix of the cycle graph expressed in Equation (1) then:

$tr(A_n^7) = 70\,n, \qquad n \geq 6$

Proof: By Using Theorem 3 and definition of trace matrix, then obtained:

$$tr(C_n^8) = tr\begin{bmatrix}
70 & 0 & 56 & 0 & 28 & 0 & 8 & 0 & \cdots & 0 & 8 & 0 & 28 & 0 & 56 & 0 \\
0 & 70 & 0 & 56 & 0 & 28 & 0 & 8 & \cdots & 1 & 0 & 8 & 0 & 28 & 0 & 56 \\
56 & 0 & 70 & 0 & 56 & 0 & 28 & 0 & \cdots & 0 & 1 & 0 & 8 & 0 & 28 & 0 \\
0 & 56 & 0 & 70 & 0 & 56 & 0 & 28 & \cdots & 0 & 0 & 1 & 0 & 8 & 0 & 28 \\
28 & 0 & 56 & 0 & 70 & 0 & 56 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 8 & 0 \\
0 & 28 & 0 & 56 & 0 & 70 & 0 & 56 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 8 \\
8 & 0 & 28 & 0 & 56 & 0 & 70 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 8 & 0 & 28 & 0 & 56 & 0 & 70 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 70 & 0 & 56 & 0 & 28 & 0 & 8 \\
8 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 70 & 0 & 56 & 0 & 28 & 0 \\
0 & 8 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 56 & 0 & 70 & 0 & 56 & 0 & 28 \\
28 & 0 & 8 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 56 & 0 & 70 & 0 & 56 & 0 \\
0 & 28 & 0 & 8 & 0 & 1 & 0 & 0 & \cdots & 28 & 0 & 56 & 0 & 70 & 0 & 56 \\
56 & 0 & 28 & 0 & 8 & 0 & 1 & 0 & \cdots & 0 & 28 & 0 & 56 & 0 & 70 & 0 \\
0 & 56 & 0 & 28 & 0 & 8 & 0 & 1 & \cdots & 8 & 0 & 28 & 0 & 56 & 0 & 70
\end{bmatrix}$$

$$= \underbrace{70 + 70 + \cdots + 70}_{n}$$

$$= 70\,n$$

Corollary 4 Given the adjacency matrix of the cycle graph expressed in Equation (1) then:

$tr(A_n^7) = 0, \qquad n \geq 6$

Proof: By Using Theorem 4 and definition of trace matrix, then obtained:

$$tr(C_n^9) = tr\begin{bmatrix} 0 & 126 & 0 & 84 & 0 & 36 & 0 & 9 & \cdots & 9 & 0 & 36 & 0 & 84 & 0 & 126 \\ 126 & 0 & 126 & 0 & 84 & 0 & 36 & 0 & \cdots & 0 & 9 & 0 & 36 & 0 & 84 & 0 \\ 0 & 126 & 0 & 126 & 0 & 84 & 0 & 36 & \cdots & 1 & 0 & 9 & 0 & 36 & 0 & 84 \\ 84 & 0 & 126 & 0 & 126 & 0 & 84 & 0 & \cdots & 0 & 1 & 0 & 9 & 0 & 36 & 0 \\ 0 & 84 & 0 & 126 & 0 & 126 & 0 & 84 & \cdots & 0 & 0 & 1 & 0 & 9 & 0 & 36 \\ 36 & 0 & 84 & 0 & 126 & 0 & 126 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 9 & 0 \\ 0 & 36 & 0 & 84 & 0 & 126 & 0 & 126 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 9 \\ 9 & 0 & 36 & 0 & 84 & 0 & 126 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 9 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 126 & 0 & 84 & 0 & 9 & 0 \\ 0 & 9 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 126 & 0 & 126 & 0 & 84 & 0 & 9 \\ 36 & 0 & 9 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 126 & 0 & 126 & 0 & 84 & 0 \\ 0 & 36 & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 84 & 0 & 126 & 0 & 126 & 0 & 84 \\ 84 & 0 & 36 & 0 & 9 & 0 & 1 & 0 & \cdots & 0 & 84 & 0 & 126 & 0 & 126 & 0 \\ 0 & 84 & 0 & 36 & 0 & 9 & 0 & 1 & \cdots & 9 & 0 & 84 & 0 & 126 & 0 & 126 \\ 126 & 0 & 84 & 0 & 36 & 0 & 9 & 0 & \cdots & 0 & 9 & 0 & 84 & 0 & 126 & 0 \end{bmatrix}$$

$$= \underbrace{0 + 0 + \cdots + 0}_{n}$$

$$= 0$$

Corollary 5 Given the adjacency matrix of the cycle graph expressed in Equation (1) then:

$tr(A_n^7) = 252\ n, \qquad n \geq 6$

Proof: By Using Theorem 5 and definition of trace matrix, then obtained:

$$tr(C_n^{10}) = tr\begin{bmatrix} 252 & 0 & 210 & 0 & 120 & 0 & 45 & 0 & \cdots & 0 & 45 & 0 & 120 & 0 & 210 & 0 \\ 0 & 252 & 0 & 210 & 0 & 120 & 0 & 45 & \cdots & 10 & 0 & 45 & 0 & 120 & 0 & 210 \\ 210 & 0 & 252 & 0 & 210 & 0 & 120 & 0 & \cdots & 0 & 10 & 0 & 8 & 0 & 28 & 0 \\ 0 & 210 & 0 & 252 & 0 & 210 & 0 & 120 & \cdots & 1 & 0 & 10 & 0 & 8 & 0 & 28 \\ 120 & 0 & 210 & 0 & 252 & 0 & 210 & 0 & \cdots & 0 & 1 & 0 & 10 & 0 & 8 & 0 \\ 0 & 120 & 0 & 210 & 0 & 252 & 0 & 210 & \cdots & 0 & 0 & 1 & 0 & 10 & 0 & 8 \\ 45 & 0 & 120 & 0 & 210 & 0 & 252 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 10 & 0 \\ 0 & 45 & 0 & 120 & 0 & 210 & 0 & 252 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 10 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 10 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 252 & 0 & 210 & 0 & 120 & 0 & 45 \\ 45 & 0 & 10 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 252 & 0 & 210 & 0 & 120 & 0 \\ 0 & 45 & 0 & 10 & 0 & 1 & 0 & 0 & \cdots & 210 & 0 & 252 & 0 & 210 & 0 & 120 \\ 120 & 0 & 45 & 0 & 10 & 0 & 1 & 0 & \cdots & 0 & 210 & 0 & 252 & 0 & 210 & 0 \\ 0 & 120 & 0 & 45 & 0 & 10 & 0 & 1 & \cdots & 45 & 0 & 210 & 0 & 252 & 0 & 210 \\ 210 & 0 & 120 & 0 & 45 & 0 & 10 & 0 & \cdots & 0 & 45 & 0 & 210 & 0 & 252 & 0 \\ 0 & 210 & 0 & 120 & 0 & 45 & 0 & 10 & \cdots & 9 & 0 & 45 & 0 & 210 & 0 & 252 \end{bmatrix}$$

$$= \underbrace{252 + 252 + \cdots + 252}_{n}$$

$$= 252\ n$$

IV. CONCLUSION

Based on the discussion that has been explained above, it is concluded that the general form of exponents of the n×n adjacency matrix in Equation (1) to the power of six to ten is found in Theorem 1 to Theorem 5. In these theorems it can be explained that the greater the power of the matrix is determined, the more matrix entries that do not have a value of 0. Meanwhile, the general form of the neighbor matrix n×n to the power of six to ten is obtained in Corollary 1 to Corollary 5. The trace values obtained can be generalized that for odd integer powers the trace value the matrix is 0, while for an even integer power, the value of the trace matrix is not zero, and the larger the power of the matrix is determined, the greater the value of the trace matrix.

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