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Strongly Binary G*-Closed Set in Binary Topological Spaces

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ARTICLE INFO	ABSTRACT
Published online: **22 July 2023**	In this paper, we introduce the concept of strongly binary g^⋆-closed sets in binary topological space and we investigate the group of structure of the set of all strongly binary g^⋆-closed sets.
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I. INTRODUCTION AND PRELIMINARIES

In 1970 Levine [7] gives the concept and properties of generalized closed (briefly g-closed) sets and the complement of g-closed set is said to be g-open set. Njasted [16] introduced and studied the concept of α-sets. Later these sets are called as α-open sets in 1983. Mashhours et.al [10] introduced and studied the concept of α-closed sets, α-closure of set, α-continuous functions, α-open functions and α-closed functions in topological spaces. Maki et.al [8, 9] introduced and studied generalized α-closed sets and α-generalized closed sets. In 2011, S.Nithyanantha Jothi and P.Thangavelu [11] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A, B) where $A{-}\mathrm{subseteq}X$ and $B{-}\mathrm{subseteq}Y$. In this paper, we introduce the concept of strongly binary $g^{\star}$-closed sets in binary topological space and we investigate the group of structure of the set of all strongly binary $g^{\star}$-closed sets.

Throughout this paper, (X, Y) denote binary topological spaces $(X, Y, \mathcal{M})$.

Let X and Y be any two nonempty sets. A binary topology [11] from X to Y is a binary structure $\mathcal{M} \subseteq \mathbb{P}(X) \times \mathbb{P}(Y)$ that satisfies the axioms namely

1. (ϕ, ϕ) and $(X, Y) \in \mathcal{M}$,
2. $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and 3. If $\{(A_\alpha, B_\alpha): \alpha \in \delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_{\alpha\in\delta} A_\alpha, \bigcup_{\alpha\in\delta} B_\alpha) \in \mathcal{M}$.

If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$. If $Y = X$ then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space.

Definition 1.1 [11] Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in \mathbb{P}(X) \times \mathbb{P}(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

Definition 1.2 [11] Let $(X, Y, \mathcal{M})$ be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary closed in $(X, Y, \mathcal{M})$ if $(X\backslash A, Y\backslash B) \in \mathcal{M}$.

Proposition 1.3 [11] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \cap \{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap \{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.

Proposition 1.4 [11] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \cup \{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and

$(A,B)^{2*} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A,B)\}$.

Definition 1.5 [11] The ordered pair $((A,B)^{1*}, (A,B)^{2*})$ is called the binary closure of (A,B), denoted by $\text{b-cl}(A,B)$ in the binary space $(X,Y,\mathcal{M})$ where $(A,B) \subseteq (X,Y)$.

Definition 1.6 [11] The ordered pair $((A,B)^{1*}, (A,B)^{2*})$ defined in proposition 1.4 is called the binary interior of of (A,B), denoted by $\text{b-int}(A,B)$. Here $((A,B)^{1*}, (A,B)^{2*})$ is binary open and $((A,B)^{1*}, (A,B)^{2*}) \subseteq (A,B)$.

Definition 1.7 [11] Let $(X,Y,\mathcal{M})$ be a binary topological space and let $(x,y) \subseteq (X,Y)$. The binary open set (A,B) is said to be a binary neighbourhood of (x,y) if $x \in A$ and $y \in B$.

Proposition 1.8 [11] Let $(A,B) \subseteq (C,D) \subseteq (X,Y)$ and $(X,Y,\mathcal{M})$ be a binary topological space. Then, the following statements hold:

1. $\text{b-int}(A,B) \subseteq (A,B)$.
2. If (A,B) is binary open, then $\text{b-int}(A,B) = (A,B)$.
3. $\text{b-int}(A,B) \subseteq \text{b-int}(C,D)$.
4. $\text{b-int}(\text{b-int}(A,B)) = \text{b-int}(A,B)$.
5. $(A,B) \subseteq \text{b-cl}(A,B)$.
6. If (A,B) is binary closed, then $\text{b-cl}(A,B) = (A,B)$.
7. $\text{b-cl}(A,B) \subseteq \text{b-cl}(C,D)$.
8. $\text{b-cl}(\text{b-cl}(A,B)) = \text{b-cl}(A,B)$.

Definition 1.9 A subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is called

1. a binary semi open set [15] if $(A,B) \subseteq \text{b-cl}(\text{b-int}(A,B))$.
2. a binary pre open set [5] if $(A,B) \subseteq \text{b-int}(\text{b-cl}(A,B))$,
3. a binary regular open set [14] if $(A,B) = \text{b-int}(\text{b-cl}(A,B))$.

Definition 1.10 A subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is called

1. a binary g-closed set [12] if $\text{b-cl}(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.
2. a binary gs-closed set [17] if $\text{b-scl}(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.
3. a binary sg-closed set [17] if $\text{b-scl}(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary semi open.
4. a binary gr-closed set [14] if $\text{b-rcl}(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.
5. a binary gsp-closed set [6] if $\text{b-}\beta\text{cl}(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.

Definition 1.11 [4] Let (A,B) be a subset of a binary topological space (X,Y). Then (A,B) is called a binary g^*-closed set if $\text{b-cl}(A,B) \subseteq (P,Q)$ whenever $(A,B) \subseteq (P,Q)$ and (P,Q) is binary g-open in (X,Y).

Definition 1.12 [2] A subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is called a binary α-open if $(A,B) \subseteq \text{b-int}(\text{b-cl}(\text{b-int}(A,B)))$.

Definition 1.13 [1] A subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is called a binary αg-closed if $\text{b-}\alpha\text{cl}(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.

II. STRONGLY BINARY G*-CLOSED SETS

Definition 2.1 Let $(X,Y,\mathcal{M})$ be a binary topological space and (A,B) be its subset, then (A,B) is strongly binary g^*-closed set if $\text{b-cl}(\text{b-int}(A,B)) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary g-open.

Theorem 2.2 Every binary closed set is strongly binary g^*-closed but not conversely.
Proof. The proof is immediate from the definition of binary closed set.

Example 2.3 Let $X = \{1,2\}$, $Y = \{a,b\}$ and $\mathcal{M} = \{(\phi,\phi), (\phi,\{a\}), (\{1\},\phi), (\{1\},\{a\}), (\{1\},\{b\}), (\{1\},Y), (\{2\},\{a\}), (X,\{a\}), (X,Y)\}$. Then the set $(\{1\},\{a\})$ is strongly binary g^*-closed set but not a binary closed in (X,Y).

Theorem 2.4 If a subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is binary g^*-closed then it is strongly binary g^*-closed in (X,Y) but not conversely.
Proof. Suppose (A,B) is binary g^*-closed in (X,Y). Let (G,H) be an binary open set containing (A,B) in (X,Y). Then (G,H) contains $\text{b-cl}(A,B)$. Now $(G,H) \supseteq \text{b-cl}(A,B) \supseteq \text{b-cl}(\text{b-int}(A,B))$. Thus (A,B) is strongly binary g^*-closed in (X,Y).

Example 2.5 Let $X = \{a,b,c\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi,\phi), (\{a\},\{1\}), (\{b\},\phi), (\{b\},\{2\}), (\{a,b\},\{1\}), (\{a,b\},Y), (X,Y)\}$. Then the set $(\{a\},\{2\})$ is strongly binary g^*-closed but not binary g^*-closed set.

Theorem 2.6 If (A,B) is subset of a binary topological space (X,Y) is binary open and strongly binary g^*-closed then it is binary closed.
Proof. Suppose a subset (A,B) of (X,Y) is both binary open and strongly binary g^*-closed. Now $(A,B) \supseteq \text{b-cl}(\text{b-int}(A,B)) \supseteq \text{b-cl}(A,B)$. Therefore $(A,B) \supseteq \text{b-cl}(A,B)$. Since $\text{b-cl}(A,B) \supseteq (A,B)$. We have $(A,B) \supseteq \text{b-cl}(A,B)$. Thus (A,B) is binary closed in (X,Y).

Corollary 2.7 If (A,B) is both binary open and strongly binary g^*-closed in (X,Y) then it is both binary regular open and binary regular closed in (X,Y).
Proof. As (A,B) is binary open $(A,B) = \text{b-int}(A,B) = \text{b-int}(\text{b-cl}(A,B))$, since (A,B) is binary closed. Thus (A,B) is binary regular open. Again (A,B) is binary open in (X,Y), b-

$cl(b\text{-}int(A,B)) = b\text{-}cl(A,B)$. As (A,B) is binary closed $b\text{-}cl(b\text{-}int(A,B)) = (A,B)$. Thus (A,B) is binary regular closed.

Corollary 2.8 If (A,B) is both binary open and strongly binary g^*-closed then it is binary rg-closed.

Theorem 2.9 If a subset (A,B) of a binary topological space (X,Y) is both strongly binary g^*-closed and binary semi open then it is binary g^*-closed.

Proof. Suppose (A,B) is both strongly binary g^*-closed and binary semi open in (X,Y). Let (G,H) be an binary open set containing (A,B). As (A,B) is strongly binary g^*-closed, $(G,H) \supseteq b\text{-}cl(b\text{-}int(A,B))$. Now $(G,H) \supseteq b\text{-}cl(A,B)$. Since (A,B) is binary semi open. Thus (A,B) is binary g^*-closed in (X,Y).

Corollary 2.10 If a subset (A,B) of a binary topological space (X,Y) is both strongly binary g^*-closed and binary open then it is binary g^*-closed set.

Proof. As every binary open set is binary semi open by the above theorem the proof follows.

Theorem 2.11 A set (A,B) is strongly binary g^*-closed iff $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ contains no non empty binary closed set.

Proof. **Necessary part:** Suppose that (E,F) is non empty binary closed subset of $b\text{-}cl(b\text{-}int(A,B))$. Now $(E,F) \subseteq b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ implies $(E,F) \subseteq b\text{-}cl(b\text{-}int(A,B)) \cap (A,B)^c$, since $b\text{-}cl(b\text{-}int(A,B)) - (A,B) = b\text{-}cl(b\text{-}int(A,B)) \cap (A,B)^c$. Thus $(E,F) \subseteq b\text{-}cl(b\text{-}int(A,B))$. Now $(E,F) \subseteq (A,B)^c$ implies $(A,B) \subseteq (A,B)^c$. Here $(E,F)^c$ is binary g-open and (A,B) is strongly binary g^*-closed, we have $b\text{-}cl(b\text{-}int(A,B)) \subseteq (E,F)^c$. Thus $(E,F) \subseteq (b\text{-}cl(b\text{-}int(A,B)))^c$. Hence $(E,F) \subseteq (b\text{-}cl(b\text{-}int(A,B))) \cap (b\text{-}cl(b\text{-}int(A,B)))^c = (\phi,\phi)$. Therefore $(E,F) = (\phi,\phi) \Rightarrow b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ contains no non empty binary closed sets.

Sufficient part: Let $(A,B) \subseteq (G,H)$, (G,H) is binary g-open. Suppose that $b\text{-}cl(b\text{-}int(A,B))$ is not contained in (G,H) then $(b\text{-}cl(b\text{-}int(A,B)))^c$ is a non empty binary closed set $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ which is a contradiction. Therefore $b\text{-}cl(b\text{-}int(A,B)) \subseteq (G,H)$ and hence (A,B) is strongly binary g^*-closed.

Corollary 2.12 A strongly binary g^*-closed set (A,B) is binary regular closed iff $b\text{-}cl(b\text{-}int(A,B)) \supseteq (A,B)$.

Proof. Assume that (A,B) is binary regular closed. Since $b\text{-}cl(b\text{-}int(A,B)) = (A,B)$,

$b\text{-}cl(b\text{-}int(A,B)) - (A,B) = (\phi,\phi)$ is binary regular closed and hence binary closed.

Conversely assume that $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ is binary closed. By above theorem $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ contains no non empty binary closed set. Therefore $b\text{-}cl(b\text{-}int(A,B)) - (A,B) = (\phi,\phi)$. Thus (A,B) is binary regular closed.

Theorem 2.13 Suppose that $(C,D) \subseteq (A,B) \subseteq (X,Y)$, (C,D) is strongly binary g^*-closed set relative to (A,B) and that both binary open and strongly binary g^*-closed subset of (X,Y) then (C,D) is strongly binary g^*-closed set relative to (X,Y).

Proof. Let $(C,D) \subseteq (G,H)$ and (G,H) be an binary open set in (X,Y). But given that $(C,D) \subseteq (A,B) \subseteq (X,Y)$, therefore $(C,D) \subseteq (A,B)$ and $(C,D) \subseteq (G,H)$. This implies $(C,D) \subseteq (A,B) \cap (G,H)$. Since (C,D) is strongly binary g^*-closed relative to (A,B),

$b\text{-}cl(b\text{-}int(C,D)) \subseteq (A,B) \cap (G,H)$.

(ie) $(A,B) \cap b\text{-}cl(b\text{-}int(C,D)) \subseteq (A,B) \cap (G,H)$. This implies $(A,B) \cap b\text{-}cl(b\text{-}int(C,D))) \subseteq (G,H)$. Thus $((A,B) \cap (b\text{-}cl(b\text{-}int(C,D)))) \cup (b\text{-}cl(b\text{-}int(C,D)))^c \subseteq (G,H) \cup (b\text{-}cl(b\text{-}int(C,D)))^c$ implies $(A,B) \cup (b\text{-}cl(b\text{-}int(C,D)))^c \subseteq (G,H) \cup (b\text{-}cl(b\text{-}int((C,D))))^c$. Since (A,B) is strongly binary g^*-closed in (X,Y), we have $(b\text{-}cl(b\text{-}int(A,B))) \subseteq (G,H) \cup (b\text{-}cl(b\text{-}int(C,D)))^c$. Also $(C,D) \subseteq (A,B) \Rightarrow b\text{-}cl(b\text{-}int(C,D)) \subseteq b\text{-}cl(b\text{-}int(A,B))$. Thus $b\text{-}cl(b\text{-}int(C,D)) \subseteq b\text{-}cl(b\text{-}int(C,D)) \subseteq (G,H) \cup (b\text{-}cl(b\text{-}int(C,D)))^c$. Therefore (C,D) is strongly binary g^*-closed set relative to (X,Y).

Corollary 2.14 Let (A,B) be strongly binary g^*-closed and suppose that (E,F) is binary closed then $(A,B) \cap (E,F)$ is strongly binary g^*-closed set.

Proof. To show that $(A,B) \cap (E,F)$ is strongly binary g^*-closed, we have to show $b\text{-}cl(b\text{-}int(A,B) \cap (E,F)) \subseteq (G,H)$ whenever $(A,B) \cap (E,F) \subseteq (G,H)$ and (G,H) is binary g-open. $(A,B) \cap (E,F)$ is binary closed in (A,B) and so strongly binary g^*-closed in (C,D). By the above theorem $(A,B) \cap (E,F)$ is strongly binary g^*-closed in (X,Y). Since $(A,B) \cap (E,F) \subseteq (A,B) \subseteq (X,Y)$.

Theorem 2.15 If (A,B) is strongly binary g^*-closed and $(A,B) \subseteq (C,D) \subseteq b\text{-}cl(b\text{-}int(A,B))$ then (C,D) is strongly binary g^*-closed.

Proof. Given that $(C,D) \subseteq b\text{-}cl(b\text{-}int(A,B))$ then $b\text{-}cl(b\text{-}int(C,D)) \subseteq b\text{-}cl(b\text{-}int(A,B))$,

$b\text{-}cl(b\text{-}int(C,D)) - (C,D) \subseteq b\text{-}cl(b\text{-}int(A,B)) - (A,B)$.

Since $(A,B) \subseteq (C,D)$. As (A,B) is strongly binary g^*-closed by the above theorem $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ contains no non empty binary closed set, $b\text{-}cl(b\text{-}int(C,D)) - (C,D)$

contains no empty binary closed set. Again by theorem 2.13, (C, D) is strongly binary g^*-closed set.

Theorem 2.16 Let $(A, B) \subseteq (U, V) \subseteq (X, Y)$ and suppose that (A, B) is strongly binary g^*-closed in (X, Y) then (A, B) is strongly binary g^*-closed relative to (U, V).

Proof. Given that $(A, B) \subseteq (U, V) \subseteq (X, Y)$ and (A, B) is strongly binary g^*-closed in (X, Y). To show that (A, B) is strongly binary g^*-closed relative to (U, V), let $(A, B) \subseteq (U, V) \cap (G, H)$, where (G, H) is binary g-open in (X, Y). Since (A, B) is strongly binary g^*-closed in (X, Y), $(A, B) \subseteq (G, H)$ implies b-cl(b-int$(A, B)) \subseteq (G, H)$.
(ie) $(U, V) \cap$ b-cl(b-int$(A, B)) \subseteq (U, V) \cap (G, H)$, where $(U, V) \cap$ b-cl(b-int$(A, B))$ is binary closure of binary interior of (A, B) in (U, V). Thus (A, B) is strongly binary g^*-closed relative to (U, V).

Theorem 2.17 If a subset (A, B) of a binary topological space (X, Y) is binary gsp-closed then it is strongly binary g^*-closed.

Proof. Suppose that (A, B) is binary gsp-closed in (X, Y), let (G, H) be binary open set containing (A, B). Then $(G, H) \subseteq$ bsp-cl(A, B), $(A, B) \cup (G, H) \supseteq (A, B) \cup$ (b-int(b-cl(b-int(A, B)))) which implies $(G, H) \supseteq$ b-int(b-cl(b-int(A, B))) as (G, H) is binary open. (ie) $(G, H) \supseteq$ b-cl(b-int$(A, B)) - (A, B)$ is strongly binary g^*-closed set in (X, Y).

Theorem 2.18 Every strongly binary g^*-closed set is an binary αg-closed set and hence binary gs-closed but not conversely.

Proof. Let (A, B) be a strongly binary g^*-closed set of $(X, Y, \mathcal{M})$. By above theorem, (A, B) is binary g-closed and binary αg-closed. Then we know that every binary g-closed set is binary gs-closed. By above theorem every strongly binary g^*-closed set is binary gs-closed.

Example 2.19 Let $X = \{a, b\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{2\}), (\{a\}, \{1\}), (\{a\}, Y), (X, \{1\}), (X, Y)\}$. Then the set $(\{a\}, \phi)$ is strongly binary g^*-closed set but not binary αg-closed.

Example 2.20 Let $X = \{a, b\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{1\}), (\{b\}, \{1\}), (X, \{1\}), (X, Y)\}$. Then the set (ϕ, Y) is strongly binary g^*-closed set but not binary gs-closed.

III. MORE ON STRONGLY BINARY g^*-OPEN SETS

Definition 3.1 Let (X, Y) be a binary topological space and $(x, y) \in (X, Y)$. A subset (J, K) of (X, Y) is said to be strongly binary g^*-neighbourhood of (x, y) if there exists an strongly binary g^*-open set (G, H) such that $(x, y) \in (G, H) \subset (J, K)$.

The collection of all strongly binary g^*-neighborhoods of $(x, y) \in (J, K)$ is called a strongly binary g^*-neighborhood system at (x, y) and shall be denoted by strongly $bg^*N(x, y)$.

Definition 3.2 Let (X, Y) be a binary topological space and (A, B) be a subset of (X, Y). (A, B) subset (J, K) of (X, Y) is said to be strongly binary g^*-neighborhood of (A, B) if there exists a strongly binary g^*-open set (G, H) such that $(A, B) \in (G, H) \subseteq (J, K)$.

Definition 3.3 Let (X, Y) be a binary topological space and (A, B) be a subset of (X, Y). A point $(x, y) \in (A, B)$ is said to be a strongly binary g^*-interior point of (A, B), if (A, B) is strongly binary $g^*N(x, y)$. The set of all strongly binary g^*-interior points of (A, B) is called a strongly binary g^*-interior points of (A, B) is called a strongly binary g^*-interior of (A, B) and is denoted by $SBG^*INT(A, B)$. $SBG^*INT(A, B) = \cup \{(G, H): (G, H)$ is strongly binary g^*-open, $(G, H) \subset (A, B)\}$.

Definition 3.4 Let (X, Y) be a binary topological space and (A, B) be a subset of (X, Y). A point $(x, y) \in (A, B)$ is said to be a strongly binary g^*-closure of (A, B). Then $SBG^*CL(A, B) = \cap \{(E, F): (A, B) \subset (E, F)$ in strongly binary g^*-closed $BG^*C(X, Y, \mathcal{M})\}$.

Theorem 3.5 A subset (A, B) of a binary topological space is strongly binary g^*-open if it is a strongly binary g^*-neighborhood of each points.

Proof. Let (G, H) be subset of a binary topological space be strongly binary g^*-open. Then for every $(x, y) \in (X, Y)$, $(x, y) \in (G, H) \subseteq (G, H)$, and therefore (G, H) is a strongly binary g^*-neighborhood of each of the points.

Theorem 3.6 Let (X, Y) be a binary topological space. If (A, B) is strongly binary g^*-closed subset of (X, Y) and $(x, y) \in SBG^*CL(A, B)$ if and only if for any strongly binary g^*-neighborhood (J, K) of (x, y) in (X, Y), $(J, K) \cap (A, B) \neq (\phi, \phi)$.

Proof. Let us assume that there is a strongly binary g^*-neighborhood (J, K) of the point (x, y) in (X, Y) such that $(J, K) \cap (A, B) = (\phi, \phi)$. There exists a strongly binary g^*-open set (G, H) of (X, Y) such that $(x, y) \in (G, H) \subseteq (J, K)$. Therefore we have $(G, H) \cap (A, B) = (\phi, \phi)$ and so $(x, y) \in (X, Y) - (G, H)$.

Then $SBG^*CL(A, B) \in (X, Y) - (G, H)$ and therefore $(x, y) \notin SBG^*CL(A, B)$, which contradicts the hypothesis that $(x, y) \in SBG^*CL(A, B)$.

Therefore $(J, K) \cap (A, B) \neq (\phi, \phi)$.

Conversely, suppose that $(x, y) \notin SBG^*CL(A, B)$. Then there exists a strongly binary g^*-closed set (G, H) of (X, Y)

such that $(A,B) \subseteq (G,H)$ and $(x,y) \notin (G,H)$. Thus $(x,y) \in (X,Y) - (G,H)$ and $(X,Y) - (G,H)$ is strongly binary g^*-open in (X,Y) and hence $(X,Y) - (G,H)$ is strongly binary g^*-open in (X,Y) and hence $(X,Y) - (G,H)$ is a strongly binary g^*-neighborhood of (x,y) in (X,Y). But $(A,B) \cap ((X,Y) - (G,H)) = (\phi,\phi)$ which is a contradiction. Hence $(x,y) \in SBG^*CL(A,B)$.

Theorem 3.7 Let (X,Y) be a binary topological space and $(x,y) \in (X,Y)$. Let strongly $bg^*N(x,y)$ be a collection of all strongly binary g^*-neighborhood of (x,y). Then

1. Strongly $bg^*N(x,y) \neq (\phi,\phi)$ and (x,y) belongs to each member of Strongly $bg^*N(x,y)$.
2. The intersection of the any two members of strongly $bg^*N(x,y)$ is again a member of strongly $bg^*N(x,y)$.
3. If $(J,K) \in$ Strongly $bg^*N(x,y)$ and $(U,V) \subseteq (J,K)$, then $(U,V) \in$ Strongly $bg^*N(x,y)$.
4. Each member $(J,K) \in$ Strongly $bg^*N(x,y)$ is a superset of a member $(G,H) \in$ Strongly $bg^*N(x,y)$ where (G,H) is a strongly binary g^*-open set.

Proof. 1. Since (X,Y) is strongly binary g^*-open set containing (p,q), it is a strongly binary g^*-neighborhood of every $(p,q) \in (X,Y)$. Hence there exist at least one strongly binary g^*-neighborhood namely (X,Y) for each $(p,q) \in (X,Y)$ there is strongly $bg^*N(p,q) \neq (\phi,\phi)$. Let $(J,K) \in$ strongly $bg^*N(p,q)$, (J,K) is a strongly binary g^*-neighborhood of (p,q), then there exists a strongly binary g^*-open set (G,H) such that $(p,q) \in (G,H) \subseteq (J,K)$, so $(p,q) \in (J,K)$. therefore (p,q) belongs to every number (J,K) strongly $bg^*N(p,q)$.

2. Let $(J,K) \in$ Strongly $bg^*N(p,q)$ and $(U,V) \in$ strongly binary $g^*N(p,q)$. There exists strongly binary g^*-open set (G,H) and (E,F) such that $(p,q) \in (G,H) \subseteq (J,K)$ and $(p,q) \in (E,F) \subseteq (U,V)$. Hence $(p,q) \in (G,H) \cap (E,F) \subseteq (U,V) \cap (J,K)$. Note that $(G,H) \cap (E,F)$ is a strongly binary g^*-open set. Therefore it follows that $(J,K) \cap (U,V)$ is a strongly binary g^*-neighborhood of (p,q). Hence $(J,K) \cap (U,V) \in$ strongly $bg^*N(p,q)$.

3. If $(J,K) \in$ strongly $bg^*N(p,q)$ then there is a strongly binary g^*-open set (G,H) such that $(p,q) \in (G,H) \subseteq (J,K)$. Since $(U,V) \subseteq (J,K)$, (U,V) is a strongly binary g^*-neighborhood of (p,q). Hence $(U,V) \in$ Strongly $bg^*N(p,q)$.

4. Let $(J,K) \in$ strongly $bg^*N(p,q)$ then there exists a strongly binary g^*-open set (G,H), such that $(p,q) \in (G,H) \subseteq (J,K)$. Since (G,H) is strongly binary g^*-open set and $(p,q) \in (G,H)$, (G,H) is strongly binary g^*-neighborhood of (p,q). Therefore $(G,H) \in$ Strongly $bg^*N(p,q)$ and also $(G,H) \subseteq (J,K)$.

Theorem 3.8 Let (X,Y) be a binary topological space. If (A,B) is strongly binary g^*-closed subset of (X,Y) and $(x,y) \in SBG^*INT(A,B)$ if and only if for any strongly binary g^*-neighborhood (J,K) of (x,y) in (X,Y), $(J,K) \cap (A,B) \neq (\phi,\phi)$.

Proof. Let us assume that there is strongly binary g^*-neighborhood (J,K) of the point (x,y) in (X,Y) such that $(J,K) \cap (A,B) = (\phi,\phi)$. There exists an strongly binary g^*-open set (G,H) of (X,Y) such that $(x,y) \in (G,H) \subseteq (J,K)$. Therefore we have $(G,H) \cap (A,B) = (\phi,\phi)$ and so $(x,y) \in (X,Y) - (G,H)$.

Then $SBG^*CL(A,B) \in (X,Y) - (G,H)$ and therefore $(x,y) \notin SBG^*(A,B)$, which is a contradiction to the hypothesis that $(x,y) \in SBG^*CL(A,B)$. Therefore $(J,K) \cap (A,B) \neq (\phi,\phi)$.

Conversely, suppose that $(x,y) \notin SBG^*CL(A,B)$, then there exists a strongly binary g^*-closed set (G,H) of (X,Y) such that $(A,B) \subseteq (G,H)$ and $(x,y) \notin (G,H)$. Thus $(x,y) \in (X,Y) - (G,H)$ and $(X,Y) - (G,H)$ is strongly binary g^*-open in (X,Y) and hence $(X,Y) - (G,H)$ is a strongly binary g^*-neighborhood of (x,y) in (X,Y). But $(A,B) \cap ((X,Y) - (G,H)) = (\phi,\phi)$ which is a contradiction. Hence $(x,y) \in SBG^*CL(A,B)$.

Proposition 3.9 If (A,B) is a subset of (X,Y), then $SBG^*INT(A,B) = \cup \{(G,H): (G,H)$ is strongly binary g^*-open, $(G,H) \subset (A,B)\}$.

Proof. Let (A,B) be a subset of (X,Y), $(x,y) \in SBG^*INT(A,B) \Leftrightarrow (x,y)$ is a strongly binary g^*-interior point of (A,B), (A,B) is a strongly binary $g^*N(x,y)$ which implies that there exists a strongly binary g^*-open set (G,H) such that $(x,y) \in (G,H) \subset (A,B)$, $(x,y) \in \cup \{(G,H): (G,H)$ is strongly binary g^*-open, $(G,H) \subset (A,B)\}$.

Hence $SBG^*INT(A,B) = \cup \{(G,H): (G,H)$ is strongly binary g^*-open, $(G,H) \subset (A,B)\}$.

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