

Total Prime Labeling of Star Related Graphs

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ARTICLE INFO

Published online:

22 July 2023

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ABSTRACT

In this paper we investigate that the total prime labeling of some star graphs .we also discuss the total prime labeling in the context of some graph operation namely corona product.

KEYWORDS: Prime labeling , vertex prime labeling, total prime labeling, corona product, star flower graph.

INTRODUCTION

Here we consider only the graphs which are finite, simple and undirected graphs. A graph G is $G = (V(G), E(G))$ where $V(G)$ denotes the vertex set and $E(G)$ denotes the edge set. The order and size of the G are denoted by 'p' and 'q' respectively. For all other terminology and notations in graph theory we follow Bondy [1].

Graph labeling where the vertices and edges are assigned real values with certain conditions. Prime labeling and vertex prime labeling are already introduced. Combining these two results a new labeling called a total prime labeling was defined by Kala and Rama Subramanian [2] and they investigated some graphs are total prime labelling graphs . By the reference we proved that the graphs wheel, gear, carona, triangular book, double comb, and planter graphs are all total prime graphs [5]. And also we proved some cycle and path related graphs [6].The star flower graphs are constructed from star graphs.

Definition (1.1)

Let $G = (V,E)$ be a graph with 'p' vertices. A bijection $f: V(G) \rightarrow \{1,2,3,\dots,p\}$ is said to be as prime labeling if for each edge $e = uv$ the labels assigned to u and v are relatively prime. A graph which admits **prime labeling** is called **prime graph**.

Definition (1.2)

Let $G = (V,E)$ be a graph with 'p' vertices and 'q' edges. A bijection $f: E(G) \rightarrow \{1,2,3,\dots,q\}$ is said to be **vertex prime labeling** if for each vertex of degree atleast two the greatest common divisor of the labels on its incident edges is one. That is the gcd of $\{f(u), f(v)\} = 1$.

Definition (1.3)

Let $G = (V, E)$ be a graph, with 'p' vertices and 'q' edges. A bijection $f: VUE \rightarrow \{1,2,3,\dots, (p+q)\}$ is said to be total prime labeling if

- (i) for each edge $e = uv$, the labels assigned to u and v are relatively prime.
- (ii) for each vertex of degree atleast two, the greatest common divisor (gcd) of the labels on the incident edge is one.

A graph which admits total prime labeling is called **total prime graph**.

Definition (1.4)

A complete bipartite graph is called a star graph S_n and it has 'n+1' vertices and 'n' edges.

Definition (1.5)

The corona of a graph G on p vertices v_1, v_2, \dots, v_p is the graph obtained from G by adding p new vertices u_1, u_2, \dots, u_p and the new edges $u_i v_i$ for $1 \leq i \leq p$ then it is denoted by $G \odot K_1$.

Definition (1.6) \longrightarrow

Star flower graph $Sfl_n(1)$ is constructed from star graph S_n ($n \geq 3, n$ is even) all alternate pendent vertices are joined by an edge separately .

Definition (1.7)

Star flower graph $Sfl_n(2)$ is constructed from star graph S_n ($n \geq 3, n$ is even) all ~~alternate~~ pendent vertices are joined with single vertex by edges.

Theorem 1:

The star flower graph $Sfl_n(1)$, (n is even) is a total prime graph .

Proof:

Let G be a star flower graph $Sfl_n(1)$, (n is even), then join all the alternate vertices by a single edge.

Let $V(G) = \{u\} \cup \{v_i / 1 \leq i \leq n\}$ and $E(G) = \{u v_i / 1 \leq i \leq n\} \cup \{v_i v_{i+2} / 1 \leq i \leq n/2\}$

Total number of vertices $p = n+1$

Total number of edges $q = 3n/2$

Define a mapping $f: VUE \rightarrow \{1, 2, 3, \dots, p+q\}$

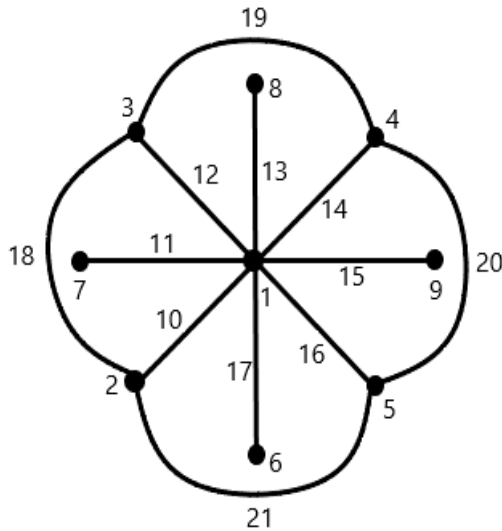
by

$f(u) = 1$

$f(v_i) = 1+i; 1 \leq i \leq n$

$f(e_i) = n+1+i; 1 \leq i \leq 3n/2$

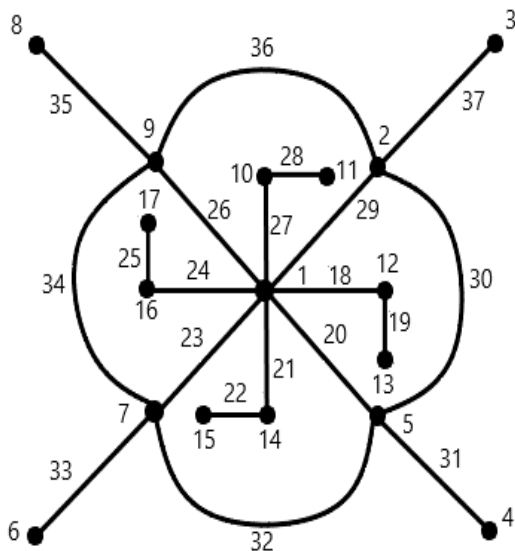
Example 1.1: Total prime graph of Sfl_8



Theorem 2:

The corona product of star flower $Sfl_n(1) \odot K_1$ is a total prime graph (excluding the center vertex).

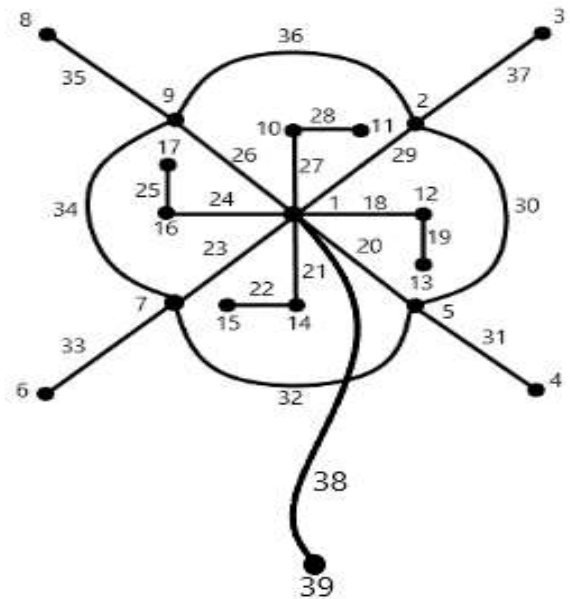
Example 1.2 : $Sfl_8(1) \odot K_1$



Theorem 3:

The corona product of star flower $Sfl_n(1) \odot K_1$ is a total prime graph.

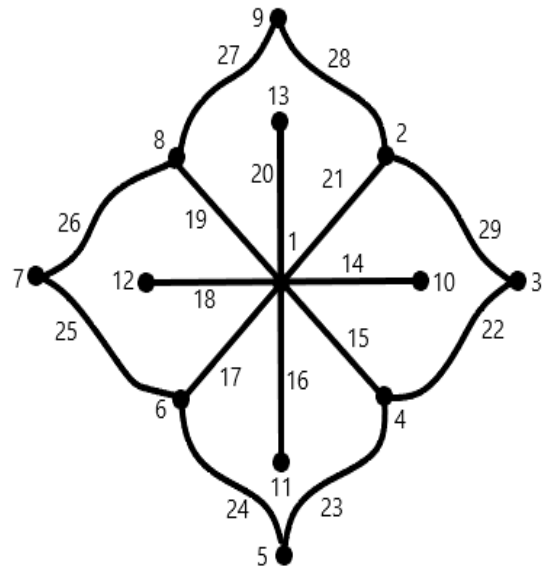
Example 1.3: $Sfl_8(1) \odot K_1$



Theorem 4:

The star flower graph $Sfl_n(2)$ is a total prime graph .

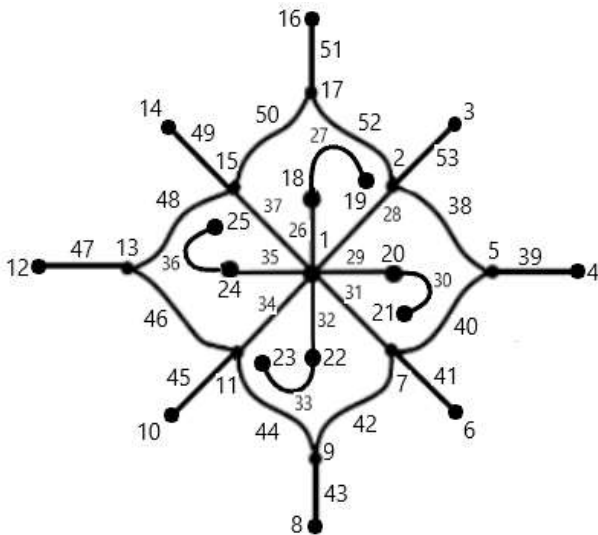
Example 1.4: $Sfl_8(2)$



Theorem 5:

The corona product of star flower $Sfl_n(2) \odot K_1$ is a total prime graph (excluding the center vertex).

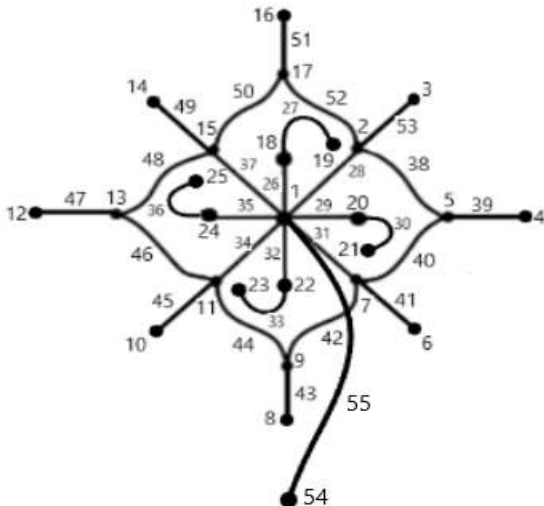
Example 1.5: $Sfl_8(2) \odot K_1$



Theorem 6:

The corona product of star flower $Sfl_n(2) \odot K_1$ is a total prime graph.

Example 1.6: $Sfl_8(2) \odot K_1$



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