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Some Contribution to Complement of Strong Neutrosophic Graphs

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ABSTRACT
The Fundamental operations on Complement of Strong Neutrosophic Graphs. We have applied the concept of strong Neutrosophic Graphs and also some graphs are connected, we explore some Particular Cases of strong Neutrosophic Graphs.

KEYWORDS: Strong Neutrosophic Graphs, Complement of Strong Neutrosophic Graphs, Union Join, and Cartesian product.
MSC2020-Mathematics Subject Classification System: 05C72, 05C76

1. INTRODUCTION

Graph theory has established itself as an important mathematical tool in a wide variety of subjects ranging from operational research, linguistic sociology and Architecture. In real word, the complexity generally arises from uncertainty in the form of ambiguity. Classical set theory also termed as crisp set theory and propounded by George Cantor,is fundamental to the study of fuzzy sets. In 1965, Lofti A. Zadeh introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. In 1975, Rosenfeld first introduced fuzzy graph theory as a generalization of Euler graph theory. Fuzzy graph is the generalization of the ordinary graph. The Fundamental operations on Complement of Strong Neutrosophic Graphs. We apply the concept of strong Neutrosophic Graphs and also some graphs are connected, we explore some Particular Cases of strong Neutrosophic Graphs.

2. PRELIMINARIES

Definition 2.1.

A Neutrosophic Graph is of the form G = < N, L > Where,

(i) $N = \{n_1, n_2, n_3, \dots n_n\}$ such that $\lambda_T : N(NG) \to [a, b]$, $\lambda_I : N(NG) \to [a, b]$ and $\lambda_F : N(NG) \to [a, b]$ denote the three ways of truth, indeterminacy and falsity, for all $n_i \in N$, with a = 0 and b = 1.

$0 \le \lambda_T(n_i) + \lambda_I(n_i) + \lambda_F(n_i) \le 3$ for each $n_i \in N(NG)$, (i = 1, 2,…n)

(ii) $L \subseteq N x N$ Where $\eta_T : NxN \to [a, b]$, $\eta_I : NxN \to [a, b]$, $\eta_F : NxN \to [a, b]$, are such that

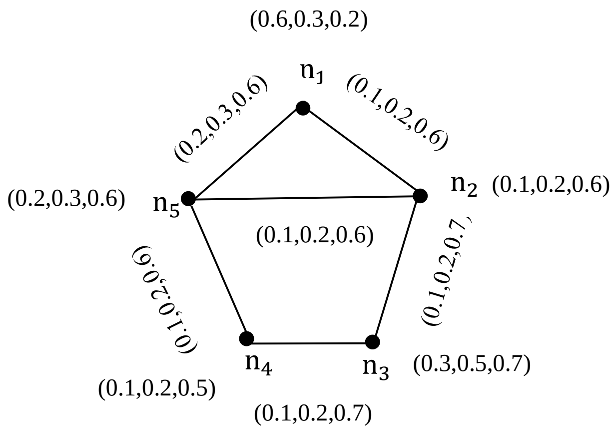
$$\eta_T(n_i n_j) \le \min[\lambda_T(n_i), \lambda_T(n_j),]$$
$$\eta_I(n_i n_j) \le \min[\lambda_I(n_i), \lambda_I(n_j)],$$
$$\eta_F(n_i n_j) \le \max[\lambda_F(n_i), \lambda_F(n_j)],$$
$$0 \le \eta_T(n_i n_j) + \eta_I(n_i n_j) + \eta_F(n_i n_j) \le 3, \text{for each } (n_i n_j) \in L(NG) \ (i,j = 1,2,\dots,n)$$

Definition 2.2.

A Neutrosophic Graph NG = < N, L > with the triplet ($\lambda_T, \lambda_I, \lambda_F$) and (η_T, η_I, η_F) is called Strong Neutrosophic Graph if

$$\eta_T(n_i n_j) = \min[\lambda_T(n_i), \lambda_T(n_j)]$$
$$\eta_I(n_i n_j) = \min[\lambda_I(n_i), \lambda_I(n_j)]$$

$\eta_F(n_i n_j) = \max[\lambda_F(n_i), \lambda_F(n_j)], \forall (n_i n_j) \in L(NG).$



Strong Neutrosophic Graph
Figure 1.

Definition 2.3.

The Complement of a Strong Neutrosophic Graph SNG = < λ,η > with is a G = (N, L) Strong Neutrosophic Graph $SNG^c = (\lambda^c, \eta^c)$ where

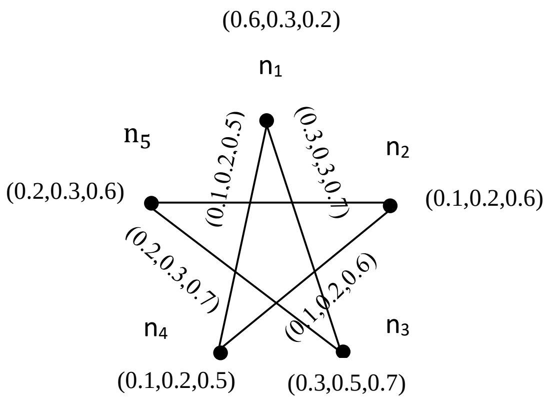
(i) $N^c = N$

(ii) $(\lambda)_T{}^C(n_i) = (\lambda)_T(n_i),\ (\lambda)_I{}^C(n_i) = (\lambda)_I(n_i),\ (\lambda)_F{}^C(n_i) = (\lambda)_F(n_i)$

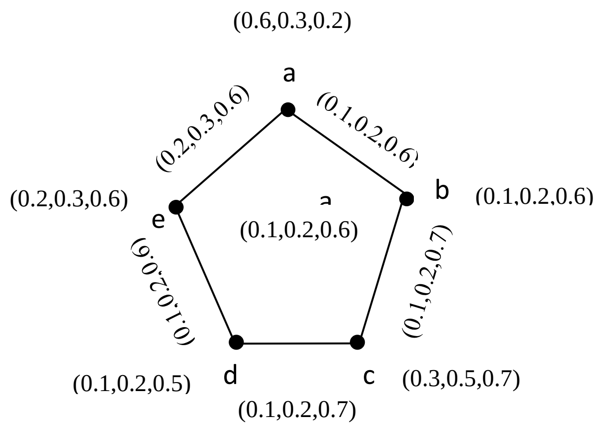
(iii) $$(\eta)_T{}^C(n_j, n_k) = \begin{cases} 0 & ;\ n_j n_k \in L \\ \min[(\lambda)_T(n_j), (\lambda)_T(n_k)] & ;\ n_j n_k \in L^C \end{cases}$$

$$(\eta)_I{}^C(n_j, n_k) = \begin{cases} 0 & ;\ n_j n_k \in L \\ \min[(\lambda)_I(n_j), (\lambda)_I(n_k)] & ;\ n_j n_k \in L^C \end{cases}$$

$$(\eta)_F{}^C(n_j, n_k) = \begin{cases} 0 & ;\ n_j n_k \in L \\ \max[(\lambda)_F(n_j), (\lambda)_F(n_k)] & ;\ n_j n_k \in L^C \end{cases}$$



Strong Neutrosophic Graph
Figure 2.



Complement of Strong Neutrosophic Graph
Figure 3.

Remarks:2.4.

If NG = < λ,η > is a Neutrosophic Graph on SNG. Then from above Definition 2.3 it follows that $(NG^C)^C = <(\lambda^C)^C, (\eta^C)^C>$ on SNG where $(\lambda^C)^C = \lambda$ and
$((\eta)_T{}^C)^C(n_j, n_k) = \min[(\lambda)_T(n_j, n_k)]$, $((\eta)_I{}^C)^C(n_j, n_k) = \min[(\lambda)_I(n_j, n_k)]$,
$((\eta)_F{}^C)^C(n_j, n_k) = \min[(\lambda)_F(n_j, n_k)]$. for all $(n_i n_j) \in L(SNG)$.

Thus $((\eta)_T{}^C)^C = (\eta)_T$, $((\eta)_I{}^C)^C = (\eta)_I$, and $((\eta)_F{}^C)^C = (\eta)_F$, the Strong Neutrosophic relation for any Neutrosophic Graph NG. SNG is Strong Neutrosophic Graph and $G \subseteq (G^C)^C$.

3. MAIN RESULTS

Definition 3.1.

Let $SNG_1 = (\lambda_1, \eta_1)$ and $SNG_2 = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graph corresponding to the crisp graph $(G_1) = (N_1, L_1)$ and $(G_2) = (V_2, E_2)$. Then the **union of two Strong Neutrosophic Graph** is defined as SNG = $(SNG_1) \cup (SNG_2) = (\lambda_1 \cup \lambda_2, \eta_1 \cup \eta_2)$ with the node set $V(SNG) = V(SNG_1) \cup V(SNG_2)$ and the line set $L(SNG) = L(SNG_1) \cup L(SNG_2)$ and
$\lambda(n_j) = (\lambda_T(n_j), \lambda_I(n_j), \lambda_F(n_j))$ and for all $n_j \in N(SNG)$;
$\eta(n_j n_k) = (\eta_T(n_j n_k), \eta_I(n_j n_k), \eta_F(n_j n_k))$ for all $n_j n_k \in L(SNG)$;

For any $n_j \in N(SNG)$;

i) $$\lambda_T(n_j) = \begin{cases} (\lambda_1)_T(n_j) & ; \text{if } n_j \in N_1 - N_2 \\ (\lambda_2)_T(n_j) & ; \text{if } n_j \in N_2 - N_1 \\ \max\left((\lambda_1)_T(n_j), (\lambda_2)_T(n_j)\right) & ; \text{if } n_j \in N_1 \cap N_2 \end{cases}$$

ii) $$\lambda_I(n_j) = \begin{cases} (\lambda_1)_I(n_j) & ; \text{if } n_j \in N_1 - N_2 \\ (\lambda_2)_I(n_j) & ; \text{if } n_j \in N_2 - N_1 \\ \max\left((\lambda_1)_I(n_j), (\lambda_2)_I(n_j)\right) & ; \text{if } n_j \in N_1 \cap N_2 \end{cases}$$

iii) $$\lambda_F(n_j) = \begin{cases} (\lambda_1)_F(n_j) & ; \text{if } n_j \in N_1 - N_2 \\ (\lambda_2)_F(n_j) & ; \text{if } n_j \in N_2 - N_1 \\ \min\left((\lambda_1)_F(n_j), (\lambda_2)_F(n_j)\right) & ; \text{if } n_j \in N_1 \cap N_2 \end{cases}$$

For any $n_j n_k \in L(SNG)$;

iv) $$\eta_T(n_j n_k) = \begin{cases} (\eta_1)_T(n_j n_k) & ; \text{if } n_j n_k \in L_1 - L_2 \\ (\eta_2)_T(n_j n_k) & ; \text{if } n_j n_k \in L_2 - L_1 \\ \max\left((\eta_1)_T(n_j n_k), (\eta_2)_T(n_j n_k)\right) & ; \text{if } n_j n_k \in L_1 \cap L_2 \end{cases}$$

v) $$\eta_I(n_j n_k) = \begin{cases} (\eta_1)_I(n_j n_k) & ; \text{if } n_j n_k \in L_1 - L_2 \\ (\eta_2)_I(n_j n_k) & ; \text{if } n_j n_k \in L_2 - L_1 \\ \max\left((\eta_1)_I(n_j n_k), (\eta_2)_I(n_j n_k)\right) & ; \text{if } n_j n_k \in L_1 \cap L_2 \end{cases}$$

vi) $$\eta_F(n_j n_k) = \begin{cases} (\eta_1)_F(n_j n_k) & ; \text{if } n_j n_k \in L_1 - L_2 \\ (\eta_2)_F(n_j n_k) & ; \text{if } n_j n_k \in L_2 - L_1 \\ \min\left((\eta_1)_F(n_j n_k), (\eta_2)_F(n_j n_k)\right) & ; \text{if } n_j n_k \in L \cap L_2 \end{cases}$$

Definition 3.2.

Let $SNG_1 = (\lambda_1, \eta_1)$ and $SNG_2 = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graph corresponding to the crisp graph $G_1 = (N_1, L_1)$and $G_2 = (N_2, L_2)$. Then the **join of two Strong Neutrosophic Graph** is defined as $(SNG) = (SNG_1) + (SNG_2) = (\lambda_1 + \lambda_2, \eta_1 + \eta_2)$ where the set of nodes are$N(SNG) = N(SNG_1) \cup N(SNG_2)$ and the set of lines are
$L(SNG) = L(SNG_1) \cup L(SNG_2) \cup L(SNG)^*$ where $L(SNG)^*$is the set of all Lines joining the Nodes of $N(SNG_1)$, $N(SNG_2)$.
With usual notation λ and η are represented by the triplet

$\lambda(n_j) = (\lambda_T(n_j), \lambda_I(n_j), \lambda_F(n_j))$ $\forall\, n_j \in N(SNG)$ and
$\eta(n_j n_k) = (\eta_T(n_j n_k), \eta_I(n_j n_k), \eta_F(n_j n_k))$ $\forall\, n_j n_k \in L(SNG)$ is defined as,
For any $n_j \in N(NG) = V(SNG_1) \cup V(SNG_2)$.

(i) $$\lambda_T(n_j) = \begin{cases} (\lambda_1)_T(n_j) & ; \text{if } n_j \in N_1 - N_2 \\ (\lambda_2)_T(n_j) & ; \text{if } n_j \in N_2 - N_1 \\ \max\left((\lambda_1)_T(n_j), (\lambda_2)_T(n_j)\right) & ; \text{if } n_j \in N_1 \cap N_2 \end{cases}$$

(ii) $$\lambda_I(n_j) = \begin{cases} (\lambda_1)_I(n_j) & ; \text{if } n_j \in N_1 - N_2 \\ (\lambda_2)_I(n_j) & ; \text{if } n_j \in N_2 - N_1 \\ \max\left((\lambda_1)_I(n_j), (\lambda_2)_I(n_j)\right) & ; \text{if } n_j \in N_1 \cap N_2 \end{cases}$$

(iii) $\lambda_F(n_j) = \begin{cases} (\lambda_1)_F(n_j) & ; \text{if } n_j \in N_1 - N_2 \\ (\lambda_2)_F(n_j) & ; \text{if } n_j \in N_2 - N_1 \\ \min\left((\lambda_1)_F(n_j), (\lambda_2)_F(n_j)\right) & ; \text{if } n_j \in N_1 \cap N_2 \end{cases}$

For any $n_j n_k \in L(SNG) = L(SNG_1) \cup L(SNG_2) \cup L(SNG)^*$

(iv) $\eta_T(n_j n_k) = \begin{cases} (\eta_1)_T(n_j n_k) & ; \text{ if } n_j n_k \in L_1 - L_2 \\ (\eta_2)_T(n_j n_k) & ; \text{ if } n_j n_k \in L_2 - L_1 \\ \max\left((\eta_1)_T(n_j n_k), (\eta_2)_T(n_j n_k)\right) & ; \text{if } n_j n_k \in L_1 \cap L_2 \\ \min\left((\lambda_1)_T(n_j), (\lambda_2)_T(n_j)\right) & ; \text{ if } n_j n_k \in L^* \end{cases}$

(v) $\eta_I(n_j n_k) = \begin{cases} (\eta_1)_I(n_j n_k) & ; \text{ if } n_j n_k \in L_1 - L_2 \\ (\eta_2)_I(n_j n_k) & ; \text{ if } n_j n_k \in L_2 - L_1 \\ \max\left((\eta_1)_I(n_j n_k), (\eta_2)_I(n_j n_k)\right) & ; \text{if } n_j n_k \in L_1 \cap L_2 \\ \min\left((\lambda_1)_I(n_j), (\lambda_2)_I(n_j)\right) & ; \text{ if } n_j n_k \in L^* \end{cases}$

(vi) $\eta_F(n_j n_k) = \begin{cases} (\eta_1)_F(n_j n_k) & ; \text{ if } n_j n_k \in L_1 - L_2 \\ (\eta_2)_F(n_j n_k) & ; \text{ if } n_j n_k \in L_2 - L_1 \\ \min\left((\eta_1)_F(n_j n_k), (\eta_2)_F(n_j n_k)\right) & ; \text{if } n_j n_k \in L_1 \cap L_2 \\ \max\left((\lambda_1)_F(n_j), (\lambda_2)_F(n_j)\right) & ; \text{ if } n_j n_k \in L^* \end{cases}$

Theorem 3.3:

Let $SNG_1 = (\lambda_1, \eta_1)$ and $SNG_2 = (\lambda_2, \eta_2)$ be two Complement of Strong Neutrosophic Graph corresponding to the crisp graph $G_1 = (N_1, L_1)$ and $G_2 = (N_2, L_2)$ respectively, and $N_1 \cap N_2 = \varphi$. Then $(SNG_1 \cup SNG_2)^c = (SNG_1)^c + (SNG_2)^c$.

Proof: To Prove that $(SNG_1 \cup SNG_2)^c = (SNG_1)^c + (SNG_2)^c$, it is enough to prove that

(i) $((\lambda_1)_T \cup (\lambda_2)_T)^c(m) = ((\lambda_1)_T)^c(m) + ((\lambda_2)_T)^c(m)$

$((\lambda_1)_I \cup (\lambda_2)_I)^c(m) = ((\lambda_1)_I)^c(m) + ((\lambda_2)_I)^c(m)$

$((\lambda_1)_F \cup (\lambda_2)_F)^c(m) = ((\lambda_1)_F)^c(m) + ((\lambda_2)_F)^c(m)$

for all $m \in N = N_1 \cup N_2$

(ii) $((\eta_1)_T \cup (\eta_2)_T)^c(mn) = ((\eta_1)_T)^c(mn) + ((\eta_2)_T)^c(mn)$

$((\eta_1)_I \cup (\eta_2)_I)^c(mn) = ((\eta_1)_I)^c(mn) + ((\eta_2)_I)^c(mn)$

$((\eta_1)_F \cup (\eta_2)_F)^c(mn) = ((\eta_1)_F)^c(mn) + ((\eta_2)_F)^c(mn)$

for all $nm \in L = L_1 \cup L_2$

Claim: To Prove

$$((\lambda_1)_T \cup (\lambda_2)_T)^c(n) = ((\lambda_1)_T)^c(n) + ((\lambda_2)_T)^c(n)$$

Consider

$$((\lambda_1)_T \cup (\lambda_2)_T)^c(n) = ((\lambda_1)_T \cup (\lambda_2)_T)(n)$$

$(\lambda_1)_T)^c(n) = (\lambda_1)_T(n)$

$(\lambda_2)_T)^c(n) = (\lambda_2)_T(n)$ (By Definition 2.3)

Assume that $\lambda_1 \cup \lambda_2 = \lambda$

$$(\lambda)_T(n) = \begin{Bmatrix} (\lambda_1)_T(n), & \text{if} \quad n \in N_1 - N_2 \\ (\lambda_2)_T(n), & \text{if} \quad n \in N_2 - N_1 \end{Bmatrix}$$ (By Definition 3.1)

$((\lambda_1)_T)^c(n) + ((\lambda_2)_T)^c(n) = (\lambda_1)_T(n) + (\lambda_2)_T(n)$ (By Definition 2.3)

$$= \begin{Bmatrix} (\lambda_1)_T(n), & \text{if} \quad n \in N_1 - N_2 \\ (\lambda_2)_T(n), & \text{if} \quad n \in N_2 - N_1 \end{Bmatrix}$$ (By Definition 3.2)

Hence $((\lambda_1)_T \cup (\lambda_2)_T)^c(n) = ((\lambda_1)_T)^c(n) + ((\lambda_2)_T)^c(n)$ for all $n \in N$ ——————— (1)

Similarly the results also apply for the intermediate and falsity values.

Thus
$$((\lambda_1)_T \cup (\lambda_2)_T)^c(n) = ((\lambda_1)_T)^c(n) + ((\lambda_2)_T)^c(n)$$
$$((\lambda_1)_I \cup (\lambda_2)_I)^c(n) = ((\lambda_1)_I)^c(n) + ((\lambda_2)_I)^c(n)$$
$$((\lambda_1)_F \cup (\lambda_2)_F)^c(n) = ((\lambda_1)_F)^c(n) + ((\lambda_2)_F)^c(n)$$

Now consider

(i) $$((\eta_1)_T \cup (\eta_2)_T)^c(nm) = \begin{Bmatrix} (\eta_1)_T(mn), \text{ if } mn \in E_1 - E_2 \\ (\eta_2)_T(mn), \text{ if } mn \in E_2 - E_1 \end{Bmatrix}^c$$

(ii) $$((\eta_1)_I \cup (\eta_2)_I)^c(nm) = \begin{Bmatrix} (\eta_1)_I(mn), \text{ if } mn \in E_1 - E_2 \\ (\eta_2)_I(nm), \text{ if } mn \in E_2 - E_1 \end{Bmatrix}^c$$

(iii) $$((\eta_1)_F \cup (\eta_2)_F)^c(nm) = \begin{Bmatrix} (\eta_1)_F(mn), \text{ if } mn \in E_1 - E_2 \\ (\eta_2)_F(mn), \text{ if } mn \in E_2 - E_1 \end{Bmatrix}^c$$

(by Definition 3.1), (by Definition 2.3)

$$(\eta_1)_T)^c(mn) = \begin{cases} 0 & ; \text{ if } mn \in E_1 \\ \min[(\lambda_1)_T(m), (\lambda_1)_T(n)] & ; \text{ if } mn \in E_1^C \end{cases}$$

$$(\eta_2)_T)^c(mn) = \begin{cases} 0 & ; \text{ if } mn \in E_2 \\ \min[(\lambda_2)_T(m), (\lambda_2)_T(n)] & ; \text{ if } mn \in E_2^C \end{cases}$$

$$(\eta_1)_T)^c(mn) \cup (\eta_2)_T)^c(mn) = \begin{Bmatrix} 0 & ; \text{ if } mn \in E^* \\ \min[(\lambda_1)_T(m), (\lambda_2)_T(n)] & ; \text{ if } mn \in E^*, m \in N_1, n \in N_2 \end{Bmatrix}$$

Similarly, the results also apply for the intermediate and falsity values

Thus

$$(i) \Rightarrow ((\eta_1)_T \cup (\eta_2)_T)^c(mn) = \begin{Bmatrix} (\eta_1)_T(mn) & ; \text{if } mn \in L_1^C \\ (\eta_2)_T(mn) & ; \text{if } mn \in L_2^C \\ \min (\lambda_1)_T(n), (\lambda_2)_T(n) & ; \text{if } mn \in L^*, m \in n_1, n \in n_2 \end{Bmatrix}$$
…………(iv)

$$(ii) \Rightarrow ((\eta_1)_I \cup (\eta_2)_I)^c(mn) = \begin{Bmatrix} (\eta_1)_I(mn) & ; \text{if } mn \in L_1^C \\ (\eta_2)_I(mn) & ; \text{if } mn \in L_2^C \\ \min (\lambda_1)_I(n), (\lambda_2)_I(n) & ; \text{ifmn} \in L^*, m \in n_1, n \in n_2 \end{Bmatrix}$$
…………………(v)

$$(iii) \Rightarrow ((\eta_1)_F \cup (\eta_2)_F)^c(mn) = \begin{Bmatrix} (\eta_1)_F(mn) & ; \text{if } mn \in L_1^C \\ (\eta_2)_F(mn) & ; \text{if } mn \in L_2^C \\ \min (\lambda_1)_F(m), (\lambda_2)_F(n) & ; \text{if } mn \in L^*, m \in n_1, n \in n_2 \end{Bmatrix}$$
…………….(vi)

Hence

$$(iv) \Rightarrow ((\eta_1)_T \cup (\eta_2)_T)^c(mn) = \begin{cases} ((\eta_1)_T \cup (\eta_2)_T)(mn) & ; \text{if } mn \in L_1^C \cup L_2^C \\ \min (\lambda_1)_T(n), (\lambda_2)_T(n) & ; \text{if } mn \in E^*, m \in N_1, n \in N_2 \end{cases}$$

$$(v) \Rightarrow ((\eta_1)_I \cup (\eta_2)_I)^c(mn) = \begin{cases} ((\eta_1)_I \cup (\eta_2)_I)(mn) & ; \text{if } mn \in L_1^C \cup L_2^C \\ \min (\lambda_1)_I(n), (\lambda_2)_I(n) & ; \text{if } mn \in E^*, n \in N_1, m \in N_2 \end{cases}$$

$$(vi) \Rightarrow ((\eta_1)_F \cup (\eta_2)_F)^c(mn) = \begin{cases} ((\eta_1)_F \cup (\eta_2)_F)(mn) & ; \text{if } mn \in L_1^C \cup L_2^C \\ \max (\lambda_1)_F(n), (\lambda_2)_F(n) & ; \text{if } mn \in E^*, m \in N_1, n \in N_2 \end{cases}$$

(By Definition 3.2)

$$((\eta_1)_T \cup (\eta_2)_T)^c(mn) = ((\eta_1)_T)^c(mn) + ((\eta_2)_T)^c(mn)$$
$$((\eta_1)_I \cup (\eta_2)_I)^c(mn) = ((\eta_1)_I)^c(mn) + ((\eta_2)_I)^c(mn)$$
$$((\eta_1)_F \cup (\eta_2)_F)^c(mn) = ((\eta_1)_F)^c(mn) + ((\eta_2)_F)^c(mn) \text{ for all } mn \in L.$$
——————— (2)

Here $L_1^C = L_1 - L_2$ and $L_1^C = L_1 - L_2$ Since $L_1 \cap L_2 = \varphi$.

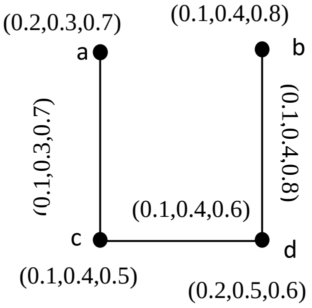
From (1) and (2) it follow that

$$(\mathrm{SNG}_1 \cup \mathrm{SNG}_2)^c = (\mathrm{SNG}_1)^c + (\mathrm{SNG}_2)^c.$$

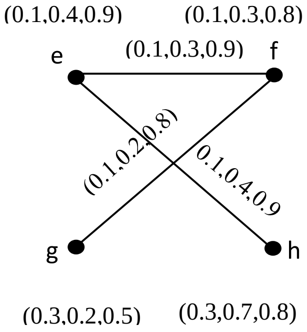
Illustration 3.4.

The validity of the **Theorem 3.3** is as follows:

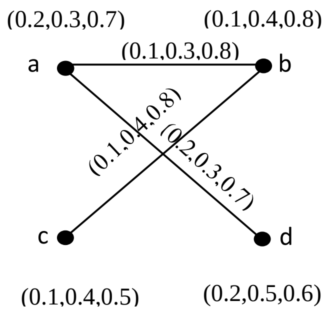
Consider the following Strong Neutrosophic Graphs SNG_1 and SNG_2 given in Figure (3.5) and Figure (3.6).



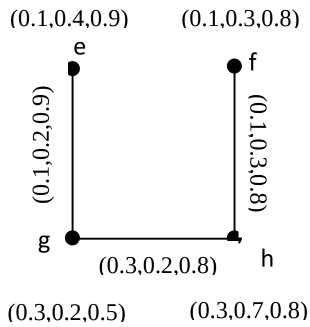
$\mathbf{SNG_1}$
Figure 3.5.



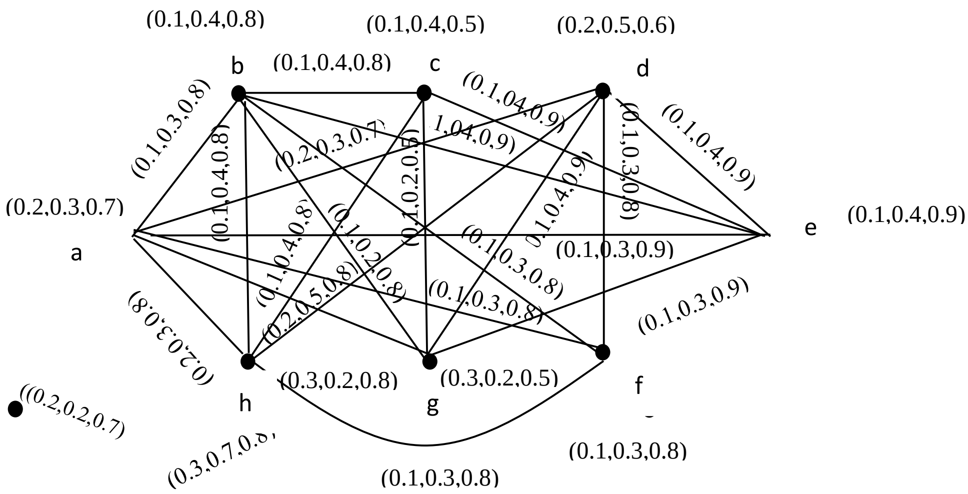
$\mathbf{SNG_2}$
Figure 3.6.



$\mathbf{(SNG_1)^c}$
Figure 3.5(a)



$\mathbf{(SNG_2)^c}$
Figure 3.6(a)



$\mathbf{(SNG_1 \cup SNG_2)^c = (SNG_1)^c + (SNG_2)^c}$.
Figure 3.7.

Theorem 3.8.

Let $\mathrm{SNG}_1 = (\lambda_1, \eta_1)$ and $\mathrm{SNG}_2 = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graph corresponding to the crisp graph $G_1 = (N_1, L_1)$ and $G_2 = (N_2, L_2)$ respectively, with $N_1 \cap N_2 = \varphi$. Then $(\mathrm{SNG}_1 + \mathrm{SNG}_2)^c = (\mathrm{SNG}_1)^c \cup (\mathrm{SNG}_1)^c$.

Proof: To Prove that $(\mathrm{SNG}_1 + \mathrm{SNG}_2)^c = (\mathrm{SNG}_1)^c \cup (\mathrm{SNG}_1)^c$, it is enough to prove that

(i)
$$((\lambda_1)_T + (\lambda_2)_T)^c(m) = ((\lambda_1)_T)^c(m) \cup ((\lambda_2)_T)^c(m)$$
$$((\lambda_1)_I + (\lambda_2)_I)^c(m) = ((\lambda_1)_I)^c(m) \cup ((\lambda_2)_I)^c(m)$$
$$((\lambda_1)_F + (\lambda_2)_F)^c(m) = ((\lambda_1)_F)^c(m) \cup ((\lambda_2)_F)^c(m)$$
for all $n \in N = N_1 \cup N_2$

(ii)
$$((\eta_1)_T + (\eta_2)_T)^c(mn) = ((\eta_1)_T)^c(mn) \cup ((\eta_2)_T)^c(mn)$$
$$((\eta_1)_I + (\eta_2)_I)^c(mn) = ((\eta_1)_I)^c(mn) \cup ((\eta_2)_I)^c(mn)$$
$$((\eta_1)_F + (\eta_2)_F)^c(mn) = ((\eta_1)_F)^c(mn) \cup ((\eta_2)_F)^c(mn)$$
for all $mn \in L = L_1 \cup L_2$

This is valid for the Intermediate and Falsity Values also.

Consider

$((\lambda_1)_T + (\lambda_2)_T)^c(m) = ((\lambda_1)_T)^c(m) \cup ((\lambda_2)_T)^c(m)$ by the definition 3.1 and 3.2.

Assume that $\lambda_1 \cup \lambda_2 = \lambda$

$$(\lambda)_T(m) = \begin{Bmatrix} (\lambda_1)_T(m), & \text{if } m \in N_1 - N_2 \\ (\lambda_2)_T(m), & \text{if } m \in N_2 - N_1 \end{Bmatrix}^c$$

$$(\lambda)_T(m) = \begin{Bmatrix} (\lambda_1)_T(m), & \text{if } m \in N_1 - N_2 \\ (\lambda_2)_T(m), & \text{if } m \in N_2 - N_1 \end{Bmatrix}$$

$$(\lambda_1)_T)^c(m) = (\lambda_1)_T(m) \text{ if } m \in N = N_1 \cup N_2$$
$$(\lambda_2)_T)^c(m) = (\lambda_2)_T(m) \text{ if } m \in N = N_1 \cup N_2$$

$$((\lambda_1)_T + (\lambda_2)_T)^c(m) = ((\lambda_1)_T + (\lambda_2)_T)(m)$$
$$= (\lambda_1)_T(m) \cup (\lambda_2)_T(m)$$
$$((\lambda_1)_T + (\lambda_2)_T)^c(m) = ((\lambda_1)_T)^c(m) \cup ((\lambda_2)_T)^c(m)$$

$$((\lambda_1)_T + (\lambda_2)_T)^c(m) = ((\lambda_1)_T)^c(m) \cup ((\lambda_2)_T)^c(m) \quad \text{for all } m \in N.$$

Similarly the result also apply for the intermediate and falsity values. (1)

Now Consider

(i) $$((\eta_1)_T + (\eta_2)_T)^c(mn) = \begin{Bmatrix} ((\eta_1)_T \cup (\eta_2)_T)(mn) \ ; & \text{if } mn \in L_1^C \cup L_2^C \\ \min (\lambda_1)_T(m), (\lambda_2)_T(n) \ ; & \text{if } uv \in E^*, m \in N_1, n \in N_2 \end{Bmatrix}$$

(ii) $$((\eta_1)_I + (\eta_2)_I)^c(mn) = \begin{Bmatrix} ((\eta_1)_I \cup (\eta_2)_I)(mn) \ ; & \text{if } mn \in L_1^C \cup L_2^C \\ \min(\lambda_1)_I(m), (\lambda_2)_I(n) \ ; & \text{if } uv \in E^*, m \in N_1, n \in N_2 \end{Bmatrix}$$

(iii) $$((\eta_1)_F + (\eta_2)_F)^c(mn) = \begin{Bmatrix} ((\eta_1)_F \cup (\eta_2)_F)(mn) \ ; & \text{if } mn \in L_1^C \cup L_2^C \\ \max(\lambda_1)_F(m), (\lambda_2)_F(n) \ ; & \text{if } mn \in E^*, m \in N_1, n \in N_2 \end{Bmatrix}$$

$$(i) \Rightarrow (\eta_1)_T + (\eta_2)_T)^c(mn) = \begin{Bmatrix} (\eta_1)_T(mn) & ; \text{if } mn \in L_1^C \\ (\eta_2)_T(mn) & ; \text{if } mn \in L_2^C \\ \min (\lambda_1)_T(m), (\lambda_2)_T(n) \ ; & \text{if } mn \in L^*, m \in N_1, n \in N_2 \end{Bmatrix}$$

……………………(iv) $(ii) \Rightarrow ((\eta_1)_I +$

$$(\eta_2)_I)^c(mn) = \begin{Bmatrix} (\eta_1)_I(mn) & ; \text{if } mn \in L_1^C \\ (\eta_2)_I(mn) & ; \text{if } mn \in L_2^C \\ \min (\lambda_1)_I(m), (\lambda_2)_I(n) \ ; & \text{if } mn \in L^*, m \in N_1, n \in N_2 \end{Bmatrix}$$

……………………(v) $(iii) \Rightarrow ((\eta_1)_F +$

$$(\eta_2)_F)^c(mn) = \begin{Bmatrix} (\eta_1)_F(mn) & ; \text{if } mn \in L_1^C \\ (\eta_2)_F(mn) & ; \text{if } mn \in L_2^C \\ \max (\lambda_1)_F(m), (\lambda_2)_F(n) \ ; & \text{if } mn \in L^*, m \in N_1, n \in N_2 \end{Bmatrix}$$

…………………(vi)

$$(iv) \Rightarrow (\eta_1)_T)^c(mn) + (\eta_2)_T)^c(mn) = \begin{Bmatrix} \begin{cases} 0 & ; \text{ if } mn \in L_1 \\ \min[(\lambda_1)_T(m), (\lambda_1)_T(n)] & ; \text{ if } mn \in L_1^C \end{cases} \\ \begin{cases} 0 & ; \text{ if } mn \in L_2 \\ \min[(\lambda_2)_T(m), (\lambda_2)_T(n)] & ; \text{ if } mn \in L_2^C \end{cases} \\ \begin{cases} 0 & ; \text{ if } mn \in L^{*C} \\ \min[(\lambda_1)_T(m), (\lambda_2)_T(n)] & ; \text{ if } mn \in L^*, m \in N_1, n \in N_2 \end{cases} \end{Bmatrix}$$

$$(v) \Rightarrow (\eta_1)_I)^c(nm) + (\eta_2)_I)^c(mn) = \begin{Bmatrix} \begin{cases} 0 & ; \text{ if } mn \in L_1 \\ \min[(\lambda_1)_I(m), (\lambda_1)_I(n)] & ; \text{ if } mn \in L_1^C \end{cases} \\ \begin{cases} 0 & ; \text{ if } mn \in L_2 \\ \min[(\lambda_2)_I(m), (\lambda_2)_I(n)] & ; \text{ if } mn \in L_2^C \end{cases} \\ \begin{cases} 0 & ; \text{ if } mn \in L^{*C} \\ \min[(\lambda_1)_I(m), (\lambda_2)_I(n)] & ; \text{ if } mn \in L^*, m \in N_1, n \in N_2 \end{cases} \end{Bmatrix}$$

$$(vi) \Rightarrow (\eta_1)_F)^c(mn)+(\eta_2)_F)^c(mn) = \left\{ \begin{array}{l} \begin{cases} 0 & ;\ \text{if}\ \ mn \in L_1 \\ \max[(\lambda_1)_F(m),(\lambda_1)_F(n)] & ;\ \text{if}\ \ mn \in L_1^C \end{cases} \\ \begin{cases} 0 & ;\ \text{if}\ \ mn \in L_2 \\ \max[(\lambda_2)_F(m),(\lambda_2)_F(n)] & ;\ \text{if}\ \ mn \in L_2^C \end{cases} \\ \begin{cases} 0 & ;\ \text{if}\ \ mn \in L^{*c} \\ \max[(\lambda_1)_F(m),(\lambda_2)_F(n)] & ;\ \text{if}\ mn \in L^*, m \in N_1, n \in N_2 \end{cases} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \begin{Bmatrix} (\eta_1)_T(mn), & \text{if}\ \ mn \in L_1 - L_2 \\ (\eta_1)_T(mn), & \text{if}\ \ mn \in L_2 - L_1 \end{Bmatrix}^c \\ \begin{Bmatrix} (\eta_1)_I(mn), & \text{if}\ \ mn \in L_1 - L_2 \\ (\eta_1)_I(mn), & \text{if}\ \ mn \in L_2 - L_1 \end{Bmatrix}^c \\ \begin{Bmatrix} (\eta_1)_F(mn), & \text{if}\ \ mn \in L_1 - L_2 \\ (\eta_1)_F(mn), & \text{if}\ \ mn \in L_2 - L_1 \end{Bmatrix}^c \end{array} \right\} =$$

$$\left\{ \begin{array}{l} \begin{Bmatrix} ((\eta_1)_T \cup (\eta_2)_T)(mn) & ; & \text{if}\ mn \in L_1 \cup L_2 \\ \min\ (\lambda_1)_T(m),(\lambda_2)_T(n) & ; & \text{if}\ uv \in L^*, m \in N_1, n \in N_2 \end{Bmatrix}^c \\ \begin{Bmatrix} ((\eta_1)_I \cup (\eta_2)_I)(mn) & ; & \text{if}\ mn \in L_1 \cup L_2 \\ \min\ (\lambda_1)_I(m),(\lambda_2)_I(n) & ; & \text{if}\ uv \in L^*, m \in N_1, n \in N_2 \end{Bmatrix}^c \\ \begin{Bmatrix} ((\eta_1)_F \cup (\eta_2)_F)(mn) & ; & \text{if}\ mn \in L_1 \cup L_2 \\ \max\ (\lambda_1)_F(m),(\lambda_2)_F(n) & ; & \text{if}\ uv \in L^*, m \in N_1, n \in N_2 \end{Bmatrix}^c \end{array} \right\}$$

$$((\eta_1)_T + (\eta_2)_T)^c(mn) = ((\eta_1)_T)^c(nm) \cup ((\eta_2)_T)^c(mn)$$
$$((\eta_1)_I + (\eta_2)_I)^c(mn) = ((\eta_1)_I)^c(nm) \cup ((\eta_2)_I)^c(mn)$$
$$((\eta_1)_F + (\eta_2)_F)^c(mn) = ((\eta_1)_F)^c(nm) \cup ((\eta_2)_F)^c(mn)$$

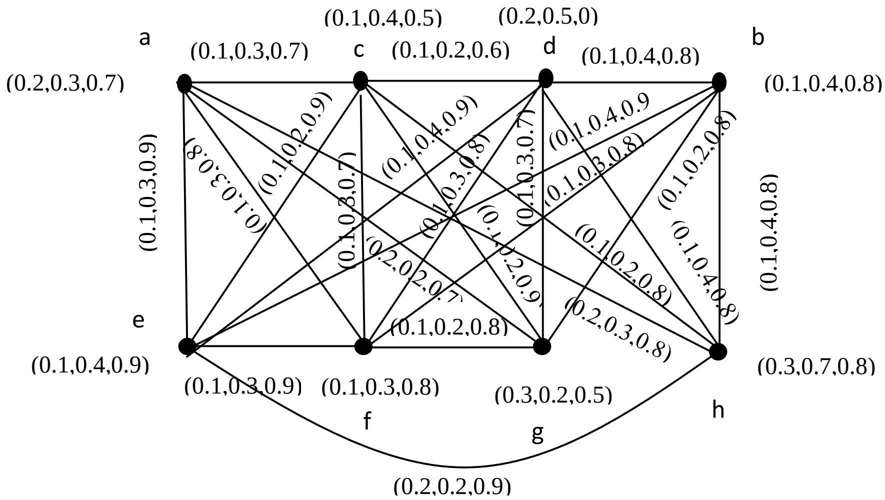
______________________ (2)

From (1) and (2) it follows that

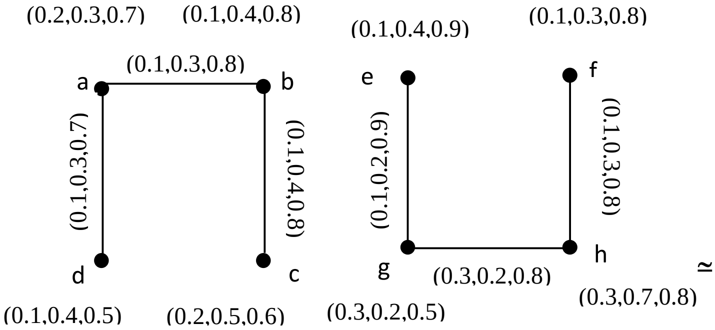
$$(SNG_1 + SNG_2)^c = (SNG_1)^c \cup (SNG_1)^c.$$

Illustration 3.9.

Consider the Strong Neutrosophic Graphs SNG_1 and SNG_2 given in Figure 3.5 and Figure 3.6.



$SNG_1 + SNG_2$
Figure 3.10



$(\mathbf{SNG_1} + \mathbf{SNG_2})^c$

Figure 3.11(a)

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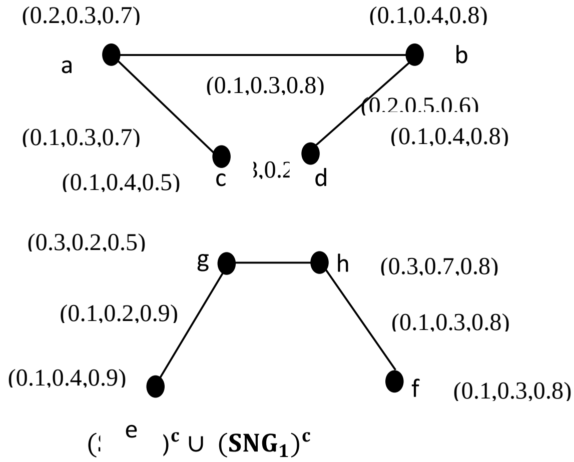


$(\quad)^c \cup (\mathbf{SNG_1})^c$

Figure 3.11(b)

Definition 3.12.

Let $SNG_1 = (\lambda_1, \eta_1)$ and $SNG_2 = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graph corresponding to the crisp graph $G_1 = (N_1, L_1)$ and $G_2 = (N_2, L_2)$ .The Product $SNG_1 \circ SNG_2$ is the pair (λ,η) of Strong Neutrosophic set defined on the Cartesian Produ $(SNG)^* = (SNG_1)^* \circ (SNG_2)^*$ such that

(i) $$\lambda_T(m_j n_k) = \min[(\lambda_1)_T(m_j), (\lambda_2)_T(n_k)],$$
$$\lambda_I(m_j n_k) = \min[(\lambda_1)_I(m_j), (\lambda_2)_I(n_k)],$$
$$\lambda_F(m_j n_k) = \max[(\lambda_1)_F(m_j), (\lambda_2)_F(n_k)], \text{ for all } (n_j, m_k) \in N_1 x N_2$$

(ii) $$\eta_T(m, m_j)(m, n_k) = \min[(\lambda_1)_T(m), (\lambda_2)_T(m_j, n_k)],$$
$$\eta_I(m, m_j)(m, n_k) = \min[(\lambda_1)_I(m), (\lambda_2)_I(m_j, n_k)],$$
$$\eta_F(m, m_j)(m, n_k) = \max[(\lambda_1)_F(m), (\lambda_2)_F(m_j, n_k)],$$
$$\text{for all } m \in N_1 \text{ and for all } (m_j n_k) \in L_2.$$

(iii) $$\eta_T(m_k, o)(n_k, o) = \min[(\lambda_1)_T(m_k, n_k), (\lambda_2)_T(o)]$$
$$\eta_I(m, m_j)(m, n_j) = \min[(\lambda_1)_I(m_k, n_k), (\lambda_2)_I(o)]$$
$$\eta_F(m, m_j)(m, n_j) = \max[(\lambda_1)_F(m_k, n_k), (\lambda_2)_F(o)]$$
$$\text{for all } (m_k n_k) \in L_1 \text{ and for all } o \in N_2.$$

Theorem 3.13.

Let $SNG_1 = (\lambda_1, \eta_1)$ and $SNG_2 = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graph corresponding to the crisp graph $G_1 = (N_1, L_1)$ and $G_2 = (N_2, L_2)$ respectively, and $SNG = (\lambda, \eta)$ be the Cartesian Product of SNG_1 and SNG_2 then $(SNG_1 \circ SNG_2)^c = (SNG_1)^c \circ (SNG_1)^c$.

Proof:

To prove that $(SNG_1 \circ SNG_2)^c = (SNG_1)^c \circ (SNG_1)^c$ it is enough to prove that

(i) $$((\lambda_1)_T \circ (\lambda_2)_T)^c(m_j n_k) = (\lambda_1)_T)^c(m_j) \circ (\lambda_2)_T)^c(n_k)$$
$$((\lambda_1)_I \circ (\lambda_2)_I)^c(m_j n_k) = (\lambda_1)_I)^c(m_j) \circ (\lambda_2)_I)^c(n_k)$$
$$((\lambda_1)_F \circ (\lambda_2)_F)^c(m_j n_k) = (\lambda_1)_F)^c(m_j) \circ (\lambda_2)_F)^c(n_k)$$
$$\forall (n_j, v_k) \in N_1 x N_2$$

(ii) $$((\eta_1)_T \circ (\eta_2)_T)^c(m, n_j)(o, n_k) = ((\eta_1)_T)^c(m, n_j) \circ ((\eta_2)_T)^c(o, n_k)$$
$$((\eta_1)_I \circ (\eta_2)_I)^c(m, n)(o, n_k) = ((\eta_1)_I)^c(m, n_j) \circ ((\eta_2)_I)^c(o, n_k)$$
$$((\eta_1)_F \circ (\eta_2)_F)^c(m, n_j)(o, n_k) = ((\eta_1)_F)^c(m, n_j) \circ ((\eta_2)_F)^c(o, n_k)$$
$$\forall n \in N_1 \text{ and for all } (v_j v_k) \in L_2$$

(iii) $$((\eta_1)_T \circ (\eta_2)_T)^c(m_i, x)(n_k, o) = ((\eta_1)_T)^c(m_i, n_k) \circ ((\eta_2)_T)^c(x, o)$$
$$((\eta_1)_I \circ (\eta_2)_I)^c(m_i, x)(n_k, o) = ((\eta_1)_I)^c(m_i, n_k) \circ ((\eta_2)_I)^c(x, o)$$
$$((\eta_1)_F \circ (\eta_2)_F)^c(m_i, x)(n_k, o) = ((\eta_1)_F)^c(m_i, n_k) \circ ((\eta_2)_F)^c(x, o)$$

$\forall\ (m_i n_k) \in\ L_1$ and $\forall\ (x, o) \in\ N_2$

by the definition 3.3 and 2.3.

(i) $((\lambda_1)_T \circ (\lambda_2)_T)^c(m_j, n_k) = \begin{cases} \min(\lambda_1)_T)^c(m_j) \circ (\lambda_2)_T)^c(n_k) \\ \min\ (\lambda_1)_T)(m_j) \circ (\lambda_2)_T)(n_k) \end{cases}$

$\forall (m_j, n_k) \in\ N_1 x N_2$

(ii) $((\lambda_1)_I \circ (\lambda_2)_I)^c(m_j, n_k) = \begin{cases} \min(\lambda_1)_I)^c(m_j) \circ (\lambda_2)_I)^c(n_k) \\ \min\ (\lambda_1)_I)(m_j) \circ (\lambda_2)_I)(n_k) \end{cases}$

$\forall (u_j, v_k) \in\ N_1 x N_2$

(iii) $((\lambda_1)_F \circ (\lambda_2)_F)^c(m_j, n_k) = \begin{cases} \min(\lambda_1)_F)^c(m_j) \circ (\lambda_2)_F)^c(n_k) \\ \min\ (\lambda_1)_F)(m_j) \circ (\lambda_2)_F)(n_k) \end{cases}$

$\forall (u_j, v_k) \in\ N_1 x N_2$

Thus $((\lambda_1)_T \circ (\lambda_2)_T)^c(m_j, n_k) = (\lambda_1)_T)^c(m_j) \circ (\lambda_2)_T)^c(n_k)$

$((\lambda_1)_I \circ (\lambda_2)_I)^c(m_j, n_k) = (\lambda_1)_I)^c(m_j) \circ (\lambda_2)_I)^c(n_k)$

$((\lambda_1)_F \circ (\lambda_2)_F)^c(m_j, n_k) = (\lambda_1)_F)^c(m_j) \circ (\lambda_2)_F)^c(n_k)$

(By Definition 3.3)

$$((\eta_1)_T \circ (\eta_2)_T)^c(m, n_j)(o, n_k) = \begin{Bmatrix} \min((\lambda_1)_T(o), (\eta_2)_T(n_j, n_k): \forall\ m = o \in N_1\ and\ n_j n_k \in L_2 \text{ or n isolated in } N_1 \\ \min((\eta_1)_T(m, o), (\lambda_2)_T(n_j): \forall\ mo \in\ L_1\ and\ n_j = n_k \in N_2 \text{ or } m_j \text{ isolated in } N_2 \\ \min((\eta_1)_T(n, l), (\eta_2)_T(m_j, m_k): \forall\ mo \in\ L_1\ and\ n_j n_k \in\ L_2 \end{Bmatrix}^c$$

(By Definition 2.3)

$$((\eta_1)_T \circ (\eta_2)_T)^c(m, n_j)(o, n_k) = \begin{Bmatrix} \min((\lambda_1)_T(o), (\eta_2)_T(n_j, n_k): \forall\ m = o \in\ N_1 \text{ and } n_j n_k \in\ L_2^C \text{or n isolated in } N_1 \\ \min((\eta_1)_T(m, o), (\lambda_2)_T(n_j): \forall\ mo \notin\ L_2 \text{ and } n_j = n_k \in\ N_2 \text{ or } n_j \text{ isolated in } N_2 \\ \min((\eta_1)_T(m, o), (\eta_2)_T(n_j, n_k): \forall\ mo \in\ L_1 \text{ and } n_j, n_k \in\ L_2^C \\ \min((\eta_1)_T(m, o), (\eta_2)_T(n_j, n_k): \forall\ mo \notin\ L_1 \text{ and } n_j n_k \in\ L_2 \end{Bmatrix}$$

$$(\eta_1)_T \circ (\eta_2)_T)^c(m, n_j)(o, n_k) = \begin{Bmatrix} \min((\lambda_1)_T(o), (\eta_2)_T(n_j, n_k): \forall\ m = o \in\ N_1 \text{ and } n_j n_k \in\ L_2 \text{ or m isolated in } N_1 \\ \min((\eta_1)_T(m, o), (\lambda_2)_T(n_j): \forall\ mo \in\ L_1 \text{ and } n_j = n_k \in\ N_2 \text{ or } n_j \text{ isolated in } N_2 \\ \min((\eta_1)_T(m, o), (\eta_2)_T(n_j, n_k): \forall\ mo \in\ L_1 \text{ and } n_j n_k \in\ L_2 \end{Bmatrix}$$

$$((\eta_1)_T)^c(n_j, n_k) = \begin{cases} 0 & ;\ \text{if}\quad n_j n_k \in L_1 \\ \min[(\lambda_1)_T(m_j), (\lambda_1)_T(n_k)] & ;\ \text{if}\quad n_j n_k \in L_1^C \end{cases}$$

$$((\eta_2)_T)^c(n_j, n_k) = \begin{cases} 0 & ;\ \text{if}\quad n_j n_k \in L_2 \\ \min[(\lambda_2)_T(m_j), (\lambda_2)_T(n_k)] & ;\ \text{if}\quad n_j n_k \in L_2^C \end{cases}$$

$$((\eta_1)_T \circ (\eta_2)_T)^c(m, n_j)(o, n_k) = \begin{cases} 0 & ;\ \text{if}\quad m = o \in\ N_1 \text{ and } n_j, n_k \in\ L_2^C \\ \min[(\lambda_1)_T(m_j), (\lambda_2)_T(m_k)] & ;\ \text{if}\quad mo \in\ L_1^C \text{and } n_j n_k \in\ L_2 \end{cases}$$

$$((\eta_1)_T \circ (\eta_2)_T)^c(m, n_j)(o, n_k) = ((\eta_1)_T)^c(m, n_j) \circ ((\eta_2)_T)^c(o, n_k)$$

Similarly the results also apply for the intermediate and falsity values.

$$(\eta_1)_T \circ (\eta_2)_T)^c(n_i, x)(n_k, o) = \begin{Bmatrix} \min((\lambda_1)_T(o), (\eta_2)_T(n_i, n_k): \forall\ x = o \in\ N_2 \text{ and } n_i n_k \in\ L_1 \text{ or } n_i \text{ isolated in } N_2 \\ \min((\eta_1)_T(x, o), (\lambda_2)_T(n_i): \forall\ xo \in\ L_1 \text{ and } n_i = n_k \in\ N_2 \text{ or } n_i \text{ isolated in } N_2 \\ \min((\eta_1)_T(x, o), (\eta_2)_T(n_i, n_k): \forall\ xo \in\ L_2 \text{ and } n_i n_k \in\ L_1 \end{Bmatrix}^C$$

$$((\eta_1)_T \circ (\eta_2)_T)^c(n_i, x)(n_k, o) \quad = \begin{cases} \min((\lambda_1)_T(o), (\eta_2)_T(n_i, n_k)\text{: } \forall\, x = o \in N_2 \text{ and } n_i n_k \in L_1^C \text{or y isolated in } N_2 \\ \min((\eta_1)_T(x, o), (\lambda_2)_T(n_j)\text{: } \forall\, xo \in E_1^C \text{and } n_i = n_k \in N_2 \text{ or } n_k \text{ isolated in } N_2 \\ \min((\eta_1)_T(x, o), (\eta_2)_T(n_i, n_k)\text{: } \forall\, xo \in L_2 \text{ and } n_i n_k \in L_1^C \\ \min((\eta_1)_T(x, o), (\eta_2)_T(n_i, n_k)\text{: } \forall\, xo \in L_2^C \text{and } n_i n_k \in L_1 \end{cases}$$

$$(\eta_1)_T \circ (\eta_2)_T)^c(n_i, x)(n_k, o) = \begin{cases} \min((\lambda_1)_T(o), (\eta_2)_T(n_i, n_k)\text{: } \forall\, x = o \in N_2 \text{ and } n_i n_k \in L_1 \text{ or } n_i \text{isolated in } N_2 \\ \min((\eta_1)_T(n, o), (\lambda_2)_T(n_i)\text{: } \forall\, xo \in L_1 \text{ and } n_i = n_k \in N_2 \text{ or } n_i \text{ isolated in } N_2 \\ \min((\eta_1)_T(x, o), (\eta_2)_T(n_i, n_k)\text{: } \forall\, xo \in L_2 \text{ and } n_i n_k \in L_1 \end{cases}$$

$$((\eta_1)_T)^c(n_i, n_k) = \begin{cases} 0 & ; \text{ if } \quad n_i n_k \in L_1 \\ \min[(\lambda_1)_T(m_i), (\lambda_1)_T(m_k)] & ; \text{ if } \quad n_i n_k \in L_1^C \end{cases}$$

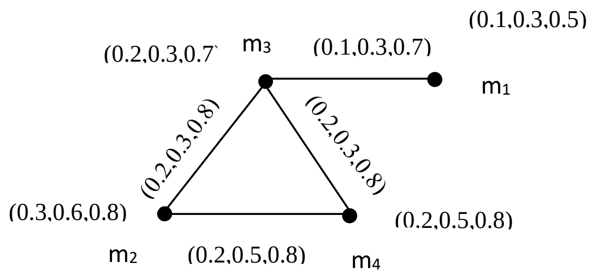
$$((\eta_2)_T)^c(n_i, n_k) = \begin{cases} 0 & ; \text{ if } \quad n_i n_k \in L_2 \\ \min[(\lambda_2)_T(m_i), (\lambda_2)_T(m_k)] & ; \text{ if } \quad n_i n_k \in L_2^C \end{cases}$$

$$((\eta_1)_T \circ (\eta_2)_T)^c(n_i, x)(n_k, o) = \begin{cases} 0 & ; \text{ if } \quad x = o \in N_2 \text{ and } n_i n_k \in L_2^C \\ \min[(\lambda_1)_T(n_i), (\lambda_2)_T(n_k)] & ; \text{ if } \quad xo \in L_2^C \text{and } n_i n_k \in L_1 \end{cases}$$

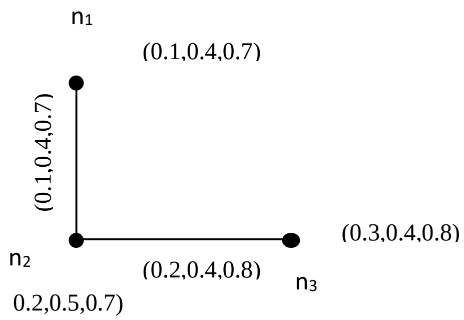
$$((\eta_1)_T \circ (\eta_2)_T)^c(n_i, x)(n_k, o) = ((\eta_1)_T)^c(n_i, n_k) \circ ((\eta_2)_T)^c(x, o)$$

Similarly the results also apply for the intermediate, falsity value the result holds.

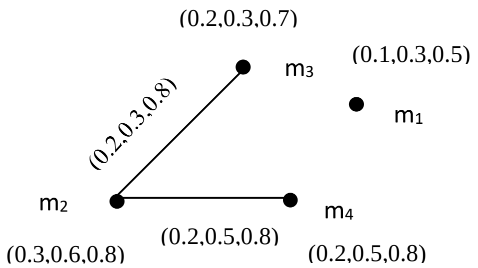
Illustration 3.14.



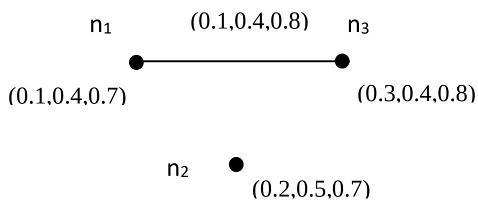
$(\mathbf{SNG_1})$
Figure 3.15 (a)



$(\mathbf{SNG_2})$
Figure 3.15 (b)



$(\mathbf{SNG_1})^{\mathbf{c}}$
Figure 3.16 (a)



$(\mathbf{SNG_2})^{\mathbf{c}}$
Figure 3.16 (b)

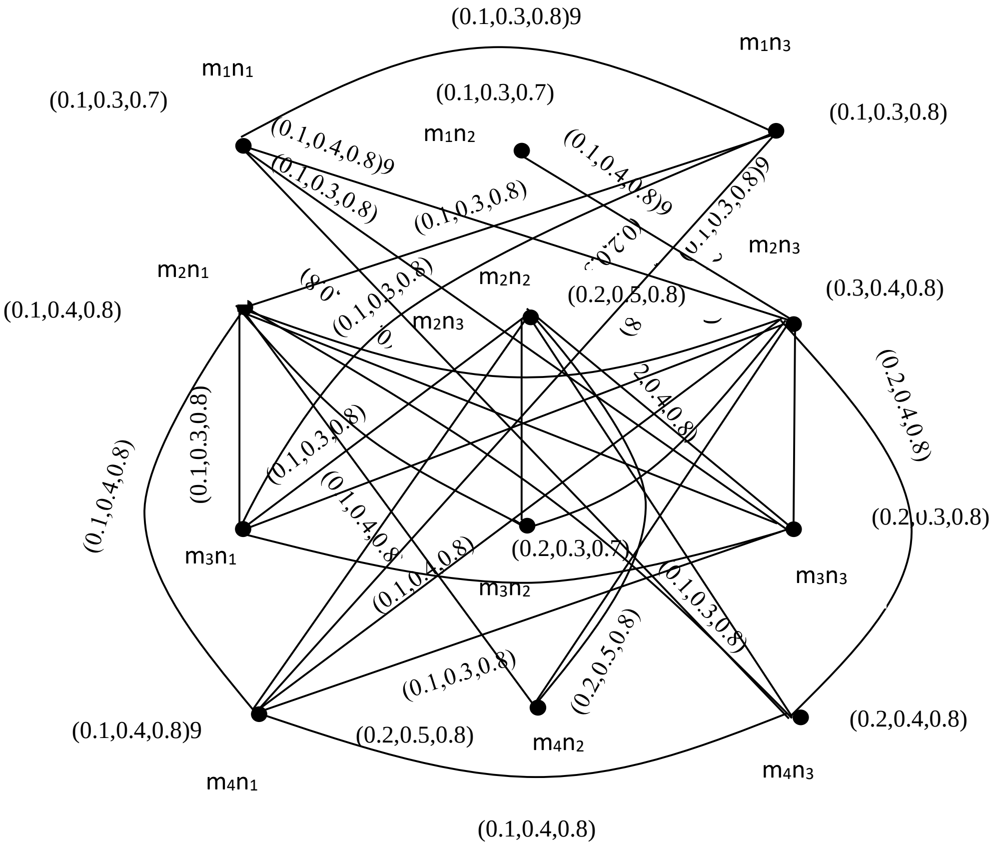
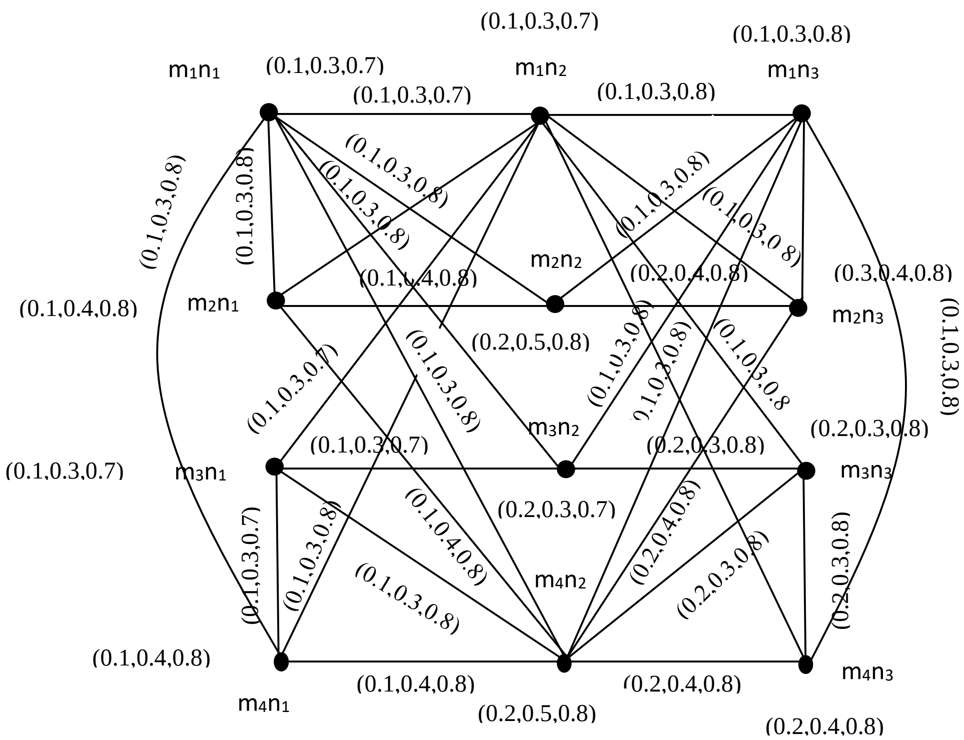


Figure 3.17

$(\mathbf{SNG_1})^c \circ (\mathbf{SNG_1})^c$



$(\mathbf{SNG_1} \circ \mathbf{SNG_2})^c$

Figure 3.18

$$(\mathbf{SNG_1})^c \circ (\mathbf{SNG_1})^c = (\mathbf{SNG_1} \circ \mathbf{SNG_2})^c$$

4. CONCLUSION

The strong neutrosophic graph is generalized information of the notion of fuzzy sets. We shall discuss the some opeartions establishes a particular cases of fuzzy set theory. The Strong Neutrosophic Graph have more precisions, Neutrosophic Graph, and Single Valued Neutrosophic Sets. We have defined for the strong Neutrosophic Graph, complement of union, join, self complementary and product of Strong Neutrosophic Graphs are also characterized.

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