



Intuitionistic Fuzzy Supra Contra Semi-Homeomorphism

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ARTICLE INFO	ABSTRACT
<p>Published online: 07 August 2023</p> <p>Corresponding Name N. Chitradevi</p>	<p>The aim of the present paper is to introduce two classes of open mappings using intuitionistic fuzzy supra open sets and semi-supra open sets in intuitionistic fuzzy supra topological spaces. It is nothing but, the image of intuitionistic fuzzy supra open set (resp. supra open) is intuitionistic fuzzy semi-supra closed set (resp. supra closed) in co-domain space. Moreover, a few of their significant properties have been studied. A necessary and sufficient condition for intuitionistic fuzzy supra contra semi-open mapping has been derived in terms of semi-supra closure and semi-supra interior. Also, the class of intuitionistic fuzzy supra contra open mappings is properly contained in that of intuitionistic fuzzy supra contra semi-open mappings has been investigated. Suitable examples have been given to establish that the reversible implications are lacking in general. A notion of intuitionistic fuzzy supra contra semi-homeomorphism has been defined and characterized.</p>
<p>KEYWORDS: intuitionistic fuzzy semi-supra open set, semi s-cl, semi s-int, intuitionistic fuzzy contra open mappings.</p> <p>MSC: AMS Classification 2010: 54A40,03F55</p>	

1 INTRODUCTION

There have been plenty of applications of fuzzy sets introduced by Zadeh [5] in 1965. One among them is intuitionistic fuzzy sets systems introduced by Atanassov [4] in 1986. In 1996, Dontchev [3] introduced the notion of contra continuity in classical topological spaces. In 1997, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. In 1999, Necla Turanh [9] introduced the concept of intuitionistic fuzzy supra topological spaces, a weaker form topological space. Manimaran et.al. studied the notion of intuitionistic fuzzy contra-open mappings. In 2015, M.Parimala et al. [8] developed the concept of intuitionistic fuzzy semi-supra open sets and its basic properties. We introduce the notion of intuitionistic fuzzy supra contra semi-open mappings which is weaker form of intuitionistic fuzzy supra contra -open mappings. Moreover, the concept of intuitionistic fuzzy supra contra -semi homeomorphism has been developed.

1.1 Preliminaries

Definition 1.1 [4] Let X be a non-empty fixed set and I be the closed interval $[0,1]$. An intuitionistic fuzzy set (in short IFS) A is an object of the following form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ where the mapping $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Clearly, every fuzzy set A on a non-empty set X is an intuitionistic fuzzy set of the following form $A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}$.

Definition 1.2 [4] Let A and B are intuitionistic fuzzy sets of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \} \text{ and}$$

$$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \} . \text{ Then}$$

- (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
- (ii) $\bar{A} = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$;
- (iii)

$$A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$$

;

(iv)

$$A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$$

;

(v) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(vi) $1. = \{ \langle x, 1, 0 \rangle, x \in X \}$ and

$$0. = \{ \langle x, 0, 1 \rangle, x \in X \};$$

Theorem 1.3 [2] Let A, A_1, A_2 and B, B_1, B_2 be an intuitionistic fuzzy sets in X and Y respectively and $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then

(i) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(ii) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(iii) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$,

(iv) $f(f^{-1}(B)) \subseteq B$ and if f is surjective, then $B = f(f^{-1}(B))$,

(v) $f^{-1}(B^c) = (f^{-1}(B))^c$

(vi) $(f(A))^c \subseteq f(A^c)$ if f is surjective. In addition,

if f is injective, then $(f(A))^c = f(A^c)$

Definition 1.4 [2] Let X and Y be two non empty sets and $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function.

If $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle : y \in Y \}$ is an

intuitionistic fuzzy set in Y , then the inverse image of B under f is denoted by $f^{-1}(B)$ is the intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle : x \in X \},$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ and

$$f^{-1}(\nu_B)(x) = \nu_B(f(x)).$$

If $A = \{ \langle y, \mu_A(y), \nu_A(y) \rangle : y \in Y \}$ is an intuitionistic fuzzy set in X , then the image of A under f is denoted by $f(A)$ is the intuitionistic fuzzy set in Y defined by

$$f(A) = \{ \langle y, f(\mu_A(y)), 1 - (f(1 - \nu_A))(y) \rangle : y \in Y \}$$

. Where,

$$(f(\mu_A))(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

$$1 - (f(1 - \nu_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

Definition 1.5 [9] A family τ of intuitionistic fuzzy set`s on

X is called an intuitionistic fuzzy supra topology(IFST in short) on X if $0. \in \tau, 1. \in \tau$, and τ is closed under arbitrary suprema. Then we call the pair (X, τ) an intuitionistic fuzzy supra topological space (IFSTS in short). Each member of τ is called an intuitionistic fuzzy supra open set and the complement of an intuitionistic fuzzy supra open set is called an intuitionistic fuzzy supra closed set. The intuitionistic fuzzy supra closure of intuitionistic fuzzy set A is denoted by $s-cl(A)$. Here, $s-cl(A)$ is the intersection of all intuitionistic fuzzy supra closed sets containing A . The intuitionistic fuzzy supra interior of A will be denoted by $s-int(A)$. Here, $s-int(A)$ is the union of all intuitionistic fuzzy supra open sets contained in A .

Definition 1.6 [8] The intuitionistic fuzzy semi-supra interior of a set A is denoted by $semi\ s-int(A) = \bigcup \{G : G \text{ is an intuitionistic fuzzy semi-supra open set in } X \text{ and } G \subseteq A\}$ and the intuitionistic fuzzy semi-supra closure of a set A is denoted by $semi\ s-cl(A) = \bigcap \{G : G \text{ is an intuitionistic fuzzy semi-supra closed set in } X \text{ and } G \supseteq A\}$.

Definition 1.7 [10] Let (X, τ) be an intuitionistic fuzzy supra topological space. An intuitionistic fuzzy set $A \in IF(X)$ is called intuitionistic fuzzy semi-supra open iff $A \subseteq s-cl(s-int(A))$, The complement of an intuitionistic fuzzy semi-supra open set is called intuitionistic fuzzy semi-supra closed set.

Theorem 1.8 [8] Every intuitionistic fuzzy supra open set is intuitionistic fuzzy semi-supra open set.

Remark 1.9 [8] $semi\ s-int(A)$ is an intuitionistic fuzzy semi-supra open set and $semi\ s-cl(A)$ is an intuitionistic fuzzy semi-supra closed set.

Definition 1.10 [8] Let (X, τ) and (Y, σ) be two intuitionistic fuzzy supra topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called intuitionistic fuzzy semi-supra continuous map if the inverse image of each intuitionistic fuzzy supra open set in Y is intuitionistic fuzzy semi-supra open in X .

Definition 1.11 [6] Let (X, τ) and (Y, σ) be an intuitionistic fuzzy topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called intuitionistic fuzzy supra contra continuous map if the inverse image of each

intuitionistic fuzzy supra open set in Y is intuitionistic fuzzy supra closed in X .

Definition 1.12 [7] Let (X, τ) and (Y, σ) be an intuitionistic fuzzy supra topological spaces (in short IFSTS). A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called intuitionistic fuzzy supra contra semi-continuous map if the inverse image of each intuitionistic fuzzy supra open set in Y is intuitionistic fuzzy semi-supra closed in X .

Definition 1.13 [1] A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy contra open mapping if $f(A)$ is an intuitionistic fuzzy closed set in (Y, σ) for each open set A in (X, τ) .

2 INTUITIONISTIC FUZZY SUPRA CONTRA SEMI-HOMEOMORPHISM

Definition 2.1 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy supra contra open mapping if the image of each open set in (X, τ) is intuitionistic fuzzy supra closed set in (Y, σ) .

Example 2.2 Let $X = \{a, b\}$,
 $A = \{ \langle a, 0.4, 0.3 \rangle, \langle b, 0.6, 0.3 \rangle \}$ and
 $\tau = \{0, 1, A\}$. Let $Y = \{u, v\}$,
 $B = \{ \langle u, 0.3, 0.6 \rangle, \langle v, 0.3, 0.4 \rangle \}$,
 $C = \{ \langle u, 0.2, 0.4 \rangle, \langle v, 0.5, 0.2 \rangle \}$,
 $D = \{ \langle u, 0.3, 0.4 \rangle, \langle v, 0.5, 0.2 \rangle \}$ and
 $\sigma = \{0, 1, B, C, D\}$. Then (X, τ) and (Y, σ) are intuitionistic fuzzy supra topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$, that is defined by $f(a) = v, f(b) = u$, is an intuitionistic fuzzy supra contra open mapping.

Definition 2.3 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an intuitionistic fuzzy supra contra semi-open mapping if for each $A \in \tau, f(A)$ is an intuitionistic fuzzy semi-supra closed set in (Y, σ) .

Example 2.4 Let $X = \{a, b\}$,
 $A = \{ \langle a, 0.4, 0.3 \rangle, \langle b, 0.5, 0.4 \rangle \}$ and
 $\tau = \{0, 1, A\}$. Let $Y = \{p, q\}$,
 $P = \{ \langle p, 0.3, 0.6 \rangle, \langle q, 0.2, 0.4 \rangle \}$
 $Q = \{ \langle p, 0.2, 0.4 \rangle, \langle q, 0.2, 0.3 \rangle \}$
 $R = \{ \langle p, 0.3, 0.4 \rangle, \langle q, 0.2, 0.3 \rangle \}$ and

$\sigma = \{0, 1, P, Q, R\}$. Now, (X, τ) and (Y, σ) are intuitionistic fuzzy supra topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$, that is defined by $f(a) = q, f(b) = p$, is an intuitionistic fuzzy supra contra semi-open mapping.

Theorem 2.5 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then the following are equivalent.

- (i) f is an intuitionistic fuzzy supra contra semi-open mapping.
- (ii) $f(A)$ is an intuitionistic fuzzy semi-supra open set in (Y, σ) for every intuitionistic fuzzy supra closed set A in (X, τ) .

Proof. (i) \Rightarrow (ii) Let A be an intuitionistic fuzzy supra closed set in (X, τ) . Then, A^c is an intuitionistic fuzzy supra open set in (X, τ) . By hypothesis, $f(A)^c = (f(A))^c$ is intuitionistic fuzzy semi-supra closed in (Y, σ) . Therefore, $f(A)$ is an intuitionistic fuzzy semi-supra open in (Y, σ) .

(ii) \Rightarrow (i) Let A be an intuitionistic fuzzy supra open set in (X, τ) . Then, A^c is an intuitionistic fuzzy supra closed set in (X, τ) . By hypothesis, $f(A)^c = (f(A))^c$ is intuitionistic fuzzy semi-supra open in (Y, σ) . Therefore, $f(A)$ is an intuitionistic fuzzy semi-supra closed in (Y, σ) . Thus, f is an intuitionistic fuzzy supra contra semi-open mapping.

Theorem 2.6 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. suppose that one of the following properties hold.

- (i) $f^{-1}(semi\ s - cl(A)) \subseteq s - int(f^{-1}(A))$ for each intuitionistic fuzzy set A in (X, τ)
- (ii) $semi\ s - cl(f(B)) \subseteq f(s - int(B))$ for each intuitionistic fuzzy set B in (Y, σ)
- (iii) $f(s - cl(B)) \subseteq semi\ s - int(f(B))$ for each for each intuitionistic fuzzy set B in (Y, σ) .

Then f is an intuitionistic fuzzy supra contra semi-open mapping.

Proof. (i) \Rightarrow (ii) Let B be any intuitionistic fuzzy set in (Y, σ) . Then $f(B) = A$ is an intuitionistic fuzzy set in

(X, τ) . By hypothesis,

$f^{-1}(semi\ s - cl(A)) \subseteq s - int(f^{-1}(A))$. This implies
 $f^{-1}(semi\ s - cl(f(B))) \subseteq s - int(f^{-1}(f(B))) \subseteq s - int(B)$
 . Thus, $semi\ s - cl(f(B)) \subseteq f(s - intB)$.

(ii) \Rightarrow (iii) : Let B be any intuitionistic fuzzy set in (Y, σ) and so B^c . By hypothesis,

$semi\ s - cl(f(B^c)) \subseteq f(s - int(B^c))$. Taking complement on both sides, we get $(f(s - int(B^c)))^c \subseteq (semi\ s - cl(f(B^c)))^c$. This implies $f(s - cl(B)) \subseteq semi\ s - int(f(B))$

(iii) \Rightarrow intuitionistic fuzzy supra contra semi-open mapping:

Let A be any intuitionistic fuzzy supra closed set in (X, τ) . Then, $A = s - cl(A)$ and so $f(A)$ is an intuitionistic fuzzy set in (Y, σ) . By hypothesis, $f(A) = f(s - cl(A)) \subseteq semi\ s - int(f(A))$. Thus, $f(A) \subseteq semi\ s - int(f(A))$. Always, $semi\ s - int(f(A)) \subseteq f(A)$. Therefore, $f(A) = semi\ s - int(f(A))$. Thus, $f(A)$ is intuitionistic fuzzy semi-supra open in (Y, σ) . Hence, f is an intuitionistic fuzzy supra contra semi-open mapping.

Theorem 2.7 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then f is an intuitionistic fuzzy supra contra semi-open mapping if $s - cl(f^{-1}(A)) \subseteq f^{-1}(semi\ s - int(A))$ for each intuitionistic fuzzy set A in (Y, σ) .

Proof. Let A be any intuitionistic fuzzy supra closed set in (X, τ) . Then, $s - cl(A) = A$ and $f(A) = B$ is an intuitionistic fuzzy set in (Y, σ) . By hypothesis, $s - cl(f^{-1}(B)) \subseteq f^{-1}(semi\ s - int(B))$. This implies $s - cl(f^{-1}(f(A))) \subseteq f^{-1}(semi\ s - int(f(A)))$. since f is injective, $A = f^{-1}(f(A))$. Therefore, $A = s - cl(A) = s - cl(f^{-1}(f(A))) \subseteq f^{-1}(semi\ s - int(f(A)))$. Now, $f(A) \subseteq f(f^{-1}(semi\ s - int(f(A))))$. Thus, $= semi\ s - int(f(A))$. Always, $semi\ s - int(f(A)) \subseteq f(A)$. Therefore, $f(A) = semi\ s - int(f(A))$. Thus, $f(A)$ is intuitionistic fuzzy semi-supra open in (Y, σ) . Hence, f is

an intuitionistic fuzzy supra contra semi-open mapping.

Theorem 2.8 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective intuitionistic fuzzy supra contra semi-open mapping, then the following conditions hold:

- (i) $semi\ s - cl(f(B)) \subseteq f(semi\ s - int(semi\ s - cl(B)))$ for every intuitionistic fuzzy supra open set (X, τ) .
- (ii) $f(semi\ s - cl(semi\ s - int(B))) \subseteq semi\ s - int(f(B))$ for every intuitionistic fuzzy supra closed set A in (X, τ) .

Proof. (i). Let B be any intuitionistic fuzzy supra open set in (X, τ) . Then, $s - int(B) = B$. By hypothesis, $f(B)$ is an intuitionistic fuzzy semi-supra closed set in (Y, σ) . That is, By [8],

$semi\ s - cl(f(B)) = f(B) = f(s - int(B))$. Always, $s - int(B) \subseteq semi\ s - int(B)$. Then $semi\ s - cl(f(B)) \subseteq f(semi\ s - int(B)) \subseteq f(semi\ s - int(semi\ s - cl(B)))$

Therefore $semi\ s - cl(f(B)) \subseteq f(semi\ s - int(semi\ s - cl(B)))$

(ii). Let B be any intuitionistic fuzzy supra closed set in (X, τ) . Then B^c is intuitionistic fuzzy supra open set in (X, τ) . By (i),

$semi\ s - cl(f(B^c)) \subseteq f(semi\ s - int(semi\ s - cl(B^c)))$. Taking complement on both sides, we get $(f(semi\ s - int(semi\ s - cl(B^c))))^c \subseteq (semi\ s - cl(f(B^c)))^c$. This implies $f(semi\ s - cl(semi\ s - int(B))) \subseteq (semi\ s - int(f(B)))$

Theorem 2.9 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-open mapping if $f(semi\ s - cl(B)) \subseteq semi\ s - int(f(B))$ for every intuitionistic fuzzy set B in (X, τ) .

Proof. Let B be any intuitionistic fuzzy supra closed set in (X, τ) . Then, $s - cl(B) = B$. By [8], every intuitionistic fuzzy supra closed set is intuitionistic fuzzy semi-supra closed set. Therefore, B is intuitionistic fuzzy

semi-supra closed set. That is, By [8] ,

$$semi\ s - cl(B) = B .$$

This implies

$$f(B) = f(semi\ s - cl(B)) .$$

By hypothesis,

$$f(B) = f(semi\ s - cl(B)) \subseteq semi\ s - int(f(B)) .$$

Therefore, $f(B) \subseteq semi\ s - int(f(B))$. Always,

$$semi\ s - int(f(B)) \subseteq f(B) .$$

Thus,

$$f(B) = semi\ s - int(f(B)) .$$

That is, $f(B)$ is

intuitionistic fuzzy semi-supra open set in (Y, σ) . Hence,

f is an intuitionistic fuzzy supra contra semi-open mapping.

Theorem 2.10 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-open mapping iff

$$f(semi\ s - cl(B)) \subseteq semi\ s - int(f(s - cl(B)))$$

for every intuitionistic fuzzy set B in (X, τ) .

Proof. Necessity : Let B be any intuitionistic fuzzy supra open set in (X, τ) . Then, $s - cl(B)$ is an intuitionistic fuzzy supra closed set in (X, τ) . By hypothesis, $f(s - cl(B))$ is an intuitionistic fuzzy semi-supra open set in (Y, σ) . That is, By [8],

$$f(s - cl(B)) = semi\ s - int(f(s - cl(B))) .$$

Always,

$$semi\ s - cl(B) \subseteq s - cl(B) .$$

Then

$$f(semi\ s - cl(B)) \subseteq f(s - cl(B)) .$$

Therefore,

$$f(semi\ s - cl(B)) \subseteq semi\ s - int(f(s - cl(B))) .$$

sufficiency : Let B be any intuitionistic fuzzy supra closed set in (X, τ) . Then, $s - cl(B) = B$. By hypothesis, $f(semi\ s - cl(B)) \subseteq semi\ s - int(f(s - cl(B))) = semi\ s - int(f(B))$.

Always, $B \subseteq semi\ s - cl(B)$, then

$$f(B) \subseteq f(semi\ s - cl(B)) \subseteq semi\ s - int(f(B)) .$$

Therefore, $f(B) \subseteq semi\ s - int(f(B))$. Always,

$$semi\ s - int(f(B)) \subseteq f(B) .$$

Thus,

$$f(B) = semi\ s - int(f(B)) .$$

Thus, $f(B)$ is an

intuitionistic fuzzy semi-supra open set in (Y, σ) . Hence

f is an intuitionistic fuzzy supra contra semi-open mapping.

Theorem 2.11 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-open mapping, then

$f(s - cl(B)) = semi\ s - int(f(s - cl(B)))$ for every intuitionistic fuzzy set B in (X, τ) .

Proof. Let B be any intuitionistic fuzzy supra open set in (X, τ) . Then, $s - cl(B)$ is an intuitionistic fuzzy supra closed set in (X, τ) . By hypothesis, $f(s - cl(B))$ is an intuitionistic fuzzy semi-supra open set in (Y, σ) . By [8], $f(s - cl(B)) = semi\ s - int(f(s - cl(B)))$. Hence, $f(s - cl(B)) = semi\ s - int(f(s - cl(B)))$.

Theorem 2.12 Every intuitionistic fuzzy supra contra open mapping is an intuitionistic fuzzy supra contra semi-open mapping.

Proof. Let A be any intuitionistic fuzzy supra open set in (X, τ) . By hypothesis, $f(A)$ is an intuitionistic fuzzy supra closed set in (Y, σ) . By [8], every intuitionistic fuzzy supra closed set is intuitionistic fuzzy semi-supra closed set. Thus, $f(A)$ is an intuitionistic fuzzy semi-supra closed set in (Y, σ) . Hence, f is intuitionistic fuzzy supra contra semi-open mapping.

Remark 2.13 From the following example, it is clear that converse of the above theorem is not true.

Example 2.14 Let $X = \{a, b\}$,

$$A = \{ \langle a, 0.3, 0.5 \rangle, \langle b, 0.4, 0.4 \rangle \}$$

and

$$\tau = \{0, 1, A\} .$$

Let $Y = \{s, t\}$,

$$B = \{ \langle s, 0.3, 0.4 \rangle, \langle t, 0.2, 0.5 \rangle \},$$

$$C = \{ \langle s, 0.2, 0.6 \rangle, \langle t, 0.7, 0.2 \rangle \},$$

$$D = \{ \langle s, 0.3, 0.4 \rangle, \langle t, 0.7, 0.2 \rangle \}$$

and

$$\sigma = \{0, 1, B, C, D\} .$$

Then, (X, τ) and (Y, σ) are

intuitionistic fuzzy supra topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$, that is defined by $f(a) = t, f(b) = s$, is an intuitionistic fuzzy supra contra semi-open mapping but not an intuitionistic fuzzy supra contra open mapping.

Theorem 2.15 Every intuitionistic fuzzy supra contra open mapping is an intuitionistic fuzzy supra contra semi-open mapping.

Proof. Let A be any intuitionistic fuzzy supra open set in (X, τ) . By hypothesis, $f(A)$ is an intuitionistic fuzzy supra closed set in (Y, σ) and hence intuitionistic fuzzy semi-closed set in (Y, σ) . Since, every intuitionistic fuzzy semi-closed set is intuitionistic fuzzy semi-supra closed set in (Y, σ) , $f(A)$ is an intuitionistic fuzzy semi-supra closed set in (Y, σ) . Therefore, f is an intuitionistic fuzzy supra

contra semi-open mapping.

Remark 2.16 The following example shows that the converse of the above theorem is not possible always.

Example 2.17 Let $X = \{a, b\}$,
 $A = \{ \langle a, 0.6, 0.4 \rangle, \langle b, 0.4, 0.3 \rangle \}$ and
 $\tau = \{0, 1, A\}$. Let $Y = \{p, q\}$,
 $B = \{ \langle p, 0.2, 0.5 \rangle, \langle q, 0.3, 0.6 \rangle \}$,
 $C = \{ \langle p, 0.3, 0.6 \rangle, \langle q, 0.5, 0.4 \rangle \}$,
 $D = \{ \langle s, 0.3, 0.5 \rangle, \langle t, 0.5, 0.4 \rangle \}$ and
 $\sigma = \{0, 1, B, C, D\}$. Then, (X, τ) and (Y, σ) are intuitionistic fuzzy supra topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$, that is defined by $f(a) = t, f(b) = s$, is an intuitionistic fuzzy supra contra semi-open mapping but not an intuitionistic fuzzy supra open mapping.

Definition 2.18 Let (X, τ) and (Y, σ) be any two intuitionistic fuzzy supra topological spaces. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy supra contra (resp. intuitionistic fuzzy supra contra semi-) bi-continuous if $f : (X, \tau) \rightarrow (Y, \sigma)$ and $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ are intuitionistic fuzzy supra contra (resp. intuitionistic fuzzy supra contra semi-) continuous.

Definition 2.19 A bijective mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy supra contra (resp. intuitionistic fuzzy supra contra semi-) homeomorphism if f is intuitionistic fuzzy supra contra bi-continuous (resp. IF supra contra semi bi-continuous) mapping.

Theorem 2.20 Every intuitionistic fuzzy supra contra homeomorphism is an intuitionistic fuzzy supra contra semi-homeomorphism.

Proof. Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy supra contra homeomorphism. By hypothesis, f is bijective and intuitionistic fuzzy supra contra bi-continuous. Since every intuitionistic fuzzy supra contra continuous is intuitionistic fuzzy supra contra semi-continuous, f is intuitionistic fuzzy supra contra semi bi-continuous. Thus, the mapping f is intuitionistic fuzzy supra contra semi-homeomorphism.

Theorem 2.21 : Let (X, τ) and (Y, σ) be an

intuitionistic fuzzy supra topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-open mapping iff

$f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is an intuitionistic fuzzy supra contra semi-continuous.

Proof. Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-open mapping. Let $A \in \tau$. By hypothesis, $f(A)$ is an intuitionistic fuzzy semi-supra closed set in (Y, σ) . That is, $((f^{-1})^{-1})(A)$ is intuitionistic fuzzy semi-supra closed set in (Y, σ) . Thus, $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is intuitionistic fuzzy supra contra semi-continuous.

Conversely, suppose $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is intuitionistic fuzzy supra contra semi-continuous. Let $A \in \tau$. By hypothesis, $((f^{-1})^{-1})(A) = f(A)$ is intuitionistic fuzzy semi-supra closed set in (Y, σ) . Therefore, $f : (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy supra contra semi-open map.

Theorem 2.22 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-homeomorphism iff

- (i) f is an intuitionistic fuzzy supra contra semi-continuous function.
- (ii) f is an intuitionistic fuzzy supra contra semi-open mapping.

Proof. It follows from the definitions.

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