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Weakly binary gα-locally closed sets in Binary Topological Space

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ABSTRACT

In this paper, we will define $wbg\alpha$-lc-sets, $wb\alpha g$-lc-sets, $wbg\alpha$-$lc^{\star}$-sets, $wb\alpha g$-$lc^{\star}$-sets, $wbg\alpha$-$lc^{\star\star}$-sets, $wb\alpha g$-$lc^{\star\star}$-sets, $bg^{\star}\alpha$-lc-sets, $bg^{\star}\alpha$-$lc^{\star}$-sets, $bg^{\star}\alpha$-$lc^{\star\star}$-sets, $wbg^{\star}\alpha$-lc-sets, $wbg^{\star}\alpha$-$lc^{\star}$-sets and $wbg^{\star}\alpha$-$lc^{\star\star}$-sets and study some of their properties. Also we will prove that $bg^{\star}\alpha$-$lc^{\star}$-sets and $bg^{\star}\alpha$-$lc^{\star\star}$-sets are stronger than $bg^{\star}\alpha$-lc-sets and weaker than binary locally closed sets.

KEYWORDS: $wbg\alpha$-lc-sets, $wb\alpha g$-lc-sets, $wbg\alpha$-$lc^{\star}$-sets, $wb\alpha g$-$lc^{\star}$-sets and $wbg\alpha$-$lc^{\star\star}$-sets

I. INTRODUCTION

In 2011, S.Nithyanantha Jothi and P.Thangavelu [8] introduced topology between two sets and also studied some of their properties. For background material, papers [1] to [15] may be perused. In this paper, we will define $wbg\alpha$-lc-sets, $wb\alpha g$-lc-sets, $wbg\alpha$-$lc^{\star}$-sets, $wb\alpha g$-$lc^{\star}$-sets, $wbg\alpha$-$lc^{\star\star}$-sets, $wb\alpha g$-$lc^{\star\star}$-sets, $bg^{\star}\alpha$-lc-sets, $bg^{\star}\alpha$-$lc^{\star}$-sets, $bg^{\star}\alpha$-$lc^{\star\star}$-sets, $wbg^{\star}\alpha$-lc-sets, $wbg^{\star}\alpha$-$lc^{\star}$-sets and $wbg^{\star}\alpha$-$lc^{\star\star}$-sets and study some of their properties. Also we will prove that $bg^{\star}\alpha$-$lc^{\star}$-sets and $bg^{\star}\alpha$-$lc^{\star\star}$-sets are stronger than $bg^{\star}\alpha$-lc-sets and weaker than binary locally closed sets.

II. WEAKLY BINARY $g\alpha$-LOCALLY CLOSED SETS AND WEAKLY BINARY αg-LOCALLY CLOSED SETS

Definition 2.1 Let (A,B) be a subset of $(X,Y,\mathcal{M})$. Then (A,B) is called a

1. weakly binary generalized α-locally closed set (briefly $wbg\alpha$-lc-set) if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $wbg\alpha$-open and (E,F) is $wbg\alpha$-closed in $(X,Y,\mathcal{M})$.
2. weakly binary α-generalized locally closed set (briefly $wb\alpha g$-lc-set) if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $wb\alpha g$-open and (E,F) is $wb\alpha g$-closed in $(X,Y,\mathcal{M})$.
3. $wbg\alpha$-$lc^{\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $wbg\alpha$-open and (E,F) is binary closed in $(X,Y,\mathcal{M})$.
4. $wb\alpha g$-$lc^{\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $wb\alpha g$-open and (E,F) is binary closed in $(X,Y,\mathcal{M})$.
5. $wbg\alpha$-$lc^{\star\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is binary open and (E,F) is $wbg\alpha$-closed in $(X,Y,\mathcal{M})$.
6. $wb\alpha g$-$lc^{\star\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is binary open and (E,F) is $wb\alpha g$-closed in $(X,Y,\mathcal{M})$.

The collection of all $wbg\alpha$-lc-sets (resp. $wbg\alpha$-$lc^{\star}$-sets, $wbg\alpha$-$lc^{\star\star}$-sets, $wb\alpha g$-lc-sets, $wb\alpha g$-$lc^{\star}$-sets and $wb\alpha g$-$lc^{\star\star}$-sets) of $(X,Y,\mathcal{M})$ will be denoted by $WBG\alpha LC(X,Y)$ (resp. $WBG\alpha LC^{\star}(X,Y)$, $WBG\alpha LC^{\star\star}(X,Y)$, $WB\alpha GLC(X,Y)$, $WB\alpha GLC^{\star}(X,Y)$ and $WB\alpha GLC^{\star\star}(X,Y)$).

Definition 2.2 Let (A,B) be a subset of $((X,Y,\mathcal{M})$. Then (A,B) is called a

1. generalized $^{\star}\alpha$-locally closed set (briefly $bg^{\star}\alpha$-lc-set) if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $bg^{\star}\alpha$-open and (E,F) is $bg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$.
2. $bg^{\star}\alpha$-$lc^{\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $bg^{\star}\alpha$-open and (E,F) is binary closed in $(X,Y,\mathcal{M})$.
3. $bg^{\star}\alpha$-$lc^{\star\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is binary open and (E,F) is $bg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$.

The collection of all $bg^{\star}\alpha$-lc-sets (resp. $bg^{\star}\alpha$-$lc^{\star}$-sets and $bg^{\star}\alpha$-$lc^{\star\star}$-sets) of $(X,Y,\mathcal{M})$ will be denoted by $BG^{\star}\alpha LC(X,Y)$ (resp. $BG^{\star}\alpha LC^{\star}(X,Y)$ and $BG^{\star}\alpha LC^{\star\star}(X,Y)$).

Definition 2.3 Let (A,B) be a subset of $(X,Y,\mathcal{M})$. Then (A,B) is called a

1. weakly binary generalized $^{\star}\alpha$-locally closed set (briefly $wbg^{\star}\alpha$-lc-set) if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $wbg^{\star}\alpha$-open and (E,F) is $wbg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$.
2. $wbg^{\star}\alpha$-$lc^{\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is $wbg^{\star}\alpha$-open and (E,F) is binary closed in $(X,Y,\mathcal{M})$.
3. $wbg^{\star}\alpha$-$lc^{\star\star}$-set if $(A,B)=(G,H)\cap(E,F)$, where (G,H) is binary open and (E,F) is $wbg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$.

The class of all $wg^{\star}\alpha$-lc-sets (resp. $wbg^{\star}\alpha$-$lc^{\star}$-sets and $wbg^{\star}\alpha$-$lc^{\star\star}$-sets) of $(X,Y,\mathcal{M})$ will be denoted by $WBG^{\star}\alpha LC(X,Y)$ (resp. $WBG^{\star}\alpha LC^{\star}(X,Y)$ and $WBG^{\star}\alpha LC^{\star\star}(X,Y)$).

Theorem 2.4 For a binary topological space $(X,Y,\mathcal{M})$, the following inclusions hold:

1. $B\alpha LC(X,Y)\subseteq BG^{\star}\alpha LC(X,Y)$.
2. $BG^{\star}\alpha LC^{\star}(X,Y)\subseteq BG^{\star}\alpha LC(X,Y)$.
3. $BG^{\star}\alpha LC^{\star\star}(X,Y)\subseteq BG^{\star}\alpha LC(X,Y)$.

Proof. (1) Assume that $(A,B)\in \alpha LC(X,Y)$. Then $(A,B)=(U,V)\cap(E,F)$, where (U,V) is $b\alpha$-open and (E,F) is $b\alpha$-closed in $(X,Y,\mathcal{M})$. Since every $b\alpha$-open set is $bg^{\star}\alpha$-open and every $b\alpha$-closed set is $bg^{\star}\alpha$-closed, we get (U,V) is $bg^{\star}\alpha$-open and (E,F) is $bg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$. Hence $(A,B)\in BG^{\star}\alpha LC(X,Y)$.

(2) and (3) follow from the fact that every binary closed set is $bg^{\star}\alpha$-closed and every binary open set is $bg^{\star}\alpha$-open.

The reverse implications need not be true as seen from the following example:

Example 2.5 Let $X=\{a,b\}$, $Y=\{1,2\}$ and $\mathcal{M}=\{(\phi,\phi),(\{a\},\{1\}),(X,Y)\}$. Then
$B\alpha LC(X,Y)=\{(\phi,\phi),\ (\phi,\{2\}),\ (\{a\},\{1\}),\ (\{a\},Y),\ (\{b\},\phi),(\{b\},\{2\}),(X,\{1\}),(X,Y)\}$.
$BG^{\star}\alpha LC(X,Y)=\{(\phi,\phi),\ (\phi,\{1\}),\ (\phi,\{2\}),\ (\{a\},\phi),\ (\{a\},\{1\}),\ (\{a\},\{2\}),\ (\{a\},Y),\ (\{b\},\phi),\ (\{b\},\{1\}),\ (\{b\},\{2\}),(X,\{1\}),(X,Y)\}$
$BG^{\star}\alpha LC^{\star}(X,Y)=\{(\phi,\phi),\ (\phi,\{2\}),\ (\{a\},\{1\}),\ (\{a\},Y),\ (\{b\},\phi),\ (\{b\},\{1\}),\ (\{b\},\{2\}),\ (X,\{1\}),\ (X,Y)\}$.
$BG^{\star}\alpha LC^{\star\star}(X,Y)=\{(\phi,\phi),\ (\phi,\{2\}),\ (\{a\},\phi),\ (\{a\},\{1\}),\ (\{a\},\{2\}),\ (\{a\},Y),\ (\{b\},\phi),\ (\{b\},\{2\}),\ (X,\{1\}),(X,Y)\}$.

Theorem 2.6 Let (A,B) be any subset of $(X,Y,\mathcal{M})$. If $(A,B)\in BG^{\star}\alpha LC(X,Y)$, then $(A,B)\in WBG\alpha LC(X,Y)$, $WB\alpha GLC(X,Y)$ and $WBG^{\star}\alpha LC(X,Y)$.

Proof. Assume that $(A,B)\in BG^{\star}\alpha LC(X,Y)$. Then $(A,B)=(U,V)\cap(E,F)$, where (U,V) is $bg^{\star}\alpha$-open and (E,F) is $bg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$. Since every $bg^{\star}\alpha$-open ($bg^{\star}\alpha$-closed) set is $wbg\alpha$-open ($wbg\alpha$-closed), $wb\alpha g$-open ($wb\alpha g$-closed) and $wbg^{\star}\alpha$-open ($wbg^{\star}\alpha$-closed), (U,V) is $wbg\alpha$-open, $wb\alpha g$-open and $wbg^{\star}\alpha$-open in $(X,Y,\mathcal{M})$ and (E,F) is $wbg\alpha$-closed, $wb\alpha g$-closed and $wbg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$. Hence $(A,B)\in WBG\alpha LC(X,Y)$, $WB\alpha GLC(X,Y)$ and $WBG^{\star}\alpha LC(X,Y)$.

The reverse implications need not be true as seen from the following example:

Example 2.7 In Example 2.5, then the subset $(X,\{2\})$ is $wbg\alpha$-lc-set, $wb\alpha g$-lc-set and $wbg^{\star}\alpha$-lc-set but it is not $bg^{\star}\alpha$-lc-set.

Remark 2.8 The following examples show that the concept of $BG^{\star}\alpha LC^{\star}(X,Y)$ and $BG^{\star}\alpha LC^{\star\star}(X,Y)$ are independent.

Example 2.9 In Example 2.5, then the subset $(\{b\},\{1\})$ is $bg^{\star}\alpha$-$lc^{\star}$-set but not $bg^{\star}\alpha$-$lc^{\star\star}$-set and also the subset $(\{a\},\phi)$ is $bg^{\star}\alpha$-$lc^{\star\star}$-set but not $bg^{\star}\alpha$-$lc^{\star}$-set.

Theorem 2.10 Let (A,B) be any subset of $(X,Y,\mathcal{M})$, then

1. $BLC(X,Y) \subseteq BG^{\star}\alpha LC(X,Y)$,
2. $BG^{\star}\alpha LC^{\star}(X,Y) \subseteq BG^{\star}\alpha LC(X,Y)$,
3. $G^{\star}\alpha LC^{\star}(X,Y) \subseteq WBG^{\star}\alpha LC(X,Y)$.

Proof. The proof follows from the fact that every binary closed set is binary α-closed, every binary α-closed set is $bg^{\star}\alpha$-closed and every $bg^{\star}\alpha$-closed set is $wbg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$.

The reverse implications need not be true as seen from the following example.

Example 2.11 Let $X=\{a,b\}$, $Y=\{1,2\}$ and $mathcalM=\{(\phi,\phi),(X,\{1\}),(X,Y)\}$. Then
$BLC(X,Y)=\{(\phi,\phi),(\phi,\{2\}),(X,\{1\}),(X,Y)\}$.
$BG^{\star}\alpha LC^{\star}(X,Y)=\{(\phi,\phi),(\phi,\{1\}),(\phi,\{2\}),(\{a\},\phi),(\{a\},\{1\}),(\{b\},\phi),(\{b\},\{1\}),(X,\phi),(X,\{1\}),(X,Y)\}$.
$BG^{\star}\alpha LC(X,Y)=BG^{\star}\alpha LC^{\star\star}(X,Y)=WBG^{\star}\alpha LC(X,Y)=\mathbb{P}(X)\times\mathbb{P}(Y)$

Definition 2.12 A binary topological space $(X,Y,\mathcal{M})$ is said to be a

1. weakly binary $g\alpha$-door (briefly $wbg\alpha$-door) space if each subset of $(X,Y,\mathcal{M})$ is either $wbg\alpha$-open or $wbg\alpha$-closed in $(X,Y,\mathcal{M})$.
2. weakly binary αg-door (briefly $wb\alpha g$-door) space if each subset of $(X,Y,\mathcal{M})$ is either $wb\alpha g$-open or $wb\alpha g$-closed in $(X,Y,\mathcal{M})$.
3. binary $g^{\star}\alpha$-door (briefly $bg^{\star}\alpha$-door) space if each subset of $(X,Y,\mathcal{M})$ is either $wg^{\star}\alpha$-open or $bg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$.
4. weakly binary $g^{\star}\alpha$-door (briefly $wbg^{\star}\alpha$) space if each subset of $(X,Y,\mathcal{M})$ is either $wbg^{\star}\alpha$-open or $wbg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$.

Remark 2.13 Let $(X,Y,\mathcal{M})$ be a binary topological space.

1. If $(X,Y,\mathcal{M})$ is $wbg\alpha$-door space, then $WBG\alpha LC(X,Y)=\mathbb{P}(X)\times\mathbb{P}(Y)$.
2. If $(X,Y,\mathcal{M})$ is $wb\alpha g$-door space, then $WB\alpha GLC(X,Y)=\mathbb{P}(X)\times\mathbb{P}(Y)$.
3. If $(X,Y,\mathcal{M})$ is $bg^{\star}\alpha$-door space, then $BG^{\star}\alpha LC(X,Y)=\mathbb{P}(X)\times\mathbb{P}(Y)$.
4. If $(X,Y,\mathcal{M})$ is $wbg^{\star}\alpha$-door space, then $WBG^{\star}\alpha LC(X,Y)=\mathbb{P}(X)\times\mathbb{Y}$.

Theorem 2.14 Let $(X,Y,\mathcal{M})$ be a ${}_{bg^{\star}\alpha}T_{bc}$-space. Then the following results hold:

1. $BG^{\star}\alpha LC(X,Y)=BLC(X,Y)$.
2. $BG^{\star}\alpha LC(X,Y)\subseteq BGLC(X,Y)$.
3. $BG^{\star}\alpha LC(X,Y)\subseteq BGLSC(X,Y)$.

Proof. (1) Since $(X,Y,\mathcal{M})$ is a ${}_{bg^{\star}\alpha}T_{bc}$-space, every $bg^{\star}\alpha$-open set is binary open and every $bg^{\star}\alpha$-closed set is binary closed in $(X,Y,\mathcal{M})$. Hence we have $BG^{\star}\alpha LC(X,Y)\subseteq BLC(X,Y)$. By Theorem 2.10, $BLC(X,Y)\subseteq BG^{\star}\alpha LC(X,Y)$.
Hence $BLC(X,Y)=BG^{\star}\alpha LC(X,Y)$. Since $BLC(X,Y)\subseteq BGLC(X,Y)$ and $BLC(X,Y)\subseteq BGLSC(X,Y)$ (1) and (3) follow.

Theorem 2.15 Let $(X,Y,\mathcal{M})$ be a ${}_{wbg^{\star}\alpha}T_{bc}$-space and let (E,F) be a subset of (X,Y), then the following statements are equivalent:

1. $(E,F)\in WBG^{\star}\alpha LC(X,Y)$.
2. $(E,F)=(G,H)\cap wbg^{\star}\alpha\text{-}cl(E,F)$ for some $wbg^{\star}\alpha$-open set (G,H).

Proof. $(1)\Rightarrow(2)$ Let $(E,F)\in WBG^{\star}\alpha LC(X,Y)$. Then $(E,F)=(G,H)\cap(S,T)$, where (G,H) is $wbg^{\star}\alpha$-open and (S,T) is $wbg^{\star}\alpha$-closed in $(X,Y,\mathcal{M})$. Since $(E,F)\subseteq(G,H)$ and $(E,F)\subseteq wbg^{\star}\alpha\text{-}cl(E,F)$, $(E,F)\subseteq(G,H)\cap wbg^{\star}\alpha\text{-}cl(E,F)$. By the definition of $wbg^{\star}\alpha$-closure, we have $wbg^{\star}\alpha\text{-}cl(E,F)\subseteq(S,T)$ and hence
$(G,H)\cap wbg^{\star}\alpha\text{-}cl(E,F)\subseteq(G,H)\cap(S,T)=(E,F)$.
Thus $(G,H)\cap wbg^{\star}\alpha\text{-}cl(E,F)=(E,F)$.

$(2)\Rightarrow(1)$ Assume that $(G,H)\cap wbg^{\star}\alpha\text{-}cl(E,F)=(E,F)$ for some $wbg^{\star}\alpha$-open set (G,H). Since $wbg^{\star}\alpha\text{-}cl(E,F)$ is $wbg^{\star}\alpha$-closed and hence $(E,F)=(G,H)\cap wbg^{\star}\alpha\text{-}cl(E,F)\in WBG^{\star}\alpha LC(X,Y)$.

Theorem 2.16 Let $(X,Y,\mathcal{M})$ be a ${}_{wbg^{\star}\alpha}T_{bc}$-space and let (E,F) be a subset of (X,Y), then the following statements are equivalent:

1. $(E,F)=(G,H)\cap b\text{-}cl(E,F)$ is $wbg^{\star}\alpha$-open set (G,H).
2. $b\text{-}cl(E,F)-(E,F)$ is $wbg^{\star}\alpha$-closed.

3. $(G,H) = (E,F) \cup ((X,Y) - b\text{-}cl(E,F))$ is $wbg^{\star}\alpha$-open.

Proof. $(1) \Rightarrow (2)$ Let $(E,F) = (G,H) \cap b\text{-}cl(E,F)$, for some $wbg^{\star}\alpha$-open set (G,H). We have $b\text{-}cl(E,F) - (E,F) = b\text{-}cl(E,F) \cap (G,H)^c$, $(G,H)^c$ is $wbg^{\star}\alpha$-closed in (X,Y), as (G,H) is $wbg^{\star}\alpha$-open. Since $(X,Y,\mathcal{M})$ is a ${}_{wbg^{\star}\alpha}T_{bc}$-space, $(G,H)^c$ is binary closed in (X,Y). Thus $b\text{-}cl(E,F) \cap (G,H)^c$ is binary closed in (X,Y). Since every binary closed set is $wbg^{\star}\alpha$-closed, $b\text{-}cl(E,F) \cap (G,H)^c$ is $wbg^{\star}\alpha$-closed in (X,Y). Hence $b\text{-}cl(E,F) - (E,F)$ is $wbg^{\star}\alpha$-closed in (X,Y).

$(2) \Rightarrow (1)$ Assume that $b\text{-}cl(E,F) - (E,F)$ is $wbg^{\star}\alpha$-closed. Let $(G,H) = (X,Y) - (b\text{-}cl(E,F) - (E,F))$. Then (G,H) is $wbg^{\star}\alpha$-open and hence $(E,F) = (G,H) \cap b\text{-}cl(E,F)$ holds.

$(2) \Rightarrow (3)$ Let $(S,T) = b\text{-}cl(E,F) - (E,F)$ be $wbg^{\star}\alpha$-closed. Then $(X,Y) - (S,T) = (X,Y) - (b\text{-}cl(E,F) - (E,F)) = (E,F) \cup ((X,Y) - b\text{-}cl(E,F))$. Since (S,T) is $wbg^{\star}\alpha$-closed, $(X,Y) - (S,T)$ is $wbg^{\star}\alpha$-open. Thus $(E,F) \cup ((X,Y) - b\text{-}cl(E,F))$ is $wbg^{\star}\alpha$-open.

$(3) \Rightarrow (2)$ Let $(G,H) = (E,F) \cup ((X,Y) - b\text{-}cl(E,F))$ be $wbg^{\star}\alpha$-open. Then $(X,Y) - (G,H) = (X,Y) - ((E,F) \cup ((X,Y) - b\text{-}cl(E,F))) = b\text{-}cl(E,F) \cap ((X,Y) - (E,F)) = b\text{-}cl(E,F) - (E,F)$. Since $(X,Y) - (G,H)$ is $wbg^{\star}\alpha$-closed, $b\text{-}cl(E,F) - (E,F)$ is $wbg^{\star}\alpha$-closed.

Theorem 2.17 Let (E,F) be a subset of $(X,Y,\mathcal{M})$ is a ${}_{wbg^{\star}\alpha}T_{bc}$-space. Then $(E,F) \in WBG^{\star}\alpha LC^{\star\star}(X,Y)$ if and only if $(E,F) = (G,H) \cap wbg^{\star}\alpha\text{-}cl(E,F)$ for some binary open set (G,H).

Proof. **(Necessity)** Let $(E,F) \in WBG^{\star}\alpha LC^{\star\star}(X,Y)$. Then $(E,F) = (G,H) \cap (O,P)$, where (G,H) is binary open in (X,Y) and (O,P) is $wbg^{\star}\alpha$-closed in (X,Y). Since $(E,F) \subseteq (G,H)$ and $(E,F) \subseteq wbg^{\star}\alpha\text{-}cl(E,F)$, we have $(E,F) \subseteq (G,H) \cap wbg^{\star}\alpha\text{-}cl(E,F)$. Since (O,P) is a $wbg^{\star}\alpha$-closed set containing (E,F), we have $wbg^{\star}\alpha\text{-}cl(E,F) \subseteq (O,P)$. This implies $(G,H) \cap wbg^{\star}\alpha\text{-}cl(E,F) \subseteq (G,H) \cap (O,P) = (E,F)$. Therefore $(E,F) = (G,H) \cap wbg^{\star}\alpha\text{-}cl(E,F)$.

(Sufficiency) Assume that $(E,F) = (G,H) \cap wbg^{\star}\alpha\text{-}cl(E,F)$ for some binary open set (G,H). Since $wbg^{\star}\alpha\text{-}cl(E,F)$ is $wbg^{\star}\alpha$-closed. Hence we have $(E,F) \in WBG^{\star}\alpha LC^{\star\star}(X,Y)$.

Theorem 2.18 For a $bg^{\star}\alpha$-closed subset (E,F) of a binary topological space $(X,Y,\mathcal{M})$ the following statements are equivalent:

1. $(E,F) \in BG^{\star}\alpha LC(X,Y)$.
2. $(E,F) = (G,H) \cap bg^{\star}\alpha\text{-}cl(E,F)$ for some $bg^{\star}\alpha$-open set (G,H) in (X,Y).

Proof. $(1) \Rightarrow (2)$ Let $(E,F) \in BG^{\star}\alpha LC(X,Y)$. Then there exists a $bg^{\star}\alpha$-open subset (G,H) and a $bg^{\star}\alpha$-closed subset (O,P) of (X,Y) such that $(E,F) = (G,H) \cap (O,P)$. Since $(E,F) \subseteq (G,H)$ and $(E,F) \subseteq bg^{\star}\alpha\text{-}cl(E,F)$, $(E,F) \subseteq (G,H) \cap bg^{\star}\alpha\text{-}cl(E,F)$. Conversely, by definition of $bg^{\star}\alpha$-closure, $bg^{\star}\alpha\text{-}cl(E,F) \subseteq (O,P)$ and hence $(G,H) \cap bg^{\star}\alpha\text{-}cl(E,F) \subseteq (G,H) \cap (O,P) = (E,F)$. Therefore $(E,F) = (G,H) \cap bg^{\star}\alpha\text{-}cl(E,F)$.

$(2) \Rightarrow (1)$ Assume that $(E,F) = (G,H) \cap bg^{\star}\alpha\text{-}cl(E,F)$ for some $bg^{\star}\alpha$-open set (G,H). Since $bg^{\star}\alpha\text{-}cl(E,F)$ is $bg^{\star}\alpha$-closed and hence $(E,F) = (G,H) \cap bg^{\star}\alpha\text{-}cl(E,F) \in BG^{\star}\alpha LC(X,Y)$.

Definition 2.19 A subset (E,F) of $(X,Y,\mathcal{M})$ is called

1. $wbg\alpha$-dense if $wbg\alpha\text{-}cl(E,F) = (X,Y)$.
2. $wb\alpha g$-dense if $wb\alpha g\text{-}cl(E,F) = (X,Y)$.
3. $bg^{\star}\alpha$-dense if $bg^{\star}\alpha\text{-}cl(E,F) = (X,Y)$.
4. $wbg^{\star}\alpha$-dense if $wbg^{\star}\alpha\text{-}cl(E,F) = (X,Y)$.

Theorem 2.20 For a binary topological space $(X,Y,\mathcal{M})$, the following results hold:

1. Every $bg^{\star}\alpha$-dense set is binary dense.
2. Every $wbg^{\star}\alpha$-dense set is $bg^{\star}\alpha$-dense.

Proof. 1. Assume that (E,F) is a $bg^{\star}\alpha$-dense set. Then $bg^{\star}\alpha\text{-}cl(E,F) = (X,Y)$. Obviously, $b\text{-}cl(E,F) \subseteq (X,Y)$. Since $bg^{\star}\alpha\text{-}cl(E,F) \subseteq b\text{-}cl(E,F)$, $b\text{-}cl(E,F) \supseteq bg^{\star}\alpha\text{-}cl(E,F) = (X,Y)$. Hence $b\text{-}cl(E,F) = (X,Y)$. Therefore (E,F) is binary dense.

2. Assume that (E,F) is a $wbg^{\star}\alpha$-dense set. Then $wbg^{\star}\alpha\text{-}cl(E,F) = (X,Y)$. Obviously, $bg^{\star}\alpha\text{-}cl(E,F) \subseteq (X,Y)$. Since $wbg^{\star}\alpha\text{-}cl(E,F) \subseteq bg^{\star}\alpha\text{-}cl(E,F)$, which implies $bg^{\star}\alpha\text{-}cl(E,F) \supseteq wbg^{\star}\alpha\text{-}$

$cl(E,F) = (X,Y)$. Hence $bg^{\star}\alpha\text{-}cl(E,F) = (X,Y)$. Therefore (E,F) is $bg^{\star}\alpha$-dense.

The converses of the above Theorem need not be true as seen from the following examples:

Example 2.21 Let $X = \{1,2\}$, $Y = \{a,b\}$ and $\mathcal{M} = \{(\phi,\phi), (\phi,\{b\}), (\{1\},\{a\}), (\{1\},Y), (X,Y)\}$. Then the subset $(\{2\},Y)$ is binary dense but not $bg^{\star}\alpha$-dense.

Example 2.22 Let $X = \{a,b\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi,\phi), (\{a\},\{1\}), (\{b\},\{2\}), (X,Y)\}$. Then the subset $(\{b\},\{1\})$ is $bg^{\star}\alpha$-dense but not $wbg^{\star}\alpha$-dense.

Theorem 2.23 Let $(X,Y,\mathcal{M})$ be a binary topological space. Then

1. Every $wbg\alpha$-dense set is a $bg^{\star}\alpha$-dense set.
2. Every $wb\alpha g$-dense set is also a $bg^{\star}\alpha$-dense set.

proof . Similar to Theorem 2.20.

Definition 2.24 A binary topological space $(X,Y,\mathcal{M})$ is called

1. $wbg\alpha$-submaximal if every $wbg\alpha$-dense subset is $wbg\alpha$-open in (X,Y).
2. $wb\alpha g$-submaximal if every $wb\alpha g$-dense subset is $wb\alpha g$-open in (X,Y).
3. $bg^{\star}\alpha$-submaximal if every $bg^{\star}\alpha$-dense subset is $bg^{\star}\alpha$-open in (X,Y).
4. $wbg^{\star}\alpha$-submaximal if every $wbg^{\star}\alpha$-dense subset is $wbg^{\star}\alpha$-open in (X,Y).

Theorem 2.25 Let $(X,Y,\mathcal{M})$ be a binary topological space. If (X,Y) is binary submaximal, then it is also $bg^{\star}\alpha$-submaximal.

Proof. Let (E,F) be a $bg^{\star}\alpha$-dense subset of $(X,Y,\mathcal{M})$. By Theorem 2.20, (E,F) is binary dense in $((X,Y,\mathcal{M})$. Since (X,Y) is binary submaximal, (E,F) is binary open. Then (E,F) is $bg^{\star}\alpha$-open, as every binary open set is $bg^{\star}\alpha$-open. Hence (X,Y) is $bg^{\star}\alpha$-submaximal.

The converse of the above Theorem need not be true as seen from the following example.

Example 2.26 Let $X = \{a,b,c\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi,\phi), (\{a\},\{1\}), (\{b\},\phi), (\{b\},\{2\}), (\{a,b\},\{1\}), (\{a,b\},Y), (X,Y)\}$. Then $(X,Y,\mathcal{M})$ is $bg^{\star}\alpha$-submaximal but not binary submaximal, since the subset $(\{a,b\},\{2\})$ is binary dense but not binary open in $(X,Y,\mathcal{M})$.

Theorem 2.27 Let $(X,Y,\mathcal{M})$ be a binary topological space. If (X,Y) is $bg^{\star}\alpha$-submaximal, then it is $wbg^{\star}\alpha$-submaximal also.

Proof. Let (E,F) be a $wbg^{\star}\alpha$-dense subset of $(X,Y,\mathcal{M})$. By Theorem 2.20, (E,F) is $bg^{\star}\alpha$-dense in $(X,Y,\mathcal{M})$. Since (X,Y) is $bg^{\star}\alpha$-submaximal, (E,F) is $bg^{\star}\alpha$-open. Since every $bg^{\star}\alpha$-open set is $wbg^{\star}\alpha$-open, (E,F) is $wbg^{\star}\alpha$-open. Hence (X,Y) is $wbg^{\star}\alpha$-submaximal.

The converse of the above Theorem need not be true as seen from the following example.

Example 2.28 In Example 2.22, then $(X,Y,\mathcal{M})$ is $wbg^{\star}\alpha$-submaximal but not $bg^{\star}\alpha$-submaximal, since the subset $(\{a\},\{2\})$ is $bg^{\star}\alpha$-dense but not $bg^{\star}\alpha$-open in $(X,Y,\mathcal{M})$.

Theorem 2.29 Let $(X,Y,\mathcal{M})$ be a binary topological space. If (X,Y) is $bg^{\star}\alpha$-submaximal, then (X,Y) is both $wbg\alpha$-submaximal and $wb\alpha g$-submaximal.

Proof. Since every $wbg\alpha$-dense set and $wb\alpha g$-dense set is also $bg^{\star}\alpha$-dense and since every $bg^{\star}\alpha$-open set is both $wbg\alpha$-open and $wb\alpha g$-open in $(X,Y,\mathcal{M})$, the proof follows.

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