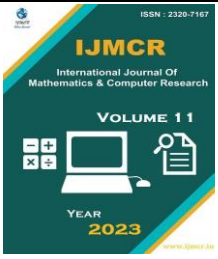
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 11 Issue 08 August 2023, Page no. 3680-3684

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v11i8.08



Gourava Domination Indices of Graphs

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ARTICLE INFO

Published Online:
26 August 2023

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ABSTRACT

In this paper, we introduce the first and second Gourava domination indices and their corresponding polynomials of a graph. Furthermore, we compute these indices and their corresponding polynomials for some standard graphs, French windmill graphs, friendship graphs and book graphs.

KEYWORDS: Gourava domination index, Gourava domination polynomial, graph.

I. INTRODUCTION

The graph *G= (V(G), E(G))*, where *V(G)* be the vertex set and *E(G)* be the edge set. $d_G(u)$ be the degree of a vertex *u*. For undefined term and notation, we refer the books [1, 2].

Graph indices have their applications in various disciplines of Science and Engineering. Recently some new graph indices were studied, for example, in [3, 4, 5, 6, 7].

The domination degree $d_d(u)$ of a vertex *u* [8] in a graph *G* is defined as the number of minimal dominating sets of *G* which contains *u*.

In [9], Kulli introduced the first and second Gourava indices of a graph and they are defined as

$$GO_1(G) = \sum_{uv\in E(G)} \left[d_G(u) + d_G(v) + d_G(u)d_G(v)\right].$$

$$GO_2(G) = \sum_{uv\in E(G)} \left(d_G(u) + d_G(v)\right)\left(d_G(u)d_G(v)\right).$$

Recently some Gourava indices were studied, for example, in [10, 11, 12].

Motivated by the work on Gourava indices, we introduce the first and second Gourava domination indices as follows:

The first and second Gourava domination indices of a graph *G* are defined as

$$GOD_1(G) = \sum_{uv\in E(G)} \left[d_d(u) + d_d(v) + d_d(u)d_d(v)\right].$$

$$GOD_2(G) = \sum_{uv\in E(G)} \left(d_d(u) + d_d(v)\right)\left(d_d(u)d_d(v)\right).$$

Considering the first and second Gourava domination indices, we introduce the first and second Gourava domination polynomials of a graph *G* and they are defined as

$$GOD_1(G,x) = \sum_{uv\in E(G)} x^{d_d(u)+d_d(v)+d_d(u)d_d(v)}.$$

$$GOD_2(G,x) = \sum_{uv\in E(G)} x^{\left(d_d(u)+d_d(v)\right)\left(d_d(u)d_d(v)\right)}$$

Ref. [8] was soon followed by a series of publications [13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. . Recently some new domination parameters were studied, for example, in [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

In this paper, we determine the first and second Gourava domination indices of some standard graphs, French windmill graphs, friendship graphs and book graphs.

II. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1. If K_n is a complete graph with *n* vertices, then

(i) $GOD_1(K_n) = \dfrac{3n(n-1)}{2}.$

(ii) $GOD_2(K_n) = n(n-1)$.

Proof: If K_n is a complete graph, then $d_d(u)$ =1.

From definition, we have

(i) $GOD_1(K_n)$

$$= \sum_{uv \in E(K_n)} \left[d_d(u) + d_d(v) + d_d(u)d_d(v)\right]$$

$$= \frac{n(n-1)}{2}\left[1+1+(1\times1)\right] = \frac{3n(n-1)}{2}.$$

(ii) $GOD_2(K_n)$

$$= \sum_{uv \in E(K_n)} \left[\left(d_d(u) + d_d(v)\right)\left(d_d(u)d_d(v)\right)\right]$$

$$= \frac{n(n-1)}{2}\left[(1+1)(1\times1)\right] = \frac{2n(n-1)}{2}.$$

Proposition 2. If S_{n+1} is a star graph with $d_d(u)$ =1, then

(i) $GOD_1(S_{n+1}) = 3n.$

(ii) $GOD_2(S_{n+1}) = 2n.$

Proposition 3. If $S_{p+1,q+1}$ is a double star graph with $d_d(u)$ =2, then

(i) $GOD_1(S_{p+1,q+1}) = 8(p+q+1).$

(ii) $GOD_2(S_{p+1,q+1}) = 16(p+q+1).$

Proposition 4. Let $K_{m,n}$ be a complete bipartite graph with $2 \leq m \leq n$. Then

(i) $GOD_1(K_{m,n}) = mn(mn+2m+2n+3).$

(ii) $GOD_2(K_{m,n}) = mn(m+n+2)(m+1)(n+1).$

Proof: Let $G=K_{m,n}$, m, $n \geq 2$ with

$d_d(u) = m+1$

$= n+1$, for all $u \in V(G)$.

From definition, we have

(i) $GOD_1(K_{m,n})$

$$= \sum_{uv \in E(K_{m,n})} \left[d_d(u) + d_d(v) + d_d(u)d_d(v)\right]$$

$$= mn\left[m+1+n+1+(m+1)(n+1)\right]$$

$$= mn(mn+2m+2n+3).$$

(ii) $GOD_2(K_{m,n})$

$$= \sum_{uv \in E(K_{m,n})} \left[\left(d_d(u) + d_d(v)\right)\left(d_d(u)d_d(v)\right)\right]$$

$$= mn\left[(m+1+n+1)(m+1)(n+1)\right]$$

$$= mn(m+n+2)(m+1)(n+1).$$

In the following proposition, by using definition, we obtain the first and second Gourava domination polynomials of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$.

Proposition 5. The first and second Gourava domination polynomials of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $$GOD_1(K_n, x) = \sum_{uv \in E(K_n)} x^{d_d(u)+d_d(v)+d_d(u)d_d(v)}$$

$$= \frac{n(n-1)}{2} x^{1+1+(1\times1)} = \frac{n(n-1)}{2} x^3.$$

(ii) $GOD_1(S_{n+1}, x) = nx^3.$

(iii) $GOD_1(S_{p+1,q+1}, x) = (p+q+1)x^8.$

(iv) $GOD_1(K_{m,n}, x) = mnx^{mn+2m+2n+3}.$

(v) $$GOD_2(K_n, x) = \sum_{uv \in E(K_n)} x^{\left(d_d(u)+d_d(v)\right)\left(d_d(u)d_d(v)\right)}$$

$$= \frac{n(n-1)}{2} x^{(1+1)(1\times1)} = \frac{n(n-1)}{2} x^2.$$

(vi) $GOD_2(S_{n+1}, x) = nx^2.$

(vii) $GOD_2(S_{p+1,q+1}, x) = (p+q+1)x^{16}.$

(viii) $GOD_2(K_{m,n}, x) = mnx^{(m+n+2)(m+1)(n+1)}.$

III. RESULTS FOR FRENCH WINDMILL GRAPHS

The French windmill graph F_n^m is the graph obtained by taking m □ 3 copies of K_n, n □ 3 with a vertex in common. The graph F_n^m is presented in Figure 1. The French windmill graph F_3^m is called a friendship graph.

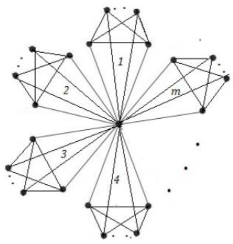

Figure 1. French windmill graph F_n^m

Let F be a French windmill graph F_n^m. Then

$d_d(u)$ =1, if *u is in center*

$=(n-1)^{m-1}$, otherwise.

Theorem 1. Let F be a French windmill graph F_n^m. Then

$$GOD_1(F) = m(n-1)\left[1+2(n-1)^{(m-1)}\right]$$
$$+[(mn(n-1)/2) - m(n-1)]$$
$$(n-1)^{(m-1)}[2+(n-1)^{(m-1)}]$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$GOD_1(F) = \sum_{uv\in E(F)} \left[d_d(u)+d_d(v)+d_d(u)d_d(v)\right]$$

$$= \sum_{uv\in E_1(F)} \left[d_d(u)+d_d(v)+d_d(u)d_d(v)\right]$$

$$+ \sum_{uv\in E_2(F)} \left[d_d(u)+d_d(v)+d_d(u)d_d(v)\right]$$

$$= m(n-1)\left[1+(n-1)^{(m-1)}+1(n-1)^{(m-1)}\right]$$

$$+[(mn(n-1)/2)-m(n-1)]$$

$$[(n-1)^{(m-1)}+(n-1)^{(m-1)}+(n-1)^{(m-1)}(n-1)^{(m-1)}]$$

$$= m(n-1)\left[1+2(n-1)^{(m-1)}\right]$$

$$+[(mn(n-1)/2)-m(n-1)]$$

$$(n-1)^{(m-1)}[2+(n-1)^{(m-1)}].$$

Corollary 1.1. Let $F_3^{\,m}$ be a friendship graph. Then

$$GOD_1\left(F_3^{\,m}\right) = 2m\left(1+2^m\right)+m2^{m-1}\left(2+2^{m-1}\right).$$

In the following theorem, by using definitions, we obtain the first Gourava domination polynomials of F_n^m and F_3^m.

Theorem 2 . The first Gourava domination polynomials of F_n^m and F_3^m are given by

(i) $GOD_1\left(F_n^m, x\right) = \sum_{uv\in E(F_n^m)} x^{d_d(u)+d_d(v)+d_d(u)d_d(v)}$

$$= m(n-1)x^{\left[1+2(n-1)^{(m-1)}\right]}$$

$$+[(mn(n-1)/2)-m(n-1)]x^{(n-1)^{(m-1)}[2+(n-1)^{(m-1)}]}$$

(ii) $GOD_1\left(F_3^{\,m}, x\right) = \sum_{uv\in E(F_3^m)} x^{d_d(u)+d_d(v)+d_d(u)d_d(v)}$

$$= 2mx^{\left(1+2^m\right)}+mx^{2^{m-1}\left(2+2^{m-1}\right)}.$$

Theorem 3. Let F be a French windmill graph F_n^m. Then

$$GOD_2(F) = m(n-1)(n-1)^{(m-1)}\left[1+(n-1)^{(m-1)}\right]$$

$$+[(mn(n-1)/2)-m(n-1)]2(n-1)^{3(m-1)}.$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$GOD_2(F) = \sum_{uv\in E(F)} \left[\left(d_d(u)+d_d(v)\right)\left(d_d(u)d_d(v)\right)\right]$$

$$= \sum_{uv\in E_1(F)} \left[\left(d_d(u)+d_d(v)\right)\left(d_d(u)d_d(v)\right)\right]$$

$$+ \sum_{uv\in E_2(F)} \left[\left(d_d(u)+d_d(v)\right)\left(d_d(u)d_d(v)\right)\right]$$

$$= m(n-1)\left[\left(1+(n-1)^{(m-1)}\right)1(n-1)^{(m-1)}\right]$$

$$+[(mn(n-1)/2)-m(n-1)]$$

$$[\left((n-1)^{m-1}+(n-1)^{m-1}\right)\left((n-1)^{m-1}(n-1)^{m-1}\right)]$$

$$= m(n-1)(n-1)^{(m-1)}\left[1+(n-1)^{(m-1)}\right]$$

$$+[(mn(n-1)/2)-m(n-1)]2(n-1)^{3(m-1)}.$$

Corollary 3.1. Let $F_3^{\,m}$ be a friendship graph. Then

$$GOD_2\left(F_3^{\,m}\right) = m2^m\left(1+2^{m-1}\right)+m2^{3m-2}.$$

In the following theorem, by using definitions, we obtain the second Gourava domination polynomials of F_n^m and F_3^m .

Theorem 4. The second Gourava domination polynomials of F_n^m and F_3^m are given by

(i) $GOD_2\left(F_n^m, x\right) = \sum_{uv\in E(F_n^m)} x^{\left(d_d(u)+d_d(v)\right)\left(d_d(u)d_d(v)\right)}$

$$= m(n-1)x^{(n-1)^{(m-1)}\left[1+(n-1)^{(m-1)}\right]}$$

$$+[(mn(n-1)/2)-m(n-1)]x^{2(n-1)^{3(m-1)}}$$

(ii) $GOD_2\left(F_3^{\,m}, x\right) = \sum_{uv\in E(F_3^m)} x^{\left(d_d(u)+d_d(v)\right)\left(d_d(u)d_d(v)\right)}$

$$= 2mx^{2^{m-1}\left(1+2^{m-1}\right)}+mx^{2^{3m-2}}.$$

IV. RESULTS FOR GoK_p

Theorem 5. Let $H=GoK_p$, where G is a connected graph with n vertices and m edges; and K_p is a complete graph. Then

(i) $GOD_1(H)$

$$= \frac{1}{2}(2m+np^2+np)(p+1)^{n-1}\left[2+(p+1)^{n-1}\right].$$

(ii) $GOD_2(H) = \frac{1}{2}(2m+np^2+np)2(p+1)^{3(n-1)}$

Proof: If $H=GoK_p$, then $d_d(u)=(p+1)^{n-1}$. In F, there are $\frac{p(p-1)}{2}$. edges. Thus H has $\frac{1}{2}(2m+np^2+np)$ edges. Thus

(i) $GOD_1(H) = \sum_{uv\in E(H)} \left[d_d(u)+d_d(v)+d_d(u)d_d(v)\right]$

$$=\frac{1}{2}(2m+np^2+np)$$

$$\left[(p+1)^{n-1}+(p+1)^{n-1}+(p+1)^{n-1}(p+1)^{n-1}\right]$$

$$=\frac{1}{2}(2m+np^2+np)(p+1)^{n-1}\left[2+(p+1)^{n-1}\right].$$

(ii)

$$GOD_2(H)=\sum_{uv\in E(H)}\left[(d_d(u)+d_d(v))(d_d(u)d_d(v))\right]$$

$$=\frac{1}{2}(2m+np^2+np)$$

$$\left[(p+1)^{n-1}+(p+1)^{n-1}\right](p+1)^{n-1}(p+1)^{n-1}$$

$$=\frac{1}{2}(2m+np^2+np)2(p+1)^{3(n-1)}$$

In the following theorem, by using definitions, we obtain the first and second Gourava domination polynomials of H.

Theorem 6. The first and second Gourava domination Polynomials of H are given by

(i) $GOD_1(H,x)=\frac{1}{2}(2m+np^2+np)x^{(p+1)^{n-1}\left[2+(p+1)^{n-1}\right]}$

(i) $GOD_2(H,x)=\frac{1}{2}(2m+np^2+np)x^{2(p+1)^{3(n-1)}}$.

V. RESULTS FOR B_n

The book graph B_n, $n\geq 3$, is a cartesian product of star S_{n+1} and path P_2.

For B_n, $n\geq 3$, we have

$d_d(u)=3$, if u is the center vertex,

$= 2^{n-1}+1$, otherwise.

Theorem 7. If B_n, $n\geq 3$, is a book graph, then

(i) $GOD_1(B_n)$

$$=15+2n(7+4\times 2^{n-1})+n(2^{n-1}+1)(2^{n-1}+3).$$

(ii) $GOD_2(B_n)$

$$=54+6n(2^{n-1}+4)(2^{n-1}+1)+2n(2^{n-1}+1)^3.$$

Proof: In B_n, there are three types of edges as follow:

$E_1 = \{uv \in E(B_n) \mid d_d(u)=d_d(v)=3\}$, $|E_1| = 1$.

$E_2 = \{uv \in E(B_n) \mid d_d(u) = 3, d_d(v)= 2^{n-1}+1\}$, $|E_2| = 2r$.

$E_3 = \{uv \in E(B_n) \mid d_d(u) = d_d(v)= 2^{n-1}+1\}$, $|E_3| = r$.

(i) By definition, we have

$$GOD_1(B_n)=\sum_{uv\in E(B_n)}\left[d_d(u)+d_d(v)+d_d(u)d_d(v)\right]$$

$$=1[3+3+(3\times 3)]+2n\left[3+(2^{n-1}+1)+3(2^{n-1}+1)\right]$$

$$+n\left[(2^{n-1}+1)+(2^{n-1}+1)+(2^{n-1}+1)(2^{n-1}+1)\right]$$

$$=15+2n(7+4\times 2^{n-1})+n(2^{n-1}+1)(2^{n-1}+3).$$

(ii) By definition, we have

$$GOD_2(B_n)=\sum_{uv\in E(B_n)}\left[(d_d(u)+d_d(v))d_d(u)d_d(v)\right]$$

$$=1[(3+3)(3\times 3)]+2n\left[(3+(2^{n-1}+1))3(2^{n-1}+1)\right]$$

$$+n\left[((2^{n-1}+1)+(2^{n-1}+1))(2^{n-1}+1)(2^{n-1}+1)\right]$$

$$=54+6n(2^{n-1}+4)(2^{n-1}+1)+2n(2^{n-1}+1)^3.$$

By using definitions, we obtain the first and second Gourava domination polynomials of B_n.

Theorem 8. The first and second Gourava domination polynomials of B_n are given by

(i) $GOD_1(B_n,x)=x^{15}+2nx^{(7+4\times 2^{n-1})}+nx^{(2^{n-1}+1)(2^{n-1}+3)}$.

(i) $GOD_2(B_n,x)=x^{54}+2nx^{3(2^{n-1}+4)(2^{n-1}+1)}+nx^{2(2^{n-1}+1)^3}$.

VI. CONCLUSION

In this study, we have defined the first and second Gourava domination indices and their corresponding polynomials of a graph. Also the first and second Gourava domination indices and their corresponding polynomials of some standard graphs, windmill graphs, book graphs are computed.

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